TWO-BRANCH DIVERSITY SIMULATION OF THE EFFECTS OF NON-ZERO SIGNAL CORRELATION ON AVERAGE FADE DURATION

Christos Kontogeorgakis and Theodore S. Rappaport
Mobile and Portable Radio Research Group
Dept of Electrical and Computer Engineering - Virginia Tech
432 New Engineering Building - Blacksburg, VA 24061-0350
Phone: (540) 231-2929 Fax: (540) 231-2968
e-mail: wireless@vt.edu

Abstract—This paper provides a simulation method that provides two Rayleigh fading signal envelopes of a desired correlation coefficient. This is based on recent work and uses an improved technique (iterative step) to achieve higher numerical accuracy between the desired and the resultant cross correlation coefficient. The average fade duration is used as a measure of the diversity scheme performance, and various diversity combining techniques (Switch combining: Pure Selection and Threshold Selection - switch and stay, switch and examine) are examined. It is shown that non-zero cross correlation affects the performance of the particular combining technique differently. A model to predict the diversity performance, for a given correlation coefficient and signal level, is examined.

I. INTRODUCTION

An important method for combating fading in mobile communication systems is the use of antenna diversity [2], [3], [5], [6], [8]. The basic principle of diversity methods is to repeat the information on two or more independent fading paths and make use of the lower probability of having a deep fade simultaneously over all the paths (diversity branches). Subsequently, using an appropriate algorithm / technique to combine the signals from the different branches (combining schemes), it is possible to obtain a signal in which the effects of fading are significantly reduced. This may prove valuable in low-cost wireless sensors.

In order to fully exploit the diversity used in a system we must have independent (uncorrelated) signals in each branch. Depending on the type of diversity used (i.e. space, polarization, angle, frequency, or time diversity) the correlation of the signals in the different branches is a function of various system and channel parameters. In practice, it is difficult to obtain independent fading [1], [4]. The consequences of correlated fading are reduction of the branch mean signal-to-noise ratio (SNR) and an increase of the signal correlation which result in an overall loss of diversity gain. For convenience, researchers simulating diversity schemes have typically assumed that the signal envelopes are uncorrelated, in part due to a lack of existence of a simple procedure for generating fading envelopes with some desired cross-correlation [1]. Such simulation results are still useful in determining an upper bound on the achievable diversity gain, but the ability to simulate the more practical situation when the envelopes are correlated is useful for estimating realistic performance of diversity schemes.

Hence, the objective of this paper is to simulate correlated signals in inexpensive two-branch diversity schemes using a new, simple method, and to study the effects of this correlation on the system performance.

II. LEVEL CROSSING RATE AND AVERAGE FADE DURATION

The level crossing rate (LCR) and average fade duration of a Rayleigh fading signal are two important statistics of channel behavior, and these impact the performance of the communication system [5].

The level crossing rate of a Rayleigh fading envelope is defined as the expected rate at which the envelope, normalized to the rms signal level, crosses a specified level in a positive-going direction, while the average fade duration is defined as the average period of time for which the received signal is below a specified level [3]. In analytical form the expression for the LCR is given by [5]:

\[ N_R = \int_0^\infty \dot{r}(\hat{R}) \, d\hat{t} = \sqrt{2\pi f_m \gamma} e^{-\gamma^2}, \]  

where \( \dot{r} \) is the time derivative of \( r(t) \), \( p(\hat{R}, \dot{r}) \) is the joint density function of \( r \) and \( \dot{r} \) at \( \hat{R} \), \( f_m \) is the maximum Doppler frequency and \( \gamma = \hat{R}/R_{\text{rms}} \) is the value of the specified level \( \hat{R} \), normalized to the local rms amplitude.

This work was sponsored by AFOSR under contract F49620-97-1-0521.
of the fading envelope. Equation (1) gives the value of $N_R$, the average number of level crossings per second at specified $R$. 

