Introduction: As mobile communication systems become more advanced, their design and optimisation become increasingly affected by the spatial parameters of the wireless channel. This Letter presents a novel definition of multipath angular spread, based on the distribution of incoming multipath power about the horizon. This definition is then related to the small-scale fading characteristics that depend on the spatial characteristics of radio waves and microwave propagation. A novel relationship between the azimuthal distribution of multipath power and narrowband small-scale fading characteristics is developed. The fundamental relationship is useful for studying adaptive arrays, smart antennas, equalisation, diversity, and any other wireless technology or concept that depends on the spatial characteristics of radio waves and microwave propagation.

Definition of angular spread: For typical terrestrial propagation, radio waves arrive at the receiver from a number of azimuthal directions about the horizon [1]. This distribution of multipath power is conveniently described by the function $p(\theta)$, where $\theta$ is the azimuthal angle. In the limit of very small $\theta$, the term $p(\theta)$ represents the power of a single multipath plane wave received from the $\theta$ direction by the receiver [1].

We propose a new method for quantifying the angular spread of multipath power which is based on the Fourier coefficients of $p(\theta)$. Let the value for angular spread, $\Lambda$, be defined by the following:

$$\Lambda = \sqrt{1 - \left| \frac{F_0}{F_0} \right|^2}$$

where $F_0$ is the $n$th complex Fourier coefficient of $p(\theta)$. The several advantages to defining angular spread in this manner. First, since angular spread is normalised by $|F_0|$ (the total amount of received power), it is invariant under changes in transmitted power. Second, $\Lambda$ is invariant under any series of rotational or linear transformations of $p(\theta)$. Finally, this definition is intuitive: angular spread ranges from 0 to 1, with 0 denoting the extreme case of a single multipath component from a single direction and 1 denoting no clear bias in the angular distribution of received power.

Basic relationship between multipath angular spread and narrowband fading in wireless channels

G. Durgin and T.S. Rappaport

A novel relationship between the azimuthal distribution of multipath power and narrowband small-scale fading characteristics is developed. The fundamental relationship is useful for studying adaptive arrays, smart antennas, equalisation, diversity, and any other wireless technology or concept that depends on the spatial characteristics of radio waves and microwave propagation.
where fading rate variance becomes

\[ \sigma^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i P_j E \left( V_i V_j \cos ( \Phi_i - \Phi_j + \tau t ) ( \delta_i - \delta_j ) \right)^2 \]  \hspace{1cm} (6)

Eqn. 6 is normalised power in \( V_i \). We define the variance of the fading rate to be

\[ \sigma^2 = E \left( \left( \frac{dP_i(t)}{dt} \right)^2 \right) - \left( E \left( \frac{dP_i(t)}{dt} \right) \right)^2 \]  \hspace{1cm} (7)

On inspection, the right term on the right-hand side of eqn. 7 is zero for the power representation in eqn. 6. The ensemble average of the remaining term is taken over the \( N \) random phases, \( \Phi_i \). The fading rate variance becomes

\[ \sigma^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i P_j E \left( \delta_i - \delta_j \right)^2 \]  \hspace{1cm} (8)

where \( P_i = V_i^2 \) and \( P_j = V_j^2; P_i \) is the power of the \( i \)th individual multipath wave received in the absence of other multipath.

To evaluate eqn. 8 further, the velocity vector needs to be described in greater detail. For convenience, the velocity will be described as having a constant magnitude \( v \), and a random direction, uniformly distributed about the azimuth. The physical interpretation of this scenario is appealing: \( \sigma^2 \) represents the mean-square fading rate a receiver will experience in a local area, without presuming an exact direction of travel. Furthermore, this definition of \( \sigma^2 \) is mathematically equivalent to the average rate of squared power change as measured along any two orthogonal tracks in the local area, assuming the same mobile receiver speed. Fig. 2 illustrates this interpretation.

![Figure 2](image-url)

**Fig. 2** Estimate of \( \sigma^2 \) obtained by averaging fading rate variance measured along two orthogonal directions as functions of space or time (with identical velocities).

Under these conditions, \( \sigma^2 = \frac{1}{2} (\sigma_\theta^2 + \sigma_\phi^2) \) regardless of azimuthal orientation of measurement.

After evaluating the final ensemble average, the fading rate variance becomes

\[ \sigma^2 = 2k^2 v^2 \sum_{i=1}^{N} \sum_{j=1}^{N} P_i P_j \sin^2 \left( \frac{\theta_i - \theta_j}{2} \right) \]  \hspace{1cm} (9)

where \( \theta_i \) and \( \theta_j \) are the arrival angles of the \( i \)th and \( j \)th multipath wave, respectively. An even more convenient form for eqn. 9 may be obtained by using the angular distribution of power for a set of propagating plane waves:

\[ p(\theta) = \sum_{i=1}^{N} P_i \delta(\theta - \theta_i) \]  \hspace{1cm} (10)

where \( \delta(\theta) \) is the unit impulse function. Eqn. 9 may be recast in terms of angular spread \( \Lambda \) (see the appendix for more detail):

\[ \sigma = k v \Lambda P \]  \hspace{1cm} (11)

where \( P \) is the average local area power seen by the mobile. Although the relationship was derived from discrete multipath, eqn. 11 is equally valid for continuous, diffuse power distributions.

