Performance Evaluation for Cellular CDMA

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Abstract—In this paper, we consider the performance of a cellular radio direct-sequence code-division multiple access system. The base-to-mobile link is modeled as a flat Rayleigh fading channel, with all signals transmitted from a given base station fading in unison. For the mobile-to-base link, we use a similar model, except that the waveforms from all users are assumed to experience independent fading. Finally, we show the effects of imperfect power control.

I. INTRODUCTION

Because of its well-known ability to both combat multipath and allow multiple users to simultaneously communicate over a channel, spread-spectrum techniques are being considered for use in cellular communication networks [1]-[3]. In this paper, we look at the performance of a code-division multiple-access (CDMA) cellular system when all users employ coherent BPSK modulation and direct-sequence (DS) spreading. Consistent with current CDMA designs, we assume that there are separate frequency bands for the mobile-to-base link and the base-to-mobile link, but otherwise there is no frequency separation. That is, we assume that all base stations in the system reuse one frequency band, and all subscribers reuse a separate band. We also assume that all transmitters, whether in bases or in mobiles, employ omnidirectional antennas. These two assumptions imply that any mobile in the system experiences interference from all base stations, but does not experience interference from the transmissions of other mobiles. Similarly, a given base station experiences interference from all mobiles in the system, but not from other base stations.

In the next section, we consider the base-to-mobile link. Under the assumption of perfect synchronization, we derive the average probability of error of a user which is straddling the boundary between adjacent cells. This particular point is chosen because it represents worst-case conditions in the following sense: A mobile on the boundary of two or more cells receives the same nominal (i.e., average unfaded) power from its own base station as it does from its immediate neighbor base stations, since the distances to the various base stations are the same. Hence, the level of multiple-access interference from those cells is maximum. We assume the propagation path loss is a function of distance, and is the same in all cells.

The channel itself is modeled as undergoing flat Rayleigh fading. That is, we assume the coherent bandwidth of the channel exceeds the spread bandwidth of the signal. This assumption, while typically incorrect for a large degree of spreading (i.e., a channel will appear frequency selective to such a DS waveform and, thus, the power over the spread bandwidth will undergo only slight variation), is made for two reasons: one is analytic simplicity, and the other is the expectation that it will lead to worst-case results, since a frequency-selective channel will not undergo the broad deep fades that the flat channel will.

In Section III, we analyze the mobile-to-base link. The same propagation model as used for the base-to-mobile link is considered, with the following exception: In the base-to-mobile link, since all waveforms from a given base traverse the same path, they all fade in unison. However, for the mobile-to-base links, each mobile signal traverses a radio path different from that of the other mobiles' signals, and thus they all are assumed to fade independently of one another.

In the system model used to generate the results summarized above, perfect power control within each base station was assumed. In Section IV, this condition is relaxed so that the sensitivity of the system to imperfect power control at the mobiles can be assessed. We make no attempt to analyze any specific scheme; rather, we allow signals to arrive at the receiver-of-interest with random power levels, and then determine the resulting degradation in performance.

Finally, Section V presents our conclusions and suggestions for additional research that is needed to allow for a full understanding of this problem.

II. BASE-TO-MOBILE LINK

It is desired to determine the performance of mobile users in a cellular CDMA system which are straddling the boundary between two adjacent cells. The significance of mobiles being at the edge of a cell is very clear; in general, due to propagation path loss, a mobile in the interior of a given cell experiences a power advantage in the reception of the signal transmitted from its own base station relative to signals received from the base stations in...
neighboring cells (which are further away from the mobile). However, when the mobile is at the boundary between two cells, this advantage disappears.

To analyze this problem, consider the following model. The two cells shown in Fig. 1 are used, and a mobile with an omnidirectional antenna is assumed to be located on the boundary of the cells and to be receiving energy from both base stations. The signal from either base is composed of \( K \) DS waveforms, all of which are asynchronous with one another. However, the composite signal from each base station is assumed to independently undergo flat fading with either a Rayleigh or log-normal distribution. That is, because we are considering the base-to-mobile link and all signals that arrive from the base at a given mobile propagate over the same path, we assume they all fade in unison.