Similarly, for the average fade duration the analytical form is given by [5]:

$$\bar{\tau} = \frac{1}{N_R} \sum_{r \leq R} \tau = \frac{e^{\gamma^2} - 1}{\gamma f_m \sqrt{2\pi}} \tag{2}$$

In the case of an $M$-branch diversity, theoretically [2], [6], [8], for all branches being statistically independent and providing uncorrelated signals, the average fade duration is reduced in length by exactly a factor of $M$:

$$\bar{\tau}(M) = \frac{\bar{\tau}}{M} \tag{3}$$

This estimation, however, is not valid for non-zero signal correlation and every combining method.

III. Combining Techniques

The methods used to combine the signal coming out from each diversity branch and produce an improved output are called combining techniques or schemes. There are basically four combining methods divided into two groups [1], [3], [5], [8]: switch combining and gain combining. In this work, only the switch combining schemes are examined, due to their simplicity in implementation compared to the gain combining methods where SNR models would be involved.

In the switch combining schemes, the method is to choose one out of $M$ received signals according to some criteria: $r = \text{one out of } \{r_1, r_2, \ldots, r_M\}$.

Pure Selection: The received signals are continuously monitored so the best one is selected. In theory, this decision is based on SNR but in practice the strongest signal plus noise is selected [8].

Threshold Selection: The received signals are scanned in a sequential order and the first signal that is found to exceed a power level (threshold) is selected. This signal remains at the output of the combiner as long as its level is above the threshold, otherwise, the scanning process is repeated. The threshold can be either fixed or variable. An effective variable threshold value is the mean signal level [8].

In case of two branches, when the signal falls below this threshold, two main strategies are used to select the signal:

- switch and stay: the other signal is selected regardless its power level
- switch and examine: the other signal is not selected unless its power level exceeds the threshold

The first approach has the disadvantage that the output signal may remain below the threshold, after the switch, and even below the "less bad" signal. On the other hand, the switch and examine approach provokes a series of noise bursts when both signals are below the threshold [2].

IV. Simulation

We simulated the generation of two correlated Rayleigh fading envelopes using a novel technique.

In order to simulate the signals we use the following parameters:

- Speed of the mobile: both 30 mph and 75 mph are tested
- Carrier frequency: 900 MHz
- Resulted maximum Doppler frequency $f_m$: 40 Hz and 100 Hz.
- Correlation coefficient: 0, 0.33, 0.66, 0.82, 0.9, and 0.99.

Then, the performance of the following combining techniques was examined:

- Pure Selection
- Threshold Selection - Switch and Stay
- Threshold Selection - Switch and Examine

Simulations included the generation of two correlated Rayleigh fading signals, the combining output calculation, and the calculation of the average fade duration for a desired correlation coefficient and $\gamma = R/R_{rms} - $ as given in equations (1) and (2) ranging from 0.1 to 2 with a step of 0.1.

A. Generation of two correlated Rayleigh fading signals

The task of generating two correlated Rayleigh fading signals has two parts. First the two Rayleigh signals are generated and then correlated.

A.1 Generation of a Rayleigh fading signal

The simulation of a fading signal can be found in various sources in the literature [1], [2], [5]. Here, the one described in [5] is used. To verify the accuracy of the simulation, the Rayleigh distribution parameters (i.e. the moments of the distribution) are calculated for both theoretical and simulation signals: agreement was exact.

A.2 Correlation of two fading signals

This procedure is described in a recent work by [1]. At first, two uncorrelated envelopes are generated (i.e. from IV-A.1 above). Then, using an expression relating the correlation coefficient of the complex Gaussian samples to the cross-correlation coefficient of the resulting Rayleigh envelopes, the required correlation matrix of the Gaussian samples is found. Let the two complex Gaussian random signals be

