Analytical Results for Capacity Improvements in CDMA

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Abstract—In this paper, we examine the performance enhancements that can be achieved by employing spatial filtering in code division multiple access (CDMA) cellular radio systems. The goal is to estimate what improvements are possible using narrow-beam adaptive antenna techniques, assuming that adaptive algorithms and the associated hardware to implement these systems can be realized. Simulations and analytical results are presented which demonstrate that steerable directional antennas at the base station can dramatically improve the reverse channel performance of multicell mobile radio systems, and new analytical techniques for characterizing mobile radio systems which employ frequency reuse are described using the wedge-cell geometry of [1]. We also discuss the effects of using directional antennas at the portable unit. Throughout this paper we will use phased arrays and steerable, fixed pattern antennas to approximate the performance of adaptive antennas in multipath-free environments.

I. INTRODUCTION

CURRENT day mobile radio systems are becoming congested due to growing competition for spectrum. Many different approaches have been proposed to maximize data throughput while minimizing spectrum requirements for future wireless personal communications services [2], [3]. One way to increase capacity without added spectrum is to reduce cell sizes [4]. For this reason, cell sizes in emerging cellular communication systems are much smaller than cells used in land mobile cellular systems designed previously. This, however, also leads to increased infrastructure (base station) costs. Furthermore, to maximize capacity in CDMA systems, power control is required [5].

The reverse link (the link from the mobile unit to the base station) presents the most difficulty in CDMA cellular systems for several reasons. First of all, the base station has complete control over the relative power of all of the transmitted signals on the forward link; however, because of different radio propagation paths between each user and the base station, the transmitted power from each portable unit must be dynamically controlled to prevent any single user from driving the interference level too high for all other users [1]. Second, transmit power is limited by battery consumption at the portable unit, therefore there are limits on the degree to which power may be controlled. Finally, to maximize performance, all users on the forward link may be synchronized much more easily than users on the reverse link [6].

Adaptive antennas at the base station and possibly at the portable unit may mitigate these problems. In the limiting case of infinitesimal beamwidth and infinitely fast tracking ability, adaptive antennas can provide for each user a unique channel that is free from interference. All users within the system would be able to communicate at the same time using the same frequency channel, in effect providing space division multiple access (SDMA) [7]. In addition, a perfect adaptive antenna system would be able to track individual multipath components and combine them in an optimal manner to collect all of the available signal energy [8]. In this paper, we will investigate the effects of spatial filtering by simulating a phased array and by simulating antenna patterns with fixed patterns but adjustable boresight angles. Furthermore, multipath is not considered.

Clearly, the perfect adaptive antenna system described above is not feasible since it requires infinitely large antennas (or alternatively, infinitely high frequencies). This raises the question of what gains might be achieved using reasonably sized antenna arrays which operate at UHF and microwave frequencies.

While both TDMA and CDMA systems have been proposed for emerging personal communication systems, CDMA is more naturally suited to the pseudo-SDMA environment. This is because co-channel users do not have to be synchronized with each other in a CDMA system. As the advantages of SDMA are realized, the interference levels seen by each simultaneous CDMA user drop, and the bit-error performance will improve for each CDMA user. On the other hand, when no SDMA is achieved, CDMA performance is no worse than the case where omnidirectional antennas are used at both the base station and the portable unit. In a single cell TDMA system, users must be reassigned to new time slots to take any advantage of SDMA.

For interference limited asynchronous reverse channel CDMA over an additive white Gaussian noise (AWGN) channel, operating with perfect power control with no interference from adjacent cells and with omnidirectional
where $K$ is the number of users in a cell and $N$ is the spreading factor. $Q(Y)$ in (1.1) is the standard $Q$-function, the probability that $Y > Y$ when $Y$ is a zero-mean, unit variance, Gaussian distributed random variable. Equation (1.1) assumes that the signature sequences are random and that $K$ is sufficiently large to allow the Gaussian approximation described in [6] to be applied.

To illustrate how directive antennas can improve the reverse link in a single cell CDMA system, consider the case in which each portable unit has an omnidirectional antenna, and the base station tracks each user in the cell using a directive beam. Assume that a beam pattern, $G(\phi)$, is formed such that the pattern has a maximum in the direction of the desired user.

Such a directive pattern can be formed at the base station using an $N$-element adaptive array illustrated in Fig. 1 [14], [16]. The array has $N$ elements, each of which has $K$ adaptive linear filters (ALF's) associated with it, if there are $K$ users in the cell. Each ALF operates on the $I$ and $Q$ components of the signal from a single antenna array element. The resulting $I$ components from all of the ALF's are summed and the $Q$ components are summed to form the signal at the array port. Each of the ALF's may be adapted using a variety of techniques such as the use of training sequences, decision directed adaptation, and property restoral algorithms [9], [7], [10]. In the case of a narrowband array, each ALF simply takes the form of a complex tap weight. For wideband arrays, each ALF may take the form of a linear transversal filter or lattice filter [9].