With the above model, the received waveform at the mobile is given by

\[
 r(t) = \sum_{i=1}^{K} A\alpha_1 d_i(t - \tau_i)P\!N_i(t - \tau_i) \cos(\omega_0 t + \theta_i) \\
+ \sum_{i=K+1}^{2K} A\alpha_2 d_i(t - \tau_i)P\!N_i(t - \tau_i) \\
\cdot \cos(\omega_0 t + \theta_i) + n_s(t)
\]  

(1)

where \( A \) represents the unfaded amplitude of any of the received signals, \( \alpha_1 \) and \( \alpha_2 \) are independent random variables representing the fading, \( d_i(t) \) is the binary data of the \( i \)th signal, \( P\!N_i(t) \) is the spreading sequence of the \( i \)th signal, and \( \tau_i \) and \( \theta_i \) represent the time delay and rf phase, respectively, of the \( i \)th signal. The delays are uniformly distributed in \([0, T]\), where \( T \) is the bit duration, the rf phases are uniformly distributed in \([0, 2\pi]\), and all delays and phases are assumed independent of one another and independent of the data. Finally, the noise \( n_s(t) \) is additive white Gaussian noise (AWGN) having two-sided power spectral density \( \eta_0/2 \).

If we assume that each user employs a long spreading sequence (i.e., one that spans many bits), that the processing gain \( L \) is such that

\[
 L = \frac{\text{chip duration}}{T_c} \gg 1
\]  

(2)

where \( T_c \) is the chip duration, and that \( K \gg 1 \), then, using the Gaussian approximation developed in [4] and discussed in many other references (see, e.g., [3], [5]–[7]), we can approximate the conditional probability of error of the system, conditioned upon \( \alpha_1 \) and \( \alpha_2 \), as

\[
P_e | \alpha_1, \alpha_2 \rangle = \phi \left( -\alpha_1 \left( \frac{\eta_0}{2E} + \frac{K}{3L} (\alpha_1^2 + \alpha_2^2) \right)^{-1/2} \right)
\]  

(3)

where

\[
\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy
\]  

(4)

and where \( E = A^2T/2 \) is the energy per bit. If we further assume that \( E/\eta_0 \gg 1 \), we can approximate (3) as

\[
P_e | \alpha_1, \alpha_2 \rangle = \phi \left( -\frac{K}{3L} \left( 1 + \frac{\alpha_2^2}{\alpha_1^2} \right)^{-1/2} \right).
\]  

(5)

Define \( u \triangleq \alpha_2/\alpha_1 \) and \( v \triangleq u^2 \). If \( \alpha_1 \) and \( \alpha_2 \) are independent and identical Rayleigh random variables, then it can be shown that the probability density of \( v \) is given by

\[
f_v(v) = \begin{cases} 
\frac{1}{(v + 1)^2} & v \geq 0 \\
0 & \text{elsewhere}
\end{cases}
\]  

(6)

Hence, the average probability of error at a subscriber receiver is given by

\[
P_e = \int_{0}^{\infty} \frac{1}{(v + 1)^2} \phi \left( -\frac{3L}{K(1 + v^2)} \right) dv.
\]  

(7)

On the other hand, if \( \alpha_i, i = 1, 2, \) has the log-normal density, then

\[
f_v(v) = \frac{\beta}{2\pi \sigma v}\exp \left[-\frac{1}{2\sigma^2} (\beta \ln (u) - d)^2 \right]
\]  

(8)

where \( \beta = 20/\ln 10 \) and where \( d \) and \( \sigma^2 \) are the standard log-normal parameters, and it can be shown that

\[
f_v(u) = \begin{cases} 
\frac{\beta}{2\pi \sigma u} \exp \left[-\frac{\beta^2}{4\sigma^2} (\ln (u))^2 \right] & u \geq 0 \\
0 & \text{elsewhere}
\end{cases}
\]  

(9)

and, hence, that

\[
P_e = \int_{0}^{\infty} \phi \left( -\frac{3K}{\sqrt{1 + u^2}} \right) f_v(u) du.
\]  

(10)