$$w_1 = y_1 + jz_1 \tag{4}$$
$$w_2 = y_2 + jz_2. \tag{5}$$

Then the envelopes of the received signals are given by

$$r_1 = |w_1| = \sqrt{y_1^2 + z_1^2} \tag{6}$$
$$r_2 = |w_2| = \sqrt{y_2^2 + z_2^2}. \tag{7}$$
The cross-correlation coefficient between \( r_1 \) and \( r_2 \) is expressed as [1]

\[
\rho_r = \frac{(1 + \lambda)E_1(\frac{\sqrt{\lambda}}{1 + \lambda}) - \frac{\pi}{2}}{2 - \frac{\pi}{2}} \tag{8}
\]

where \( \lambda \) is a parameter that depends on the correlation between the Gaussian samples, and \( E_1(\eta) \) denotes the complete elliptical integral of the second kind with modulus \( \eta \). It can be shown [1] that the desired Rayleigh fading samples \( z \) are given by

\[
z = Lw, \tag{9}
\]

where

\[
L = \begin{bmatrix}
\frac{\sigma_s}{\sqrt{\lambda}} & 0 \\
\frac{\sigma_s}{\sqrt{\lambda}}(1 + i) & \sigma_s\sqrt{1 - \lambda^2}
\end{bmatrix}, \tag{10}
\]

and \( \sigma_s^2 \) is the desired signal power.

The simulation following this procedure for long blocks of samples (3192) gives satisfying results. The difference of simulated and theoretical correlation coefficients is within 5%. This discrepancy is mainly due to the fact that computer generated random samples are not completely independent. Nevertheless, in this paper, a way to further reduce this inaccuracy is given. It consists of one iterative step: After the two correlated signals are generated, the simulation calculates an intermediate resultant correlation coefficient \( \rho' \), and solves equation (8) to find a corresponding \( \lambda' \) and form the corresponding \( L' \) using equation (10). Then, any offset effect is canceled, by calculating:

\[
z = L(L')^{-1}Lw. \tag{11}
\]

This iterative method leads to simulated values that are within 0.1% of theoretical values. Also, the correlation coefficient does not depend on the signal power, so this method can be used for simulation of non-equal power fading signals, too.

**B. Combining techniques**

Figure 1 shows a few samples of a simulation of the pure selection method using two Rayleigh fading signals \( r_1(t) \) and \( r_2(t) \) of zero correlation coefficient and 1 ms sampling time. The signal with maximum envelope level is selected at every point. Figure 2 shows the simulation results for the threshold selection, switch and stay, combining technique. The slow moving line (squares) corresponds to the (variable) threshold. The threshold at any point is implemented as the mean of the combining output for up to that point. The resulting output is depicted with the circles. In this case, the output starts as one of the signals and switches to the other only in every transition from above to below the current threshold (switch and stay). In the case of switch and examine, Figure 3, it switches every time the current level is below the threshold. We can see how the switch and examine method gives rise to noise bursts while resulting in a slightly higher mean value than for the switch and stay method.

**V. Results**

The simulation process generates two correlated fading signals and the corresponding combining method output signal. Next, the average fade duration is calculated for various values of \( \gamma \) and the correlation coefficient \( \rho_r \), both theoretically, based on equation (2), and experimentally, based on the simulation results. In order to test the dependence of average fade duration on different Doppler frequencies, it is also calculated for \( f_m = 40Hz \) (30mph, 900MHz) and \( f_m = 100Hz \) (75mph, 900MHz). Some
representative results for the single-branch (no diversity) case are shown in Table I for various values of \( \gamma \) and different Doppler frequencies. We notice that the simulation results are close to theoretical, and can be accepted as reliable. A good criterion to see how well a diversity scheme works is to compare its performance to the one of equation (3). In other words, to see how close to the value of 2 the ratio of the theoretical (no-diversity) average fade duration over the simulated (diversity) one is. This ratio is calculated for all the three combining schemes, the two Doppler frequencies and the set of correlation coefficient \([0, 0.33, 0.66, 0.82, 0.9, 0.99]\). In all cases, it was found that the differences due to the use of different Doppler frequency were negligible and therefore in the following illustrations only one frequency (40 Hz) is referred to.

