ABSTRACT

This paper presents bit error rate (BER) results for \( \pi/4 \) DQPSK with Nyquist pulse shaping in flat and frequency-selective fading channels. BER results are shown for a wide range of multipath delays, receiver speeds and power ratios (C/D) between the main ray and the delayed ray in a two ray channel model. Co-channel interference effects are also considered. In addition, the instantaneous BER for \( \pi/4 \) DQPSK in various fading channels are simulated. The simulation is first performed off line, and then bit error patterns are replayed in real time to allow subjective evaluation of link quality between a source and sink. This method is being used to provide real time bit error simulation of U.S. Digital Cellular data channels using simple computer hardware.

I. INTRODUCTION

Frequency-selective fading and flat fading cause bit errors in digital radio communications. The accurate prediction of average and instantaneous BERs in fading channels will become increasingly important as system design as demand grows for digital wireless communication systems. Accurate BER simulations allow designers to determine acceptable modulation methods and coding techniques in the operating environment. This research uses both flat and frequency-selective fading channels to predict and simulate BER performance for \( \pi/4 \) DQPSK modulation with Nyquist pulse shaping.

For data transmission systems, error bursts due to signal nulls are a primary concern, and understanding the burstiness of the channel is necessary to implement successful antenna diversity or coding techniques. With accurate BER computer simulations in fading environments, it becomes possible to test digital radio communication systems by using a simple baseband digital hardware BER simulator between the data source and sink as shown in Figure 1. Digital hardware BER simulators, which operate on an applied digital data stream, are commercially available and can be programmed in real time to alter the BER. The techniques presented in this paper can be used to simulate the modulation, filtering, propagation, and detection, and lend themselves directly to the control of hardware BER simulators. This paper presents the results of a simulation study that provides insight into the simulation methodology of \( \pi/4 \) DQPSK in flat and frequency-selective fading radio channels, and produces error patterns that can be applied to a hardware BER simulator in real time.

II. COMMUNICATION SYSTEM MODEL

A block diagram of the \( \pi/4 \) DQPSK system used in the analysis is shown in Figure 2. All bandpass signals and channels described in this paper are represented by low pass complex envelope forms. Referring to Figure 2, a pseudorandom binary bit stream, \( d(t) \), is sent through the simulator program for BER simulation. The binary bit stream \( d(t) \) is stored in computer memory and compared with the received bits \( d'(t) \) at the output of the simulated receiver for determination of bit errors. The number of bits sent is controlled by the bounded binomial sampling method [1].

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Let $R_i$ be the $i$th generated pseudorandom binary number which has equal probability of being 0 or 1, then $d(t)$ is a bit stream of $R_0 \ldots R_i R_{i+1} \ldots$. For $\pi/4$ DQPSK [2], a symbol is formed from two consecutive bits, $R_i$ and $R_{i+1}$, in the bit stream. Each of the four combinations of $R_i$ and $R_{i+1}$ (i.e., 00, 10, 11, 01) represents a specific phase shift, $\phi_n$, with reference to the phase of the previous symbol. The modulated waveform of the $n$th symbol represented in baseband I and Q components is given by

$$x(t) = \sum_{n=0}^{L/2-1} A \cos(\theta_n) \delta(t - nT) \quad \text{I channel}$$

$$x(t) = \sum_{n=0}^{L/2-1} A \sin(\theta_n) \delta(t - nT) \quad \text{Q channel}$$

where $A$ is the amplitude and

$$\theta_n = \theta_{n-1} + \phi_n \quad (2)$$

$$\phi_n = \begin{cases} \pi/4 & R_i = 0 \quad R_{i+1} = 0 \\ 3\pi/4 & R_i = 1 \quad R_{i+1} = 0 \\ 5\pi/4 & R_i = 1 \quad R_{i+1} = 1 \\ 7\pi/4 & R_i = 0 \quad R_{i+1} = 1 \end{cases} \quad n = 0, 1, \ldots, L/2 - 1$$

$$l = 0, 2, \ldots, L-1$$

$\theta_n$ and $\theta_{n-1}$ are the phases of the $n$th and $(n-1)$th symbols, respectively. $L$ is the number of symbols sent in simulation.