When the ALF's are implemented digitally, it is possible to use a different set of ALF filter coefficients for each desired user, giving each desired user a distinct beam pattern. Each element of the array would have $K$ ALF's associated with it, for a total of $NK$ ALF's. The ALF coefficients for each of the $K$ sets of $N$ ALF's are adapted independently for each desired user.

Assume that a beam pattern, $G(\phi)$, with no variation in the $\theta$ direction, such as that illustrated in Fig. 2, can be formed by the array. The pattern, $G(\phi)$, can be steered through 360° in the horizontal ($\phi$) plane such that the desired user (user 0) is always in the main beam of the pattern.

We assume that $K$ users in the single cell CDMA system are uniformly distributed throughout a two-dimensional cell (in the horizontal plane, $\theta = \pi/2$). On the reverse link, the power received from the desired mobile signal is $P_{r,0}$. The powers of the signals incident at the base station antenna from the $K - 1$ interfering users are given by $P_{r,i}$ for $i = 1 \cdot \cdot \cdot K - 1$. Then the average total interference power, $I$, seen by a single desired user, measured in the received signal at the array port (as shown in Fig. 1) of the base station antenna array, which is steered to user 0, is given by

$$I = E \left\{ \sum_{i=1}^{K-1} G(\phi_i)P_{r,i} \right\} \text{ (1.2)}$$

where $\phi_i$ is the direction of the $i$th user in the horizontal plane, measured from the $x$-axis. No interference from outside the cell contributes to total received interference in (1.2). If perfect power control is applied such that the power incident at the base station antenna from each user is the same, then $P_{r,i} = P_i$ for each of the $K$ users, and the average interference power seen by user 0 is given by

$$I = P_i E \left\{ \sum_{i=1}^{K-1} G(\phi_i) \right\} \text{ (1.3)}$$

Assuming that users are independently and identically distributed throughout the cell, the average total interference power received at the central base station may be

$^1$While this work considers adaptive antennas at the base station, power control could be implemented using a reference omnidirectional antenna at the base station to receive all mobile signals.
expressed as

\[ I = P_c (K - 1) \int_0^{2\pi} f(r, \phi) G(\phi) \, d\phi \, dr \tag{1.4} \]

where \( f(r, \phi) \) is the probability density function describing the geographic distribution of users throughout the cell. Assuming that users are uniformly distributed in the cell, we have

\[ I = P_c (K - 1) \int_0^{2\pi} G(\phi) \, d\phi. \tag{1.5} \]

The directivity of an antenna which has no variation in the \( \theta \) direction is [11]

\[ D = \frac{2\pi}{\int_0^{2\pi} G(\phi) \, d\phi}. \tag{1.6} \]

Therefore the average total interference seen by a user in the central cell is given by

\[ I = P_c (K - 1) \cdot D. \tag{1.7} \]

In order to develop simple bit error rate expressions for simultaneous asynchronous interference limited CDMA users when directive antennas are used, we assume that the bit-error-rate expression of (1.1) can be expressed as

\[ P_b = Q(\sqrt{3N \times CIR}) \tag{1.8} \]

where \( N \) is the spreading factor, and \( CIR \) is the ratio of the power of the desired signal to the total interference. In (1.8), it is assumed that \( M \) interfering users, each with a received power level of \( P/M \), have the same effect on bit error performance as one interfering user with a received power \( P \). This assumption is known to be inaccurate when the powers of users are widely different and when the number of users is small [12]; however, it provides first order approximation for the case of a large number of users.

Using the fact that the power of the desired signal, weighted by the array pattern, is \( P_c \), and using (1.7), the bit error rate for user 0 is given by

\[ P_b = Q(\sqrt{3DN} / \sqrt{K - 1}). \tag{1.9} \]

Thus, (1.9) holds for any single cell system with perfect power control when base station antenna pattern which has no variation in the \( \theta \) direction. Equation (1.9) is useful in showing that the probability of error for a CDMA system is related to the beam pattern of a receiver. If we use the idealized antenna pattern illustrated in Fig. 2 to approximate a realizable directive antenna pattern then it is immediately apparent that the gain of the antenna directly contributes to the performance of a CDMA system. For instance, if \( K = 250 \) and \( N = 511 \), with omnidirectional antennas at the base station, an average bit error rate of \( 6.6 \times 10^{-3} \) is obtained per user. Using the flat-top beam pattern shown in Fig. 2 with a side lobe level of 0.25 and a main beamwidth of \( 60^\circ \), the directivity of the antenna is 2.67 or 4.3 dB. The bit error rate with the directive antenna at the base station is \( 2.5 \times 10^{-3} \), a BER improvement of two orders of magnitude.