Eqn. 11 is a remarkable result; it states that as multipath power becomes more spread out on the horizon, the average fading rate in a local area will increase exactly proportional to the angular spread. This type of fading behaviour is expected [2], although a simple relationship such as eqn. 11 has not previously existed to describe the phenomenon.

**Conclusions:** This Letter has presented a fundamental relationship between the angular distribution of power in a multipath channel and the fading in a local area. The simple relationship is exact for small-scale fading, regardless of the complexity of the angular power distribution. Eqn. 11 has many implications for analysis and measurement in wireless communications. For example, propagation measurements that obtain angle-of-arrival information can yield immediate insight into local area fading characteristics. Conversely, by studying the fading rate of power measurements made along orthogonal tracks, it is possible to infer basic angle-of-arrival channel characteristics.

**Appendix:** Eqn. 9 may be rewritten as follows:

\[ \sigma^2 = 2k^2 v^2 \sum_{i=1}^{N} \sum_{j=1}^{N} P_i P_j \int_{0}^{2\pi} \sin^2 \left( \theta_i - \theta_j \right) \]  \hspace{1cm} (12)

which may be regrouped, using eqn. 10 and the identity \( \sin(x/2) = 1/2 - 1/2 \cos(x) \):

\[ \sigma^2 = k^2 v^2 \int_{0}^{2\pi} \int_{0}^{2\pi} p(\theta_1)p(\theta_2) \sin(\theta_1 - \theta_2) d\theta_1 d\theta_2 \]  \hspace{1cm} (13)

Using the identity \( \cos(\theta_1 - \theta_2) = \cos(\theta_1) \cos(\theta_2) + \sin(\theta_1) \sin(\theta_2) \) and rearranging terms leads to

\[ \sigma^2 = k^2 v^2 (|F_0|^2 - |F_0|^2) \]  \hspace{1cm} (14)

where \( F_0 \) and \( F_1 \) are the zeroth and first complex Fourier coefficients of the angular distribution of power, \( p(\theta) \). Note that the average local area power \( P_L \) is equal to \( |F_0|^2 \). Eqn. 11 is the final result in terms of angular spread \( \Lambda \).

**Acknowledgments:** This work is sponsored by a Bradley Fellowship in Electrical Engineering and the MPRG Industrial Affiliates Program.

© IEE 1998

Electronics Letters Online No: 19981689

G. Durgin and T.S. Rappaport (Mobile and Portable Radio Research Group, Bradley Department of Electrical Engineering, Virginia Polytechnic Institute and State University, 432 NEB, Blacksburg, VA 24061-0530, USA)

References


---

**Conclusions:** This Letter has presented a fundamental relationship between the angular distribution of power in a multipath channel and the fading in a local area. The simple relationship is exact for small-scale fading, regardless of the complexity of the angular power distribution. Eqn. 11 has many implications for analysis and measurement in wireless communications. For example, propagation measurements that obtain angle-of-arrival information can yield immediate insight into local area fading characteristics. Conversely, by studying the fading rate of power measurements made along orthogonal tracks, it is possible to infer basic angle-of-arrival channel characteristics.

**Appendix:** Eqn. 9 may be rewritten as follows:

\[ \sigma^2 = 2k^2 v^2 \sum_{i=1}^{N} \sum_{j=1}^{N} P_i P_j \int_{0}^{2\pi} \sin^2 \left( \theta_i - \theta_j \right) \]  \hspace{1cm} (12)

which may be regrouped, using eqn. 10 and the identity \( \sin(x/2) = 1/2 - 1/2 \cos(x) \):

\[ \sigma^2 = k^2 v^2 \int_{0}^{2\pi} \int_{0}^{2\pi} p(\theta_1)p(\theta_2) \sin(\theta_1 - \theta_2) d\theta_1 d\theta_2 \]  \hspace{1cm} (13)

Using the identity \( \cos(\theta_1 - \theta_2) = \cos(\theta_1) \cos(\theta_2) + \sin(\theta_1) \sin(\theta_2) \) and rearranging terms leads to

\[ \sigma^2 = k^2 v^2 (|F_0|^2 - |F_0|^2) \]  \hspace{1cm} (14)

where \( F_0 \) and \( F_1 \) are the zeroth and first complex Fourier coefficients of the angular distribution of power, \( p(\theta) \). Note that the average local area power \( P_L \) is equal to \( |F_0|^2 \). Eqn. 11 is the final result in terms of angular spread \( \Lambda \).

**Acknowledgments:** This work is sponsored by a Bradley Fellowship in Electrical Engineering and the MPRG Industrial Affiliates Program.

© IEE 1998

Electronics Letters Online No: 19981689

G. Durgin and T.S. Rappaport (Mobile and Portable Radio Research Group, Bradley Department of Electrical Engineering, Virginia Polytechnic Institute and State University, 432 NEB, Blacksburg, VA 24061-0530, USA)

References