B. Triangular Taper

This excitation is given by

\[ w(x) = \begin{cases} 
(2/L)x + 1, & \text{if } x > 0 \\
(2/L)x + 1, & \text{if } x < 0 \\
1, & \text{if } x = 0.
\end{cases} \]  

The exact radiated fields, for \( N = 11 \) elements, are plotted in Fig. 2.

Using (10) the following expression is obtained:

\[ f_{\text{tri}}(\theta) = \frac{1}{2(N-1)} \cos \left( \frac{\pi (N-1)}{2} \sin \theta \right) - 1 \int x \cot \frac{1}{2} \sin \theta \]  

which is a closed form for the field due to (14) whose plot is identical to Fig. 2.

In this case the complexity is not much affected because the critical point is at the origin with zero phase (i.e., real) and does not include any additional multiplication.

V. CONCLUSION

In this communication, the expansion of the Fourier sum has been introduced. This expansion has been applied to linear phase antenna arrays in order to compute their radiation patterns. Although the expansion is an infinite series, a few terms (in some cases only one term) are enough for computing the field due to many classes of excitation functions.

Since a small number of terms suffice, the expansion may be considered as a fast algorithm for computing the field levels and it plays, in radiation pattern computation, the same role as the fast Fourier transform in signal processing.

REFERENCES


A Single-Hop \( F_2 \) Propagation Model for Frequencies Above 30 MHz and Path Distances Greater than 4000 km

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Abstract—Standard textbook explanations of ionosphere propagation suggest that 30 MHz and 4000 km are limiting frequencies and distances for single-hop communications. During the peak of sunspot cycle 22, however, there have been hundreds of reports of \( F_2 \) communications exceeding great circle distances of 4200 km at 50 MHz. We show that parabolic \( F_2 \) ionospheric models explain this propagation irregularity. Textbooks and reference books fail to show that such long distances at VHF are possible. As low earth orbit satellite and meteor burst systems proliferate, models which account for sunspot peaks will be needed.

I. INTRODUCTION

Long-distance ionospheric propagation models at frequencies above 30 MHz are not described in the literature, but promise to become increasingly important for interference and frequency allocation studies with the proliferation of commercial VHF meteor burst and low earth orbit (LEO) satellite systems. Trial systems are already using spectrum between 30 and 70 MHz. Well-known references on HF and VHF propagation would lead one to believe that such future systems would not be subject to long haul propagation such as is experienced on HF channels. However, recent reports by amateurs indicate the viability of \( F_2 \) propagation over distances of 4200 to over 6500 km using modest equipment [1].

During 1989, the one-year smoothed sunspot number approached 200, and provided 10-cm solar flux levels greater than 160 for the first time in 20 years. For the first time in many years, sunspot cycles have been near maximum, and the result is an increase in \( F_2 \) ionospheric activity. By the turn of the decade many countries are beginning to provide \( F_2 \) communication services to the public, and a number of foreign countries are also launching satellite and meteor burst systems which will demand a better understanding of \( F_2 \) propagation at VHF.

Textbooks and reference books fail to show that such long distances at VHF are possible. As low earth orbit satellite and meteor burst systems proliferate, models which account for sunspot peaks will be needed.

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periods of several days at a time. A indices were below 15. During such conditions, midwestern U.S. radio amateurs reported continuous reception of 48 MHz British television aural signals, and hundreds of low-power amateur stations throughout the world reported contacts ranging from 4200 to over 6500 km great circle distance [1]. Southwestern U.S. stations reported 50 MHz contacts into Australia/New Zealand. These transmissions covered distances in excess of 10 000 km and would be explained by a two-hop model, each hop covering a distance on the order of 5000 km.

II. PHYSICAL IONOSPHERE MODEL

To account for the unusually large distances at VHF frequencies, we investigate the necessary free electron density and shape of the electron density profile needed to support such long distance communications. The electron density in the \( F_2 \) layer is modeled using the well-known parabolic model [2, 3]

\[
N(h) = N_0 \left[ 1 - \left( \frac{h - h_0}{2H} \right)^2 \right], \quad h_0 - 2H < h < h_0 + 2H
\]