This example illustrates the possible improvements that can be achieved using adaptive antennas at the base station. In the remainder of this paper, we remove the constraint that users in adjacent cells do not interfere with the received signal, and develop a general analysis technique which is confirmed by simulation.

Section II describes analytical techniques used to determine bit error rates in cellular CDMA systems employing adaptive antennas.

Section III presents simulations in which we compare the performance of five base station antenna configurations, three of which use adaptively steerable antennas at the base station. It is assumed that the portable units use omnidirectional antennas. We also compare the simulation results with the analytical results developed in Section II.

In Section IV, the effects of adaptive antennas at the portable unit are examined using several different base station configurations. Furthermore, we demonstrate the two distinctly different effects achieved by using directive antennas at the portable unit versus using directive antennas at the base station. Finally, Section V summarizes the results of this paper.

II. REVERSE CHANNEL PERFORMANCE WITH ADAPTIVE ANTENNAS AT THE BASE STATION

The use of adaptive antennas at the base station receiver is a logical first step in improving capacity for several reasons. First of all, space and power constraints are not nearly as critical at the base station as they are at the portable unit. Second, the physical size of the array does not pose as much difficulty at the base station as at the portable unit.

Note that adaptive antennas may also be used at the base station for directing energy in the forward channel, in which case the analysis is similar to the reverse channel case because of the perfect power control assumption. The only difference on the forward link is that interferers are other base stations, rather than portable users. Since the transmitter and receiver typically operate in two different frequency bands in a duplex manner, the adaptive antennas at the base station transmitter would be adjusted by performing a transformation on the tap weights adapted for the receiver, and copying the new weights to the transmitting antennas [9]. This is reasonable if an assumption of retrodirectivity on similar frequency bands is appropriate. If the multipath components arriving in the reverse channel do not have the same angles of arrival as those in the forward channel, then it is no longer appropriate to derive the transmitter tap weights from the received signal.

Equation (1.9) is only valid when a single cell is con-
sidered. To consider the effects of adaptive antennas when CDMA users are simultaneously active in several adjacent cells, we must first define the geometry of the cell region. For simplicity, we consider the geometry proposed in [1] with a single layer of surrounding cells, as illustrated in Figs. 3 and 4.

Let \( d_{i,j} \) represent the distance from the \( i \)th user to base \( j \) as illustrated in Fig. 3. Let \( d_{i,0} \) represent the distance from the \( i \)th user to base station 0, the center base station.

Assume that path loss in dB between user \( i \) and base \( j \) is given by a simple distance dependent path loss relationship such that the power received at base station \( j \), from the transmitter of user \( i \), \( P_{r,i,j} \), is given by

\[
P_{r,i,j} = P_{r,i} \left( \frac{\lambda}{4\pi d_{i,0}} \right)^2 \left( \frac{d_{i,j}}{d_{i,0}} \right)^n \quad (2.1)
\]

where \( n \) is the path loss exponent typically ranging between 2 and 4, and \( d_{i,0} \) is a close-in reference distance [1].

If we assume that perfect power control is applied to the \( i \)th user, and all other users in cell \( j \), by base \( j \), such that power \( P_{c,i} \) is received as base \( j \), then the power transmitted by user \( i \), \( P_{t,i} \), is given by

\[
P_{t,i} = P_{r,i} \left( \frac{\lambda}{4\pi d_{i,0}} \right)^2 \left( \frac{d_{i,j}}{d_{i,0}} \right)^n \quad (2.2)
\]

The power received at base station 0 from user \( i \), \( P_{r,i,0} \) is given by

\[
P_{r,i,0} = P_{r,i} \left( \frac{\lambda}{4\pi d_{i,0}} \right)^2 \left( \frac{d_{i,j}}{d_{i,0}} \right)^n \quad (2.3)
\]

Substituting (2.2) into (2.3), the power received at base 0 from user \( i \), in adjacent cell, \( j \), is given by

\[
P_{r,i,0} = P_{r,i} \left( \frac{d_{i,j}}{d_{i,0}} \right)^n \quad (2.4)
\]

To analyze (2.4), we consider the geometry shown in Fig. 4.