At initialization, $\theta_{-1}$ is arbitrarily set to $\pi/4$. $x(t)$ is discretized for computer simulation at a rate of $T_s = T/N$, where $T$ is the symbol period and $N$ is the number of samples per symbol. $N$ is 13 in our simulation. $x(t)$ is represented as $x(kT_s)$, where $k$ is a time index and $k = 0, 1, \ldots, N - 1, \ldots, (L \cdot N) - 1$.

By placing a square root raised cosine filter in both the transmitter and the receiver, we can have both a Nyquist pulse and a matched filter (in flat fading channels). By performing the inverse Fourier Transform of the transfer function of the square root raised cosine filter, the impulse response of the filter in the time domain is obtained. In computer simulation, the impulse response is a collection of amplitudes as a function of time, $t(kT_s)$, stored in a file. $x(kT_s)$ is then convolved with the impulse response of the square root raised cosine filter and the signal at the output of the filter $s(kT_s)$ is sent through the simulated channel. $s(kT_s)$ is convolved with the impulse response of the channel and white Gaussian noise is added to the signal at the output of the channel. In the simulation, the mean signal level is held constant while the added noise power is changed for BER analysis as a function of signal to noise ratio. The matched filter in the receiver is another square root raised cosine filter. In flat fading channels, the output of this matched filter will be Nyquist pulses with no ISI and with the signal to noise ratio maximized. In the receiver, differential detection is used to recover the bit stream.

III. CHANNEL DESCRIPTION

The impulse response of the channel is expressed by the two ray model

$$h(t) = A e^{j\theta(t)} + A e^{-j\theta(t)}$$

where $\theta_1$ and $\theta_2$ are independent and Rayleigh distributed, $\theta_1$ and $\theta_2$ are independent and uniformly distributed on $0$ to $2\pi$, and $r$ is the time delay between the two rays. The sum of $E(\sigma_1^2)$ and $E(\sigma_2^2)$ is set to unity in the simulation so the channel has unity gain. The ratio of $E(\sigma_1^2)$ to $E(\sigma_2^2)$ is the power ratio of the main ray to the delayed ray and is denoted as C/D. A flat fading channel is formed by setting $\sigma_2$ to zero.

A software fading simulator similar to [3] has been used to generate the amplitude and phase of each ray in the two ray model. However, we use the baseband (not RF) power spectrum of the fading envelope given in [4] to simulate the Rayleigh fading spectrum found in mobile radio. Specifically, the spectra of two independent white Gaussian processes are shaped to the baseband fading spectrum given in [4]. The shaped spectra are then inverse Fourier-transformed to the time domain, yielding in-phase and quadrature baseband fading signals. The distribution of the envelope of the resulting processes in the time domain is Rayleigh and the phase is uniform. Correlation is preserved from the Doppler spectrum.

The delay between two rays is implemented in the carrier frequency is assumed to be fixed at 850 MHz and receiver speeds of 40 km/hr to 120 km/hr are used. A typical fading envelope generated by the simulator with the maximum Doppler frequency of 76 Hz (receiver speed = 100 km/hr, $\lambda = 0.35$) is shown in Figure 3. The simulator has been thoroughly tested and the cumulative distribution and the normalized level crossing rate agree very well with theoretical results.
errors (em) was set to the same order of magnitude of analytical result [5] or outcomes described in [1]. The median unbiased interval on BER was approximately 25% (the exact upper and lower limit depended on the number of bits required to produce the interference signal power which is denoted as C/I).