Finally, if we assume an \((n, k)\) block code capable of correcting all combinations of \( e \) and fewer errors, in conjunction with ideal interleaving, are used at the output of the demodulator, then the final decoded average bit error rate (BER) can be approximated by

\[
P_b = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{n}{i} \right) P_e^i (1 - P_e)^{n-i}.
\]  

(11)
Equations (7) and (10) were evaluated numerically to obtain values for $P_e$, as a function of $K$, and those values were used in (11) to obtain the average decoded BER at a mobile receiver. Typical performance results are shown in Fig. 2, wherein BER is plotted against the number of active users in each of the two cells, $K$. There are three curves shown in Fig. 2, one corresponding to Rayleigh fading and the other two corresponding to log-normal fading. In all cases, the processing gain was $L = 511$, and the forward error correction was provided by the $(23, 12)$ Golay code, for which $e = 3$.

If we assume that, for satisfactory voice communications, a decoded BER of $10^{-3}$ or better is required, we see that for Rayleigh fading, $K$ can be about 100; for log-normal fading with $\sigma = 3.16$ (i.e., 10 dB variance) $K$ can be about 170, while if $\sigma = 2$, $K$ can increase to about 200.

Note that these results do not account for speaker activity factors. If, for example, the active users are only transmitting, in some average sense, 50% of the time, the above values of $K$ will double. Also note that these results do not account for diversity reception.

Suppose we now extend the above model to include additional cells. From Fig. 3, it is clear the worst-case location for a mobile is at the point of intersection of all four cells; for this case, the received average power from the base stations of each of the three interfering cells equals that from the mobile’s own base station.

Following the same procedure as before, we now have a conditional probability of error of

$$P(e | \alpha_1, \alpha_2, \alpha_3, \alpha_4) = \phi \left( - \frac{\alpha_1 AT}{\eta_0 T + \frac{4^2 K T^2}{3 L} (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)^{1/2}} \right)$$

= $\phi \left( - \frac{\alpha_1}{\frac{\eta_0}{2 E} + \frac{K}{3 L} (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)^{1/2}} \right)$.

(12)

Again, if we assume $E/\eta_0 >> 1$, we have

$$P(e | \alpha_1, \alpha_2, \alpha_3, \alpha_4) = \phi \left( - \frac{\alpha_1}{\frac{K}{3 L} (1 + \frac{\alpha_2 + \alpha_3 + \alpha_4}{\alpha_1})^{1/2}} \right).$$

(13)

Letting

$$x \triangleq \frac{\alpha_2}{\alpha_1} + \frac{\alpha_3}{\alpha_1} + \frac{\alpha_4}{\alpha_1}$$

(14)

it is clear that $x$ is a chi-squared random variable with six degrees-of-freedom, and hence the density function of $x$ is given by

$$f(x) = \frac{x^2}{16\sigma^5} \exp \left( -x/2\sigma^2 \right) x \geq 0$$

$$0 \quad \text{elsewhere.}$$

(15)

Consequently, with $z \triangleq x/\alpha_1$, it is straightforward to show that

$$f(z) = \frac{3z^2}{(z + 1)^3}$$

(16)

and, hence, the average probability of error for three interfering cells is given by

$$P_e = \int_0^\infty \phi \left( - \frac{K}{3 L (1 + z)^{1/2}} \right) \frac{3z^2}{(z + 1)^3} dz.$$
The decoded BER for this case is shown in Fig. 4 for a Rayleigh fading channel. Also shown in Fig. 4 is the curve of Fig. 2 for the Rayleigh channel when only a single interfering cell is considered, as well as results which correspond to two interfering cells. From Fig. 4, it can be seen that, with one interfering cell, for an average BER of $10^{-5}$, 105 users are simultaneously supported. However, $K$ drops to 61 users for two interfering cells and to 50 users for three interfering cells. It is clear that the increased degradation is quite significant. However, this latter situation of three adjacent interfering cells is probably too pessimistic since, in actual cell layouts, the cell geometry is closer to a hexagon than it is to a square. Hence, only three cells can intersect at a point, not the four cells considered above.