From the law of cosines,

\[
d_{i,j}^2 = (2R)^2 + (d_{i,0})^2 - 2(2R)d_{i,0} \cos \varphi_{i,0} \quad (2.5)
\]

Substituting (2.5) into (2.4), the power received at base 0 from user \( i \) is given by

\[
P_{r,i,0} = P_{r,i} \left( 1 + \frac{(2R)^2}{d_{i,0}^2} - \frac{4R}{d_{i,0}} \cos \varphi_{i,0} \right)^{n/2} \quad (2.6)
\]

To determine the average out-of-cell interference power incident on the central base station, we assume that users are uniformly distributed in a typical adjacent cell from \( r = R \) to \( r = 3R \) and from \( \varphi = -\pi/8 \) to \( \pi/8 \). Thus, we use a modified geometry from [1] where eight equal area cells surround the center cell. The probability density function (pdf) for the spatial distribution of users in a single adjacent cell is given by

\[
f(r, \varphi) = \frac{r}{\pi R^2} \quad R < r < 3R; \quad -\pi/8 < \varphi < \pi/8.
\]

(2.7)

Let \( \chi \) represent the expected value of the interference power from a single user in one of the adjacent cells when omnidirectional basestation antennas are used.

\[
\chi = P_{c,i} \int_R^{3R} \int_{-\pi/8}^{\pi/8} f(r, \varphi) \left( 1 + \frac{(2R)^2}{r^2} - \frac{4R}{r} \cos \varphi \right)^{n/2} r \, dr \, d\varphi \quad (2.8)
\]

If it is assumed that all nine base stations control power such that \( P_{c,i} = P_c \), then given a value of \( n \), we can express the expected value of central cell interference power for a single adjacent cell user as

\[
\chi = \beta P_c \quad (2.9)
\]
where

\[
\beta = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(r, \varphi) \left(1 + \left(\frac{2R}{r}\right)^2 - \frac{4R}{r} \cos(\varphi)\right)^{n/2} dr d\varphi
\]

(2.10)

Table I lists the values of \( \beta \) for several values of \( n \).

When omnidirectional antennas are used at both the base station and the portable unit, \( \beta \) is related to the reuse factor, \( f \), which is defined in [1], for a single layer of adjacent cells, as

\[
f = \frac{N_0}{N_0 + N_nM_1}
\]

(2.11)

where \( N_0 \) is the total interference, seen by a desired user in the central cell, at the central base station on the reverse link, \( N_n \) is the total interference seen by the desired central cell user from all users in a single adjacent cell, \( M_1 \) is the number of cells which are immediately adjacent to the central cell, which is always eight for the geometry considered in this paper.

This reuse factor is a measure of the impact of users in adjacent cells on the performance of the link between a user in the central cell and the central base station.

When power control is performed as described in this section, such that the power received from each mobile unit in the base station controlling that unit is \( P_c \), then (2.11) may be expressed as

\[
f = \frac{(K - 1)P_c}{(K - 1)P_c + 8K\beta P_c} = \frac{1}{1 + 8\beta}
\]

for \( K \gg 1 \)

(2.12)

where we have assumed that there are \( K \) users in each of the nine cells. For \( n = 4 \), from Table I, \( \beta = 0.05513 \), and, from (2.12), \( f = 0.693 \), implying that 31% of the interference power received at the central base station is due to users in adjacent cells. Note that, when omnidirectional antennas are used at both the base station and the portable unit, the value of the reuse factor, \( f \), is determined by the cell geometry, the power control scheme, and the path loss exponent.

When omnidirectional antennas are used at both the base station and the portable unit, the total interference seen on the reverse link by the central base station is the sum of the interference from users within the central cell, \( (K - 1)P_c \), and users in adjacent cells, \( 8K\beta P_c \).

\[
I = (K - 1)P_c + 8K\beta P_c.
\]

(2.13)

Let us assume that for the \( m \)th user in the central cell, an antenna beam from the base station with pattern, \( G(\varphi) \), may be formed with maximum gain in the direction of user \( m \). It is assumed that perfect power control is applied such that all base stations controlling reverse link received power to the same level, \( P_c \). The average interference power contributed by a single user in the central cell

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.14962</td>
</tr>
<tr>
<td>3</td>
<td>0.08228</td>
</tr>
<tr>
<td>4</td>
<td>0.05513</td>
</tr>
</tbody>
</table>

is thus given by

\[
E[P_{r,0} | 0 < r < R]
\]

\[
= P_c \int_{0}^{\infty} \int_{0}^{\infty} \frac{r}{2\pi} G(\varphi) dr d\varphi = \frac{P_c}{D}
\]

(2.14)

where \( D \) is the directivity of the beam with pattern \( G(\varphi) \) and the average received power at the base, \( P_{r,0} \), from an interfering user in the central cell is directly a function of the base station directive gain. Then the average interference power at the array port of the antenna array at the base station, as shown in Fig. 1, due to a single user in an adjacent cell is given by

\[
E[P_{r,0} | R < r < 3R]
\]

\[
= \frac{1}{8} \sum_{p=0}^{7} \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \frac{r}{2\pi} G(\varphi + \frac{p\pi}{4}) \frac{r}{\pi R^2} d\varphi
\]

\[
\cdot P_c \left(1 + \left(\frac{2R}{r}\right)^2 - \frac{4R}{r} \cos(\varphi)\right)^{n/2} dr d\varphi
\]