IV. BER CALCULATION

We used the bounded binomial sampling method described in [1] to set confidence levels on our results. This method tells how many bits must be sent in a BER simulation from parameters provided. It also provides double boundaries in simulated BER results so that the true BER lies within certain relative precision (in terms of percentage for low bit error rates; e.g. $10^{-6}$) and certain absolute precision (for high bit error rates; e.g. $10^{-4}$). In our simulation, the confidence level, $\alpha$, was 99%. Relative precision, $\delta_r$, was 25% and absolute precision, $\delta_a$, was $10^{-2}$. From Figure 1 in [1], the minimum number of bit errors ($c_m$) was set to 106 so that our confidence interval on BER was approximately 25% (the exact upper and lower limit depended on the number of bit errors observed in a particular simulation run) of the median unbiased BER with 99% confidence. For an absolute lower bound on the required number of bits seeded at high BER, Figure 2 in [1] shows the minimum number of bits ($n_r$) required is at least $2 \times 10^4$ for an absolute precision of $10^{-4}$ BER. The maximum number of bits ($n_u$) sent in a BER simulation depends on the BER estimated before the actual run. We assumed the BER would be on the same order of magnitude of analytical result [5] or made rough estimates when analytical results were not available. We set the maximum number of bits to approximately 10 times the bits required to produce one bit error. Our BER simulation for a particular run will terminate due to one of three possible outcomes described in [1] and the median unbiased BER estimate are calculated according to [1].

V. SIMULATION RESULTS

Figure 4 shows the BER simulation results for $\pi/4$ DQPSK in flat fading channels. Results are found for receiver speeds of 40 km/hr and 120 km/hr. An irreducible bit error floor exists at $E_b/N_0$ above 60 dB. The irreducible bit error floor is caused by random FM and is a function of the receiver speed. For low $E_b/N_0$ ($E_b/N_0 < 30$ dB), bit errors are caused by the additive white Gaussian noise and changing of receiver speed has minimal effect on the BER.

Figure 5 shows BER as a function of $E_b/N_0$ in frequency-selective fading channels for different $r/T$. At low $E_b/N_0$, additive white Gaussian noise and ISI cause most of the bit errors. At high $E_b/N_0$, frequency-selective fading dominates the BER and BER becomes a function of $r/T$.

BER performance as a function of $r/T$ in frequency-selective fading channels is shown in Figure 6. The influence of receiver speed and C/D are also shown in the figure. The results show that BER increases with the increase of $r/T$. In our simulation, $r/T$ ranged from 0.077 to 1.46, and Figure 6 shows that BER is not a strong function of the receiver speed for $r/T$ in that range. There is a BER ceiling for $r/T$ greater than 1.0 which is determined by the C/D ratio.

Figures 7 and 8 show BER vs C/D in frequency-selective fading channels for two different receiver speeds and $r/T$ ratios. As C/D increases, the channels become less frequency-selective and the random FM (fast fading) places a lower limit on the BER. By comparing the bit error floors in Figures 7 and 8, the lower BER limit is determined by the receiver speed. The BER limits are about $2 \times 10^{-6}$ and $2 \times 10^{-4}$ for receiver speeds of 40 km/hr and 120 km/hr, respectively. For C/D below 40 dB, the BER is dominated by frequency-selective fading (ISI). By comparing the results for C/D below 25 dB in Figures 7 and 8, the variation of the receiver speed appears to have little effect on the BER. We have compared our simulation results with Liu and Feher’s [5] analytical results in the figures for the same receiver speeds and slightly different $r/T$. As shown in Figures 7 and 8, our simulation results agree closely with results given in [5].

The effect of co-channel interference in flat fading channels is shown in Figure 9. The BER decreases as C/I increases from 10 dB to 40 dB. Three different receiver speeds, 40, 70 and 120 km/hr, were used in our study. We found that the variation of receiver speed makes no difference on the BER performance for a given C/I ratio. This agrees with the observation by Malupin and McNair [6]. Experimental results found in [6] from actual field measurements and analytical results for flat fading channels [5] are also shown in Figure 9 for comparison. Results in [6] have a higher irreducible bit error at large C/I due to the frequency-selective fading nature of the channels. In Figure 10, the impact of co-channel interference in frequency-selective fading channels for receiver speeds of 40 and 120 km/hr is shown. Again we found that receiver speed has no effect on the BER performance for a given C/I ratio in frequency-selective fading channels.
VI. SIMULATION OF INSTANTANEOUS BER