Consider now the effect of diversity on the flat fading channel we are considering. Assume, for simplicity, the two-cell model of Fig. 1, and assume second-order space diversity. The system block diagram is shown in Fig. 5, and the received waveforms $r_1(t)$ and $r_2(t)$ are given by

$$
r_j(t) = \sum_{i=1}^{K} A_{\alpha_{1j}} d_i(t - \tau_{1j}) PN(t - \tau_{1j}) \cos (\omega_0 t + \theta_{ij}) + n_{wj}(t)
+ \sum_{i=K+1}^{2K} A_{\alpha_{2j}} d_i(t - \tau_{2j}) PN(t - \tau_{2j}) \cdot \cos (\omega_0 t + \theta_{ij}) + n_{wj}(t)
$$

respectively. The two AWGN processes, $n_{w1}(t)$ and $n_{w2}(t)$, have spectral density $\eta_0/2$ and are assumed to be statistically independent, and the four Rayleigh random variables which represent the fading, $\alpha_{11}$, $\alpha_{12}$, $\alpha_{21}$, and $\alpha_{22}$, are taken to be independent and identically distributed with parameter $\sigma^2$.

If we define

$$I_j(t) = \int_{\tau_{1j}}^{T + \tau_{1j}} PN(t - \tau_{1j}) d_i(t - \tau_{1j}) PN(t - \tau_{1j}) dt,$$

and

$$r_j(t) = \sum_{i=1}^{K} A_{\alpha_{1j}} d_i(t - \tau_{1j}) PN(t - \tau_{1j}) \cos (\omega_0 t + \theta_{ij})
+ \sum_{i=K+1}^{2K} A_{\alpha_{2j}} d_i(t - \tau_{2j}) PN(t - \tau_{2j}) \cdot \cos (\omega_0 t + \theta_{ij}) + n_{wj}(t)$$

then the test statistics out of the two integrators in Fig. 5 are given by

$$g_1 = AT\alpha_{11} + \alpha_{11} \sum_{i=2}^{K} A_{i1} \cos \phi_{i1}
+ \alpha_{21} \sum_{i=K+1}^{2K} I_{i1} \cos \phi_{i1} + n_1$$

and

$$g_2 = AT\alpha_{12} + \alpha_{12} \sum_{i=2}^{K} A_{i2} \cos \phi_{i2}
+ \alpha_{22} \sum_{i=K+1}^{2K} I_{i2} \cos \phi_{i2} + n_2.$$

Assuming perfect maximal-ratio combining (i.e., assuming $g_1$ and $g_2$ are appropriately aligned in time and weighted by $\alpha_{11}$ and $\alpha_{12}$, respectively), the final test statistic is given by

$$g = \alpha_{11}g_1 + \alpha_{12}g_2 = AT(\alpha_{11}^2 + \alpha_{12}^2) + N$$
If we now define
\[ Z \triangleq \alpha_{11}^2 \alpha_{21}^2 + \alpha_{12}^2 \alpha_{22}^2 \] (28)

it is straightforward to show that
\[
\begin{align*}
f_{Z|\alpha_{11}, \alpha_{12}}(Z|\alpha_{11}, \alpha_{12}) &= \frac{1}{2\sigma^2 \sigma_{11}^2} \exp \left(- \frac{Z}{2\sigma^2 \sigma_{11}^2} \right) \\
&\quad - \exp \left(- \frac{Z}{2\sigma^2 \sigma_{11}^2} \right).
\end{align*}
\] (29)

Hence, the average probability of error, \( P_e \), is given by
\[
P_e = \int_0^{\infty} \int_0^{\infty} \phi \left( -\frac{X_1^2 + X_2^2}{\sqrt{\gamma_1} \left[ X_1^2 + X_2^2 + X_3 + \frac{\gamma_3}{\gamma_1 \sigma^2} (X_1^2 + X_2^2) \right]^{1/2}} \right) \\
\cdot \exp \left(- \frac{X_3^2}{2X_1^2} \right) - \exp \left(- \frac{X_3^2}{2X_2^2} \right) \frac{X_1 \exp \left(- \frac{X_1^2}{2X_1^2} \right) X_2 \exp \left(- \frac{X_2^2}{2X_2^2} \right)}{2(X_1^2 - X_2^2)} dX_1 dX_2 dX_3
\] (30)

where
\[
\gamma_1 = \frac{K}{3L}
\] (31)
and
\[
\gamma_3 = \frac{\eta_0}{2E_a}
\] (32)