(2.15)

Here a special case is considered. If \( G(\varphi) \) is piecewise constant over the region \((2p - 1)(\pi/8) < \varphi < (2p + 1)(\pi/8)\) for \( p = 0 \cdots 7 \), then the antenna pattern may be expressed as

\[
G(\varphi) = \sum_{p=0}^{7} G_p U(\varphi - \frac{p\pi}{4})
\]

(2.16)

where

\[
U(\varphi) = \begin{cases} 
1 & |\varphi| < \pi/8 \\
0 & |\varphi| \geq \pi/8
\end{cases}
\]

(2.17)

Substituting (2.16) into (2.15), we obtain,

\[
E[P_{r,0} | R < r < 3R]
\]

\[
= P_c \sum_{p=0}^{7} G_p \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \frac{r}{2\pi} R^2 \left(1 + \left(\frac{2R}{r}\right)^2 - \frac{4R}{r} \cos(\varphi)\right)^{n/2} dr d\varphi
\]

(2.18)

The directivity of the antenna pattern described by (2.16) is

\[
D = \frac{8}{\sum_{p=0}^{7} G_p}
\]

(2.19)
Therefore, (2.18) may be rewritten, using (2.10) and (2.19), as

\[ E[P_{R,0} | R < r < 3R] = \frac{P_e \beta}{D}. \] (2.20)

It can be shown that (2.20) remains valid when the beam pattern, \( G(\phi) \), is rotated in the \( \phi \) plane. Therefore (2.20) is appropriate when \( G(\phi) \) is piecewise constant over \( (2p - 1)(\pi/8) < \phi - \phi_d < (2p + 1)(\pi/8) \) for any angle \( \phi_d \) between \( -\pi/8 \) and \( \pi/8 \).

Using (2.20) with (1.7), the total interference power at the array port (in Fig. 1) of the center base station receiver is given by

\[ I = \frac{(K - 1)P_c + 8KP_e \beta}{D}. \] (2.21)

Substituting (2.21) into the (1.8), using the fact that the desired signal power at the array port is \( P_c \), we obtain an average bit-error probability for the CDMA system employing a piecewise constant directive beam:

\[ P_b = Q\left(\frac{3ND}{\sqrt{K(1 + 8\beta)} - 1}\right). \] (2.22)

For \( K >> 1 \), \( P_b \) is approximated by

\[ P_b = Q\left(\frac{3ND}{\sqrt{K(1 + 8\beta)}}\right). \] (2.23)

Equation (2.23) relates the probability of error to the number of users per cell, the directivity of the base station antenna, and the propagation path loss exponent through the value of \( \beta \). It is assumed that perfect power control is applied as described in Section I, with all base stations controlling reverse link received power to the same level, \( P_c \).

III. SIMULATION OF ADAPTIVE ANTENNAS AT THE BASE STATION FOR REVERSE CHANNEL PERFORMANCE

To explore the utility for (2.23) and to verify its accuracy, we considered five base station antenna patterns which are illustrated in Fig. 5. These antenna patterns are assumed to be directed such that maximum gain is in the direction of the desired mobile users. The first-base station antenna pattern is an omnidirectional pattern which models that used in traditional cellular systems. The second configuration, illustrated in Fig. 5(b), used \( 120^\circ \) sectorization at the base station. In our model, the base station used three sectors, one covering the region from 30° to 150°, the second covering the region from 150° to 270°, and the third covering the region from -90° to 30°. The first sector is illustrated in Fig. 5(b) since this sector would be active when the desired user is at an angle of 60°.

The third simulated base station configuration, shown in Fig. 5(c), used a "flat-topped" beam pattern similar to that shown in Fig. 2. The main beam was 30° wide with uniform gain in the main lobe. Side lobes were simulated by assuming a uniform side lobe gain which was 6 dB below the main beam gain. From (1.6), the directivity of this beam is 5.1 dB.

The fourth configuration, which used a simple three element linear array, is illustrated in Fig. 5(d). This is the beam pattern formed by a binomial phased array with elements spaced a half wavelength apart. The axis of the array is in the \( \phi = 0^\circ \) direction. Like all linear arrays, this array exhibits a pattern which is symmetric about the axis of the array (the X-axis, as shown in Fig. 2), therefore a
mirror image of the main beam is also present as illustrated in Fig. 5(d). This array is not capable of adaptively nulling interfering signals; therefore we expect the performance of this array to be poorer than that of a truly adaptive system. On the other hand, we did assume that the array was able to direct the one of the two main beam components in the direction of the desired user. For each desired user, the phase was computed for each element of the array and the new beam pattern was formed at the center cell base station. While the three-dimensional gain of a binomial phased array is constant at 4.3 dB regardless of scan angle, the two-dimensional gain defined by (3.1), which is more appropriate for comparison given our assumption of users in the horizontal plane only, varies between 2.6 and 6.0 dB, depending on scan angle, with the higher gain corresponding to broadside scan angles.