### Table I

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( f_m = 40 )</th>
<th>( f_m = 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.00</td>
<td>1.38</td>
</tr>
<tr>
<td>0.4</td>
<td>4.30</td>
<td>4.32</td>
</tr>
<tr>
<td>0.7</td>
<td>8.96</td>
<td>9.62</td>
</tr>
<tr>
<td>1.0</td>
<td>17.04</td>
<td>16.77</td>
</tr>
<tr>
<td>1.3</td>
<td>33.72</td>
<td>33.19</td>
</tr>
<tr>
<td>1.6</td>
<td>73.89</td>
<td>73.41</td>
</tr>
<tr>
<td>1.9</td>
<td>187.74</td>
<td>182.40</td>
</tr>
</tbody>
</table>

A. Pure Selection

The simulation results for average fade duration ratio, when using the pure selection combining technique, are shown in Figure 4. We see that for most values of \( \gamma \) the average fade duration ratio varies uniformly between 2 (for \( \rho_r = 0 \) as predicted) and 1 (for \( \rho_r = 1 \) as expected). For very small values of \( \gamma \), however, the simulation fails to achieve reliable results. This is mainly due to the numerical inaccuracy resulting from the relatively insufficient number of samples used; for low signal levels (small \( \gamma \)) many more samples per time would be required for a realistic estimation of the level crossing rate.

B. Threshold Combining - Switch and Stay

Figure 5 shows the average fade duration ratio that results by simulating threshold selection combining with the switch and stay scheme. We see that with this technique, the ratio is about 1 for values of \( \gamma < 0.9 \). This technique, as expected, does not perform very well since with the "switch-and stay" strategy the signal may remain in fade while there is a branch with high signal level. The maximum occurs naturally around the threshold value, as it can be observed in Figure 2, where the larger number of level crossings is expected (equation (2) shows that a larger number of level crossings yields a smaller average fade duration). Nevertheless, for \( \gamma > 1 \), there is improvement of the average fade duration which depends on the correlation coefficient of the two envelopes.

C. Threshold Combining - Switch and Examine

Simulation results for threshold selection combining using the switch and examine scheme, are shown in Figure 6. This appears to be the most interesting technique among the ones simulated here. The ratio reaches a value as high as 4 for uncorrelated signals and is still above 2 even for correlation of 0.82. However, these high numbers occur only in the region of \( 0.2 < \gamma < 0.8 \). This is
expected because there is a region below the threshold in which noise bursts occur (see Figure 3) and consequently the level crossings are overly increased. For \( \gamma > 1 \), the results seem to follow an expected order varying from a ratio of 2 (for \( \rho_r = 0 \)) to a ratio of 1 (\( \rho_r = 1 \)).

**D. Average fade duration ratio dependence on the correlation coefficient**

In Figure 4 through Figure 6, the dependence on the correlation coefficient is shown as a family of curves. Figure 7 shows how the average fade duration ratio, at \( \gamma = 1 \), changes with respect to the correlation coefficient \( \rho_r \). As expected, all three curves, corresponding to each combining scheme, converge to 1 as \( \rho_r \) goes to 1 (equivalent to a no-diversity scheme). However, the transition of their slopes is very smooth showing a possibility of modeling this variation with some empirical analytical function.

**VI. CONCLUSIONS**

The simulation of two Rayleigh fading signals of a desired correlation is demonstrated here. This is applied with a simulation of switch combining techniques to estimate the effect of the signal correlation on the average fade duration improvement. The quantity used to express this improvement is the ratio between the theoretical value of the average fade duration without diversity (one branch) over the results of two-branch diversity. A series of figures demonstrating fading and statistics are given here. Further work to develop a curve fit or closed form expression from Figures (4)-(6) will allow analysis and comparison to experimental work [7]. This work maybe useful for low power sensors employing inexpensive diversity methods.

**REFERENCES**