For simplicity, assume \( \cos \phi_i \) is uncorrelated with \( \cos \phi_{i'} \), due, in part, to independent noise sources in the respective carrier recovery loops, as well as independent channel phases in the two diversity links which are not completely tracked out by the carrier recovery loops. Using this approximation and assuming further that \( K \gg 1 \), then, conditioned on \( \alpha_{11}, \alpha_{12}, \alpha_{21}, \) and \( \alpha_{22}, N \) is approximately Gaussian with zero-mean and conditional variance
\[
\begin{align*}
\text{var} (N|\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}) &= \frac{A^2LT^2K}{3} \left[ \alpha_{11}^4 + \alpha_{12}^4 + \alpha_{21}^4 + \alpha_{22}^4 \right] \\
&\quad + (\alpha_{11}^2 + \alpha_{22}^2) \eta_0 T.
\end{align*}
\] (26)

The conditional probability of error is then
\[
P_e (\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}) = \phi \left( -\frac{\alpha_{11}^2 + \alpha_{12}^2}{\frac{K}{3L} \left( \alpha_{11}^4 + \alpha_{12}^4 + \alpha_{21}^4 \alpha_{22}^4 + \alpha_{11}^2 \alpha_{22}^2 + \alpha_{12}^2 \alpha_{21}^2 \right) + \frac{\eta_0}{2E_a} \left( \alpha_{11}^2 + \alpha_{12}^2 \right)^{1/2}} \right). 
\] (27)

As a cross-check on (30), note that if we let \( \gamma_1 = 0 \) (i.e., we remove the multiple access interference), (30) reduces to
\[
P_e = \frac{\gamma_3}{\gamma_1 \sigma^2} \exp \left(- \frac{X_3^2}{2X_1^2} \right) - \exp \left(- \frac{X_3^2}{2X_2^2} \right) dX_1, dX_2, dX_3
\] (33)

If we now change to polar coordinates, it can be shown in a straightforward manner that
\[
P_e = \frac{1}{2} \frac{d}{\sqrt{1 + d^2}} - \frac{1}{4} \frac{d}{(1 + d^2)^{3/2}}
\] (34)

where
\[
d \triangleq \frac{2E_a^2}{\eta_0}.
\] (35)

Equation (34) is the well-known result for second-order diversity (see, e.g., [8]).
TABLE I
BER with Second-Order Diversity

<table>
<thead>
<tr>
<th>K</th>
<th>BER Without Diversity</th>
<th>d = 20</th>
<th>d = 200</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>9.1 x 10^{-4}</td>
<td>3.9 x 10^{-4}</td>
<td>9 x 10^{-7}</td>
</tr>
<tr>
<td>200</td>
<td>9.5 x 10^{-5}</td>
<td>1.9 x 10^{-4}</td>
<td>1.3 x 10^{-4}</td>
</tr>
<tr>
<td>300</td>
<td>2.6 x 10^{-2}</td>
<td>2 x 10^{-3}</td>
<td>1.4 x 10^{-3}</td>
</tr>
<tr>
<td>400</td>
<td>5.7 x 10^{-4}</td>
<td>7.3 x 10^{-3}</td>
<td>5.7 x 10^{-3}</td>
</tr>
<tr>
<td>500</td>
<td>8.6 x 10^{-2}</td>
<td>1.6 x 10^{-2}</td>
<td>1.4 x 10^{-2}</td>
</tr>
</tbody>
</table>

Returning to (30), because there does not appear to be a way to significantly simplify it, it was evaluated numerically by making repeated use of a one-dimensional Gaussian quadrature formula. For the special case of \( \gamma_1 = 0 \), the numerical result was checked against (34) and was found to be in excellent agreement.

To see what is the effect of dual diversity, consider Table I. The entries in this table were obtained by assuming the \((23, 12)\) Golay code was used in conjunction with the dual-order diversity (again, assuming ideal interleaving). It is seen that now one can increase the number of active users by about a factor of three (i.e., from about 100 with no diversity to about 300 with diversity and with \( d = 200 \)).