The pattern for the fifth simulated base station configuration, a sectorized adaptive antenna, is shown in Fig. 5(e). Beginning with the sectorizing system whose pattern is illustrated in Fig. 5(b), we added a three element linear phased array to each sector. The linear array for each sector is aligned such that the broadside direction is in the same direction as the center of the sector. This base station configuration actually uses a total of nine elements, however, only three elements are used to track any given user. For example, in Fig. 5(e), the desired user is at an angle of $\phi = 60^\circ$, therefore the first sector (30$^\circ$ to 150$^\circ$) was active. The three-element linear array used for the first sector was used to further reduce beamwidth of the pattern. It was assumed that the backlobe of each of the antenna for each sector was negligible so that only users within the 120$^\circ$ wedge of each sector were illuminated by the beam of the phased array. This accounts for the sharp cutoff at $\phi = 30^\circ$. Due to the elimination of the backlobes, this pattern has a significantly higher gain than the beam pattern shown in Fig. 5(d). At broadside (for a user in the center of a sector), the gain of this pattern is approximately 10.7 dB.

To evaluate the performance of these systems, a simulation was designed using the simple wedge geometry illustrated in Fig. 3. Users were randomly placed throughout the region with an average of $K$ users per cell. Each user was assigned to one of the nine cells based on geographical location. The path loss from each user to the base station assumed to follow (2.1). Perfect power control was applied to each user within its own cell, as described in Section II, such that the incident power from each user at the in-cell base station antenna was a constant.

The carrier to interference ratio was calculated for each user in the central cell and the bit error rate was determined for each in-cell user by assuming that all users were asynchronous and by applying the Gaussian approximation. We define $P_{x,k,i}$ as the component of the received power at the array port (shown in Fig. 1) of the base station antenna array (weighted by the array pattern) at the $k$th base station from the $i$th user associated with cell $j$. The CIR for the $i$th user in the central cell (cell $j = 0$) was calculated from

$$CIR_i = \frac{\sum_{n=0}^{K-1} P_{n,0,i} + \sum_{m=1}^{K-1} \sum_{n=0}^{K-1} P_{n,m,i}}{\sum_{n=0}^{K-1} P_{n,0,i} + \sum_{m=1}^{K-1} \sum_{n=0}^{K-1} P_{n,m,i}}$$

The bit error rate for the $i$th user in cell 0 on the reverse link was determined by first calculating the CIR for the $i$th user from (3.2) then using that value in (1.8), which is restated here:

$$P_{b,i} = Q(\sqrt{3N \times CIR_i})$$

where $N$ is the spreading factor. For each of the simulations performed in this study, a spreading factor of $N = 511$ was used. It was assumed that any portable unit in the nine-cell region (except for the desired user) contributed to the interference level of the desired user in the central cell.

This calculation was carried out for every user in the central cell and the resulting bit error rates were averaged to obtain an average bit error rate for the cell. For instance, if there were 2700 users in the nine cells and 300 users in the central cell, then the bit error rate was determined for the 300 users in the central cell, and 2699 interfering users contributed to each CIR computation. Each base station configuration was simulated for user densities ranging from 25 to 500 users per cell, in steps of 25.

Fig. 6 shows average bit error rates resulting from the simulation for the five previously described antenna patterns for several values of path loss exponent, $n$. The three element linear array, whose pattern is shown in Fig. 6(d), was able to achieve almost an order of magnitude improvement in BER despite the large backlobe. By eliminating the large backlobe, but still retaining significant side lobes, the flat-top pattern, shown in Fig. 6(c), achieves a BER which is better than two orders of magnitude less than the BER when omnidirectional antennas are used at the base station, with fewer than 200 users per cell.

The average bit error rate alone is not a sufficient metric of system performance. Rather, the distribution of BER's over the user population is a second-order measure which provides insight about the performance of a CDMA cellular system. Fig. 5 relates the average BER to the BER which is not exceed by 50, 90, 95, and 99% of the users. Note that for a given bit error rate, two to four times as many users many be supported using directional antennas as for omnidirectional antennas. It is useful to note that these increases in performance were made by applying relatively modest requirements to the base-station adaptive antenna. The flat top antenna was specified to have a 30$^\circ$ beamwidth and a side lobe level that was only 6 dB below the main lobe.