The use of channel models which have accurate second order fading statistics (i.e. level crossing rate) is important for the accurate prediction of burst errors in real channels. This can be seen in Figure 11. Figure 11 illustrates the instantaneous BER for \( \pi/4 \) DQPSK operating at \( E_b/N_0 \) of 10 dB in a flat fading channel. Figure 3 shows the corresponding flat fading envelope which yields the results in Figure 11. It can be seen in Figure 11 that the times at which instantaneous BER increases is directly coupled to the spatial location of the receiver given in Figure 3. That is, the high instantaneous BER values at the elapsed times of 50 ms, 60 ms, 70 ms and 180 ms in Figure 11 correspond exactly to the deep fades or fades with long duration at the same moment in Figure 3. Results in Figure 11 were generated from simulation and the instantaneous BER was computed for each block of 243 consecutive symbols using a data rate of 24,300 symbols per second. While 243 symbols provides a relatively coarse estimate of the instantaneous BER, it preserves the bursty nature of real world channels. The time interval over which the instantaneous BER was computed was 10 ms, although this can be adjusted in simulation. By computing a string of instantaneous BER values as a function of time and \( E_b/N_0 \), through computer simulation, it is simple to clock out the actual bit error patterns to drive a hardware BER simulator. Such simulators, when used in conjunction with computer simulations that use highly accurate channel models, will be advantageous in evaluating the performance of emerging digital communication services.

VII. SUMMARY

This paper presents the methodology and implementation of software BER simulation for \( \pi/4 \) DQPSK in flat and frequency-selective fading channels. The BER performances for flat fading as a function of \( E_b/N_0 \) and receiver speed were found. The flat fading sets an irreducible bit error floor which is determined by the receiver speed. BER results in frequency-selective fading channels were also found for a range of \( E_b/N_0 \), C/D and receiver speeds. For low C/D (C/D < 25 dB), BER is determined by \( r/T \). For high C/D, BER is determined by the receiver speed. Results show that the receiver speed has minimal effect on BER for \( r/T \) in the range of 0.077 to 1.46. The effect of co-channel interference in flat and frequency-selective fading channels was also studied. Receiver speed has an negligible effect on the BER for a given C/D in flat and frequency-selective fading channels, and \( r/T \) dominates BER at high \( r/T \). Our simulation results agree with recent work [5,6]. By using instantaneous bit errors generated by simulations in conjunction with a hardware BER simulator, an inexpensive real time bit error simulation can be performed in the laboratory. The real time baseband bit error simulation between a data source and a data sink has been successfully demonstrated in our laboratory and is a useful technique for evaluating data transmission products for various system parameters and channel conditions.

VIII. ACKNOWLEDGEMENT

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REFERENCES


![Figure 4. BER vs \( E_b/N_0 \) in flat fading channels with receiver speed = 40 and 120 km/hr. C/I \( \approx \) 100 dB.]

![Figure 5. BER vs \( E_b/N_0 \) in frequency-selective fading channels. Receiver speed = 120 km/hr, \( r/T \) = 0.077, 0.154, 0.539 and 1.46. C/D = 0 dB, C/I \( \approx \) 100 dB.]
Figure 6. BER vs $\tau/T$ in frequency-selective fading channels for various receiver speeds and C/D ratios. $E_b/N_0 = 100$ dB. C/I = 100 dB.

Figure 7. BER vs C/D in frequency-selective fading channels. Receiver speed = 40 km/hr. $E_b/N_0 = 100$ dB. C/I = 100 dB.

Figure 8. BER vs C/D in frequency-selective fading channels. Receiver speed = 120 km/hr. $E_b/N_0 = 100$ dB. C/I = 100 dB.

Figure 9. BER vs C/I in flat fading channels for various receiver speeds. $E_b/N_0 = 100$ dB.

Figure 10. BER vs C/I in frequency-selective fading channels for various $\tau/T$. Receiver speed = 40 and 120 km/hr. $E_b/N_0 = 100$ dB. C/D = 0 dB.

Figure 11. Instantaneous BER for $\pi/4$ DQPSK in a flat fading channel shown in Figure 3. Note that the instantaneous BER is directly a function of deep fade as the mobile moves. $E_b/N_0 = 10$ dB.