III. MOBILE-TO-BASE LINK

Consider now the mobile-to-base link and, for simplicity, consider initially just two adjacent cells as shown in Fig. 1. Assuming \( K \) users per cell, we model the received waveform at base station \#1 as

\[
r(t) = \sum_{i=1}^{K} A_{ai} d_i(t - \tau_i) P_N(t - \tau_i) \cos(\omega_0t + \theta_i) + \sum_{i=K+1}^{2K} \left( \frac{d_i}{d_1} \right)^{\gamma/2} \alpha_i d_i(t - \tau_i) P_N(t - \tau_i) \cos(\omega_0t + \theta_i) + n_w(t) \tag{36}
\]

where the notation is the same as in the previous section, except now each transmitted mobile signal is assumed to experience independent fading. Specifically, we model the \( \{\alpha_i\} \) as independent Rayleigh random variables, each with parameter \( \sigma^2 \). Also, \( d_2 \) is the distance from the \( i \)-th mobile in cell \#2 to its own base station, while \( d_1 \) is the distance from the same mobile to the base station in cell \#1; thus, \( d_2 \leq d_1 \). Finally, \( r \) is the path loss exponent which describes how the received power falls off with distance [9].

Using the receiver of Fig. 6, we see that the test statistic of receiver \#1 is given by

\[
g_1(T) = A \alpha_1 T + A \sum_{i=2}^{K} \alpha_i I_i(T) \cos \theta_i + A \sum_{i=K+1}^{2K} \left( \frac{d_i}{d_1} \right)^{\gamma/2} I_i(T) \cos \theta_i + N(T) \tag{37}
\]

where, as before, \( N(T) \) is a zero-mean Gaussian random variable with variance \( \eta_0 T \), and

\[
I_i(T) = \int_0^T d(t - \tau_i) P_N(t - \tau_i) dt \tag{38}
\]

and where \( \theta_i \) and \( \tau_i \) have been set equal to zero. If we now define

\[
\beta_i = \left( \frac{d_i}{d_1} \right)^{\gamma/2} \quad K + 1 \leq i \leq 2K
\]

we can express (19) as

\[
g_1^r(T) = A \alpha_1 T + A \sum_{i=2}^{2K} \alpha_i \beta_i I_i(T) \cos \theta_i + N(T). \tag{39}
\]

Consider the term

\[
g_1(T) = \sum_{i=2}^{2K} \alpha_i \beta_i I_i(T) \cos \theta_i. \tag{40}
\]

It is shown in the Appendix that, as \( K \) becomes arbitrarily large, \( g_1(T) \) can be taken to be a Gaussian random variable with zero-mean and variance given by

\[
\sigma^2 = \frac{\sigma^2}{3} L T^2 \left[ K - 1 + \sum_{i=K+1}^{2K} \beta_i^2 \right]. \tag{41}
\]

Therefore, the conditional probability of error, conditioned upon \( \alpha_i \), is approximated by

\[
P(e | \alpha_i) = \Phi \left( -\frac{A \alpha_1 T}{\eta_0 + \frac{\sigma^2}{3} \left( K - 1 + \sum_{i=K+1}^{2K} \beta_i^2 \right)^{1/2}} \right) = \Phi \left( -\frac{\sigma^2}{2E} \left[ \frac{\eta_0}{2E} + \frac{\sigma^3}{3L} \left( K - 1 + \sum_{i=K+1}^{2K} \beta_i^2 \right)^{1/2} \right] \right). \tag{42}
\]

Upon averaging (24) over the density of \( \alpha_i \), we obtain the average probability of error as

\[
P_e = \frac{1}{2} \left[ 1 - \left( 1 + \frac{\eta_0}{2\sigma^2} + \frac{1}{3L} \left( K - 1 \right) + \sum_{i=K+1}^{2K} \beta_i^2 \right)^{1/2} \right]. \tag{43}
\]

Finally, the extension of (25) to any number of cells is straightforward. For example, consider the expanded cell

...
If we want to include the interference effects of, say, $M_I$ cells in addition to the cell-of-interest, the term
\[
\sum_{i=K+1}^{2K} \beta_i^2
\]
in the denominator of the $\phi(\cdot)$ function is replaced with the term
\[
\sum_{i=K+1}^{(M_I+1)K} \beta_i^2
\]
where the $K \beta_i$'s that correspond to, say, the $j$th cell, $2 \leq j \leq M_I + 1$, are defined by
\[
\beta_i = \frac{d_{\text{cell}}}{d_j}.
\]
As before, $d_{\text{cell}}$ is the distance from the $i$th mobile, in this case assumed located in the $j$th cell, to the base station in cell #1, and $d_j$ is the distance from that same mobile to the base station in the $j$th cell.