It should be noted that these bit-error-rate improve-
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TABLE II

<table>
<thead>
<tr>
<th>Base Station Pattern</th>
<th>( P_{e,50} )</th>
<th>( P_{e,95} )</th>
<th>( P_{e,99} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Omni</td>
<td>3.0e-2</td>
<td>3.1e-2</td>
<td>3.2e-2</td>
</tr>
<tr>
<td>Sectorized</td>
<td>6.1e-4</td>
<td>5.3e-4</td>
<td>1.0e-3</td>
</tr>
<tr>
<td>Adaptive</td>
<td>2.9e-3</td>
<td>2.6e-3</td>
<td>6.4e-3</td>
</tr>
<tr>
<td>Flat-topped</td>
<td>4.9e-4</td>
<td>3.9e-4</td>
<td>5.2e-4</td>
</tr>
<tr>
<td>Adaptive-Sectorized</td>
<td>1.5e-7</td>
<td>6.5e-8</td>
<td>3.0e-7</td>
</tr>
</tbody>
</table>

The relationship between the average bit error rate, \( P_e \), and \( P_{e,x} \), where \( P_{e,x} \) is defined such that \( x\% \) of the users in the central cell have a bit error rate which is less than \( P_{e,x} \). This is for the case of \( K = 200 \), and a path loss exponent of \( n = 2 \). Note that this is a much wider range of bit error rates for the higher gain antennas. For example, 2 users or 1\% of the user population experienced a BER which was worse than 1.5e-3 when the sectorized antenna pattern was considered.

Fig. 6. BER using adaptive antennas at the base station for (a) \( n = 2 \), (b) \( n = 3 \), and (c) \( n = 4 \). These results were developed through simulation by averaging the BER of every user in the center cell.

Fig. 7. Plots of analytical results using equation 2.23 with two-dimensional directivities of 1.0, 2.67, 3.0, and 3.2 for the omni, adaptive, sectorized and flat-topped patterns, respectively.

Fig. 8. BER for the omni and flat-topped beam systems as a function of \( n \).
expONENT. This is reasonable to expect since, when the CIR is large, the bit error rate is more sensitive to relatively small changes in interference power.

IV. SIMULATION OF ADAPTIVE ANTENNAS AT THE PORTABLE UNIT TO IMPROVE REVERSE CHANNEL PERFORMANCE

In this section, we examine how the reverse channel is affected by using adaptive antennas at a portable transmitter. A flat-topped beam shape, as illustrated in Fig. 2, was used to model an adaptive antenna at the portable transmitter. Since space is extremely limited on the portable unit, the gain achievable by the portable unit antenna will be considerably less than that at the base station. For this study, it was assumed that the portable unit could achieve a beamwidth of 60° with a side lobe level that was 6 dB down from the main beam. This corresponds to an antenna with a directivity of 4.3 dB. The pattern is similar to that shown in Fig. 5(c) except that the beamwidth is wider in this case.

It was assumed that each portable unit was capable of perfectly aligning the boresight of its adaptive antenna with the base station associated with that portable unit. In this manner, portable units could radiate maximum energy to the desired base station, while reducing battery power proportional to the directivity of the portable antenna.

Portable units with adaptive antennas were simulated for each of the five base station patterns described in Section III. As in Section III, average values of $P_b$ were found by averaging the bit error rates of each user in the central cell, subjected to interference from the central cell and all immediately adjacent cells. The resulting bit error rates for these systems are shown in Fig. 9. Note that, comparing Fig. 6 and Fig. 9, the bit error rates for the reverse channel are improved when directive antennas are used at the portable unit. For omnidirectional base stations, the BER is only decreased by a small amount (20% or less) for $K > 200$ when steerable directive antennas are used at the portable unit. However, for highly directive base station antenna patterns such as the adaptive-sectorized pattern, the BER was decreased by an order of magnitude for $K > 300$.

In Fig. 10, we have defined the BER factor as the ratio of the BER with adaptive antennas at all portable units to the BER without adaptive antennas at the portable units. A small BER factor indicates that adding adaptive antennas improved the BER significantly. For example, a BER factor of 0.5 indicates that using an adaptive antenna at the mobile unit resulted in a reduction in BER of 50% compared with the case of omnidirectional antennas at the mobile unit.

As shown in Fig. 10, the adaptive sectorized base station pattern improved greatly by adding adaptive antennas at the portable unit. The resulting BER for this base station configuration when using adaptive antennas at the portable unit was decreased by an order of magnitude compared with the BER when omnidirectional antennas were used at the portable unit. In general, the more directive base station configurations benefitted more from adding adaptive antennas at the portable unit. Using a 60° beamwidth flat-topped pattern with a $-6$ dB side lobe level at the portable unit, the reverse channel BER for omnidirectional base stations was only improved slightly over the case of omnidirectional antennas at the portable. For directive antennas at the base station, the improvements were more dramatic, as illustrated in Fig. 10.