where \( h_0 - 2H \) is the physical height of the lower boundary of the \( F_2 \) layer, \( h_0 \) is the physical height in km at which the maximum free electron density \( N_0 \) occurs, and 2 \( H \) is the parabolic semithickness. In [3], \( h_0 \) is the height of the \( F_2 \) lower boundary, the semithickness is \( \gamma_m \), and \( h_0 = h_m - \gamma_m \). The poor fit of the parabolic model at heights more than 2 \( H \) below the height of peak electron density is of little consequence since it can be shown that the refractive index is very close to unity for the frequencies of interest. To study \( F_2 \) propagation reported in [1], we assume three model electron density profiles having the form of (1) with \( N_0 = 3(10)^{12} \) with the parameters: 1) \( h_0 = 325 \text{ km}, H = 75 \text{ km}; \) 2) \( h_0 = 350 \text{ km}, H = 100 \text{ km}; \) and 3) \( h_0 = 375 \text{ km}, H = 125 \text{ km}. These are realistic profiles based on measurements during previous sunspot maxima [2, 3]. Using an exact analysis of standard single-hop path geometry, we have computed the great circle distances as a function of elevation angle of transmitter and receiver antenna. We find under such conditions, exact analysis explains the observed propagation whereas commonly used geometrical approximations do not.

The refractive index is unity at real heights less than the lowest height of the parabolic \( F_2 \) model. As in [4], let \( i \) denote the angle a propagating wave makes with the ionosphere at some height \( x \) above earth with respect to a line perpendicular to the earth surface, and let \( r \) denote the angle at height \( x \) of the refracted wave as it propagates into a higher layer of the ionosphere (\( \rho \) is the angle made at the lowest \( F_2 \) layer boundary). Using the law of sines and assuming an \( F_2 \) layer that is comprised of many thin curved slabs of ionosphere, each a few kilometers thick and with discrete and decreasing refractive indices for increasing height, it can easily be shown that if \( n_x \) is the refractive index at height \( x \) and \( R_0 \) is the distance from the lowest \( F_2 \) layer to the earth's center, then

\[
n_x R_x = n_x (6370 + h) = K
\]

and

\[
K = n_x R_0 \sin r_0 = (6370 + h_0 - 2H) \sin (90^\circ - \gamma) = (6370 + h_0 - 2H) \cos (\gamma)
\]

must hold since \( n_x \) must equal one for internal reflection within the ionosphere. In (2), \( h \) ranges between \( h_0 - 2H \) and \( h_0 \) for a parabolic \( N(h) \) profile, \( n_x \) is less than or equal to one as dictated by \( N(h) \), and \( K \) is a function of the antenna launch angle \( \gamma \), in degrees. For specific \( N(h) \), the existence of single-hop propagation paths at some junction frequency \( f \) and launch angle \( \gamma \) is given as solutions to (3).

\[
(6370 + h_0 - 2H) \cos (\gamma) = (6370 + h) \sqrt{\sqrt{1 - \frac{81N_0}{1 - \frac{h - h_0}{2H}}} \frac{2}{f^2}}. \tag{3}
\]

Fig. 1 shows curves of great circle path distance \( D_r \) versus launch angle \( \gamma \) as a function of operating frequency, based on numerical solutions to (3). See in Fig. 1 that single-hop distances (\( D_r \)) of 4200-7000 km over \( F_2 \) paths are derived from ionosphere models that are comparable to those measured during past sunspot maxima. Thus, these models seem to explain the great distances reported. In [3], an approximate (and commonly used) expression for \( D_r \) given by

\[
D_r = 8R(h_0 - 2H), \quad R = 6370 \text{ km} \tag{4}
\]

was derived by neglecting certain first- and second-order terms. The approximate solution (4) was shown to be within 2.5% of the exact solution up to \( i_0 = 73^\circ \). However, with very low launch angles and the models presented here, \( i_0 \) can be as great as 78°, in which case (4) predicts maximum great circle path distances of only between 2700 and 3000 km, at frequencies no greater than about 30 MHz for the model profiles. Note Fig. 1 indicates that the exact solutions to (3) correctly indicate that during typical "good" conditions \( (N_0 = 2.5 \times 10^{12}, h_0 = 400 \text{ km}, H = 75 \text{ km}) \), propagation is not possible above 40 MHz or over distances greater than about 4,200 km. (4) predicts a maximum path distance of 3570 km. Under more typical conditions, such as are generally given in textbooks and reference books, (3) and (4) give identical solutions.

REFERENCES