If we now assume these results correspond to the channel error rate of a receiver employing, as before, the (23, 12) Golay code, we can use (11) to obtain an approximation to the decoded BER. Figs. 8 and 9 show curves of decoded BER versus $K$, the number of active users, for $E/I_0 = 30$ dB and $L = 511$. The curves are parameterized by $r$, and Fig. 8 corresponds to just a single layer of cells around the cell-of-interest, while Fig. 9 corresponds to two layers of interfering cells. It is clear from the figures that for those scenarios where $r = 2$ applies, accounting for only a single layer of cells yields results which are much too optimistic; alternately, if $r = 4$ is applicable, the difference in performance between ac-
counting for only a single layer and accounting for both layers is insignificant.

IV. EFFECTS OF IMPERFECT POWER CONTROL

Up to this point, we have assumed that the power control [2, 3, 10] at each mobile within each cell is perfect. To obtain some perspective on the sensitivity of the system to imperfect power control, consider again the mobile-to-base link, and consider rewriting (36) as

\[ r(t) = \sum_{i=1}^{K} A_i \alpha_i d(t - \tau_i) PN(t - \tau_i) \cos(w_0 t + \theta_i) \]

\[ + \sum_{i=K+1}^{M_1} A_i \left( \frac{d_i}{d_i} \right)^{\tau/2} \alpha_i d(t - \tau_i) PN(t - \tau_i) \cdot \cos(w_0 t + \theta_i) + n_i(t) \]  

(45)

where now \( A_i \) representing the received unfaded amplitude of the \( i \)th user at his own base station, is no longer a constant.

The test statistic now becomes

\[ g_i(T) = A_i \alpha_i T + \sum_{i=2}^{K} A_i \alpha_i I_i(T) \cos \theta_i \]

\[ + \sum_{i=K+1}^{M_1} A_i \alpha_i \beta_i I_i(T) \cos \theta_i + N(T) \]  

(46)

where \( M_1 \), as before, is the number of cells contributing nonnegligible interference, and \( \beta_i \) is defined by (44).

To proceed further requires a more specific description of the \( \{A_i\} \). For our purposes, assume that \( A_i = A \) and \( A_i = \lambda_i A, i > 1 \), where the \( \{\lambda_i\} \) are i.i.d random variables. In particular, we assume they are uniformly distributed around the desired value \( A \). That is, for some value \( V \), let the density of \( \lambda_i \) be given by

\[ f_\lambda(\lambda) = \begin{cases} 
\frac{1}{2V} & A - V \leq \lambda \leq A + V \\
0 & \text{elsewhere}. 
\end{cases} \]  

(47)

With these assumptions, it is straightforward to show that the central-limit theorem used in the Appendix still applies, and that the average probability of error is now given by

\[ P_e = \frac{1}{2} \left\{ 1 - \left[ 1 + \frac{\eta_0}{2E\sigma^2} + \frac{1 + \frac{\Gamma}{3}}{3L} (K - 1) \right] \right\}^{(M_1 + 1)K} \]

(48)

where

\[ \Gamma \triangleq \frac{V}{A}. \]  

(49)

In Fig. 10, the performance of the system versus the propagation law exponent is shown for the case of 100 users, each operating with a processing gain of 511 and a signal-to-noise ratio of 30 dB. The three curves correspond to \( V = 0 \) (i.e., perfect power control), \( V = 0.707 \), and \( V = 1 \). Note that \( V = 0.707 \) corresponds to the average power of any user varying by ±50% about its nominal value, whereas \( V = 1 \) corresponds to a variation of ±100%.