The relatively small improvements obtained by using adaptive antennas at the portable unit can be explained by the fact that when omnidirectional antennas are used at the mobile unit, no more than 1-0.455, or 0.545, of the total interference power is due to users in adjacent cells (see Table III where $f = 1/(1 + 80)$). When using adaptive antennas at the mobile unit, all users in the central cell will appear no different to the central base station than if they had used omnidirectional antennas. Thus, adaptive
antennas at the portable unit will only reduce out-of-cell interference levels. Therefore, the maximum improvement in CIR, on the reverse link, that can be achieved by using adaptive antennas rather than omnidirectional antennas at the portable unit is only 3.5 dB.

Table III shows several values of the reuse factor, $f$, defined in (2.12) as the ratio of in-cell interference to total interference, for several base station patterns when omni-directional antennas are used at the portable unit. Similarly, Table IV shows values of $f$ when steerable, directional antennas, with directivities of 4.3 dB, are used at the portable units.

Comparing Tables III and IV, it can be concluded that the use of adaptive antennas at the base station does nothing to improve the reuse factor, $f$; however the use of adaptive antennas at the portable unit does allow $f$ to be improved. When omni-directional antennas are used at the portable unit, $f$ is entirely determined by the cell geometry, the power control scheme, and path loss exponent, $n$, which is a function of propagation and not easily controlled by system designers. Using adaptive antennas at the portable unit, it is possible to tailor $f$ to a desired value which is greater than the reuse factor obtained using omni-directional antennas at the portable unit. Ideally, driving $f$ to unity would allow system design to much less sensitive to the intercell propagation environment, when perfect power control is assumed.

This is an important result for CDMA cellular systems because it indicates that use of adaptive antennas at the portable unit could help to allow greater capacity through more efficient reuse, and for more frequent reuse of signature sequences throughout a large coverage area.

### V. CONCLUSIONS

It was shown in this study that adaptive antennas, with relatively modest bandwidth requirements, and no interference nulling capability, both at the base station and at the portable, can provide large improvements in BER, as compared to omnidirectional systems. Analytical expressions which relate the average BER of a CDMA user to the antenna directivity and propagation environment were derived and used to determine capacity improvements offered by a number of antenna patterns. It was demonstrated in Section III that the linear phased array provided an order of magnitude of improvement over the omnidirectional base station. The low-gain (5.1 dB) flat-top pattern provided almost two orders of magnitude of improvement over the omnidirectional system. In addition, it was shown that up to three orders of magnitude of improvement can be achieved by adding a simple three element linear array to a three-sector base station. In terms of capacity, the results of Section III indicate that using adaptive antennas at the base station can allow the number of users to increase by a factor of 2 to 4, while maintaining an average BER of $10^{-3}$ on the reverse link.

The bit error rate on the reverse channel is further improved by adding adaptive antennas at the portable unit. Using a 4.3 dB gain antenna at the portable, the bit error rate for the directive base station configurations (but not the omnidirectional base station) was at least half of the bit error achieved without directive antennas at the portable unit. For the highly directive adaptive sectorized base station, the improvement was over an order of magnitude for user densities less than 425 users/cell when each user employed an adaptive antenna.

Since the directivity of portable unit adaptive antennas is limited by the size of a handheld device, improvements achieved on the reverse channel at the portable are not as dramatic as gains achieved by adaptive antennas at the base station. In addition, cost issues may limit the application of portable unit adaptive antennas. However, the reduction in reverse channel BER may be critical in extremely high traffic environments. In addition, the portable unit is required to track the only current base station, while adaptive antennas at the base station must track every user in the cell. It should be noted, however, most importantly, Tables III and IV showed the increase in reuse efficiency which portable adaptive antennas provide. By using modest gains at the portable unit, such antennas ameliorate the loss in capacity due to intercell propagation through interference control.

In short, adaptive antennas at the base station can have a major effect on bit-error-rate performance, but cannot impact the reuse factor, $f$. Conversely, it has been shown in this paper that adaptive antennas at the portable unit can provide no more than a 3.5 dB improvement in reverse channel CIR; however, they allow the reuse factor,
f, to be altered. It should be noted, however, that the use of directional antennas at the portable unit can only result in an increase in reuse factor of approximately 1/3. It was assumed throughout this study that the adaptive algorithms and hardware could be designed to meet the specified requirements on beamwidth, side lobe level, and tracking ability. It should be noted that, unlike the arrays discussed in this paper, a properly designed adaptive array can null out interference. Conversely, tracking a large number of users with an adaptive array is nontrivial, and it was assumed that each of the base station arrays described here were able to track all of the portable units without error.

The multipath channel was not considered in detail in this study; however, it will be significant in developing algorithms for successful adaptive antenna steering. Rather than tracking users, the adaptive array in a multipath environment must track the angle of arrival of multipath components in order to distinguish the maximum signal. This problem is currently under investigation. Furthermore, efforts are currently underway to develop bit error rate expressions which are accurate for small numbers of simultaneous CDMA users with non-identical power levels.

REFERENCES


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