V. CONCLUSIONS

In this paper, we have evaluated the performance of both the base-to-mobile link and the mobile-to-base link of a cellular CDMA system. The channel was modeled as a flat fading Rayleigh channel, and the maximum number of users per cell was determined such that the average probability of error did not exceed some predetermined level. For example, if a decoded error rate of \( 10^{-3} \) is required and the propagation law exponent is taken to be 3, then with perfect power control, a processing gain of 511 allows about 120 users to simultaneously access the channel (again, as pointed out above, this does not include effects of voice activity). On the other hand, if imperfect power control effects are considered, then, assuming the received power can vary by 50% about its nominal value, the number of simultaneous users drops to about 105.

To extend these results further, there are several obvious topics from which to choose. The channel should be modeled as exhibiting frequency selectivity, there should be a more accurate model to account for imperfect power control, and better path loss models are required. Finally, other coding schemes should be considered to provide an overall optimization of performance.
APPENDIX

It is desired to show that the random variable

\[ g(T) = \sum_{i=1}^{\infty} \alpha_i I(T) \cos \theta_i \]  

(41)

of (40) is asymptotically normal. Recall the following:

a) The \( \{\alpha_i\} \) are independent Rayleigh random variables with parameter \( \sigma^2 \).

b) The \( \{\theta_i\} \) are i.i.d random variables uniformly distributed in \([0, 2\pi]\).

c) The term \( I(T) \) is given by (20). The data \( d(t) \) is assumed to be an independent random binary sequence for each \( i \). If the period of each spreading sequence is sufficiently large, it can also be accurately modeled as a random binary sequence. Hence, if \( T/T_i \) is an integer, we can ignore the data (i.e., it can be combined with the chips each of the spreading sequence) and we can represent \( I(t) \) as

\[ I(T) = \sum_{j=0}^{L} [b_{j} - r + b_{j}(T_c - r)] \]  

(42)

where the \( \{b_j\} \) are i.i.d binary random variables taking values \( \pm 1 \) (i.e., if, in a given \( T_c \)-second interval, for a given user, the data symbol is \( d \) and the chip symbol is \( c \), then \( b = dc \) and \( r \) is uniformly distributed in \([0, T]\).

To show that \( g(T) \) is asymptotically normal, we invoke the Liapounoff version of the Central Limit Theorem [1], which states that

\[ z \triangleq \sum_{i=1}^{J} x_i \]  

(43)

is asymptotically normal if the following condition holds:

\[ \gamma = \frac{\left[ \sum_{i=1}^{J} E(\{ x_i - \mu_i \}^3) \right]^{1/3}}{\left[ \sum_{i=1}^{J} E(\{ x_i - \mu_i \}^2) \right]^{1/2}} \to 0 \]  

(44)

where \( \mu_i \triangleq E(x_i) \).

Note that \( \alpha_i, I(T), \) and \( \cos \theta_i \) are all statistically independent, and their statistics (e.g., their moments) are independent of \( i \). Consequently, since both the variance and third-central moment of all three are finite, we can express \( \gamma \) of (44) as

\[ \gamma = C \frac{\left( \sum_{i=2}^{K} \sigma_i^3 \right)^{1/3}}{\left( \sum_{i=2}^{K} \sigma_i^2 \right)^{1/2}} \]  

(45)

where \( C \) is a constant which depends upon the statistics of \( \alpha_i, I(T), \) and \( \cos \theta_i \), but is independent of \( i \). Finally, we define \( d_{2,\min} \) as the smallest value of \( d_2 \), and further assume that \( d_{2,\min} > 0 \) (i.e., we assume the shortest distance in a cell between any mobile and its own base station is greater than zero), and note that \( \beta_{\max} = 1 \). Then, with \( \beta_{\min} \) associated with \( d_{2,\min} \), we have:

\[ \gamma \leq \frac{\left( \sum_{i=2}^{K} \beta_{\max}^3 \right)^{1/3}}{\left( \sum_{i=2}^{K} \beta_{\min}^2 \right)^{1/2}} = \frac{(2K - 1)^{1/3}}{(2K - 1)^{1/2} \beta_{\min}} \]  

(46)

REFERENCES


Laurence B. Milstein

Laurence B. Milstein (S'66-M'68-SM'77-F'85), for a photograph and biography, see this issue, p. 667.

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