APPLICATION OF SECOND-ORDER STATISTICS FOR AN INDOOR RADIO CHANNEL MODEL

Scott Y. Seidel, Koichiro Takamizawa, Theodore S. Rappaport

Bradley Department of Electrical Engineering
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061

ABSTRACT

The secondary statistics for an indoor radio impulse response model in the factory environment are investigated. Under the assumption that multipath signal amplitudes are jointly log-normally distributed over local areas and excess delay, application of second order statistics (correlation) into a statistical model is presented. In addition, the distributions of received power within a particular excess delay interval are found for both global and local areas. We show that the log-normal distribution describes the fading of multipath components over local areas for particular excess delays. Moreover, individual multipath signal strengths are log-normal about a mean power law of the form d^n. The log-normal distribution is attractive for modeling the impulse response amplitudes because correlation data is easily incorporated. Conditional probabilities of path occupancy are presented which show the effect power control will have on indoor radio systems.

I. INTRODUCTION

During the summer of 1987, empirical radio wave propagation measurements were made at UHF within five factories in Indiana [1]. The measurements apparatus was capable of operating in either a pulsed or CW mode, and provided the basis for narrow-band (flat fading) models for path loss (as a function of T-R separation and topography), small scale fading at a receiver, signal fluctuations for stationary transmitter and receiver, and typical shadowing losses [2]. Wideband measurements have revealed typical and worst case time delay spreads [3,4] and time delay jitters [5]. The assumption that multipath signal components arrive with identical and independent phase angles has been shown to agree with empirical data [6]. While the above works are useful in that they present typical characteristics of indoor factory radio channels, they fail to provide much insight into useful wideband (impulse response) channel models. Channel models which could accurately recreate the empirical data would be valuable for the analysis of novel modulation and anti-multipath techniques.

Stochastic processes are completely described in terms of n-th order statistics. To more accurately model the radio channel in manufacturing environments, it is desirable to determine second order statistics for the channel impulse response. The primary statistics previously determined [6,7] have shown the average number of multipath components in factories as well as the large scale signal amplitude distribution at various excess delays. These statistics are an important part of the channel model as they provide a method for predicting signal strengths and delay spreads for random locations. However, they are not enough to describe the relationship between signals over time and space within a local (small scale) area.

II. POWER DISTRIBUTION

The power in a received multipath profile measured at a particular transmitter-receiver (T-R) separation of d meters has a path loss which is well described by the log-normal distribution about a mean power law of d^n. Free-space path loss is assumed for the first 2.3 meters (10J) and values of n (a positive) are given in [2,3,8,9]. It is shown here that not only is the total path loss log-normally distributed, but the path loss within a particular excess delay interval is also log-normally distributed about some mean d^n power law. This is similar to the simulation of urban radio propagation (SURP) channel model [10-13]. At first glance, this appears to contradict the results presented in [6,7] that individual multipath amplitudes are Rayleigh distributed. However, it is important to remember that the results in [6,7] assumed no information about T-R separation. The multipath component amplitudes have been shown to be Rayleigh distributed if the transmitter and receiver are placed at a random separation. It is shown here that a log-normal distribution is a good model for multipath component amplitudes when the T-R separation is known.

Figure 1 shows the cumulative distribution function of the received signal power relative to the local mean within the particular excess delay interval 23.4-31.2 ns. The figure, which is based on the entire ensemble of power delay profile measurements in [1], also shows the CDF of the log-normal distribution with the least mean-square error fit to a mean d^n path loss law and standard deviation about the mean for measured data in LOS topographies. This figure is typical of the data, and demonstrates that for a particular T-R separation and topography, the log-normal distribution for received signal power within a particular excess delay interval models the measured data reasonably well.

![Figure 1. CDF of received signal power and CDF of log-normal distribution fit to the experimental mean and variance for LOS topographies.](image-url)
III. AUTOCORRELATION COEFFICIENT FUNCTION

Intuition leads us to believe that amplitudes of multipath components which exist at various locations and time delays are correlated. This has been found to be the case for urban mobile radio channels [10-13]. The general correlation coefficient is assumed to be a function of both space and time. Computation of the correlation of multipath amplitudes for all locations and time delays, although desirable, is not possible due to the limited amount of measured data and the difficulty of identifying the location and orientation of scatterers in an indoor radio channel. Therefore, it is necessary to assume that temporal and spatial correlations are independent. The space and time correlation function can be written as

$$r_{sa}(x_1, t_1, x_2, t_2) = K r_{sa}(A(x_1), A(x_2), t_1) r_{ta}(A(t_1), A(t_2); x)$$

(1)

where $A$ is the multipath component signal amplitude for a particular spatial location $x$ and excess delay $t$, $r_s$ is the space correlation, and $r_t$ is the temporal correlation. Assuming independence, autocorrelation coefficients for received signal levels are estimated individually over space and time.

The jointly normal probability density function of random variables $A(x_1)$ and $A(x_2)$ is defined as [11,15,16]

$$f_{x_1, x_2}(x_1, x_2) = \frac{1}{2\pi \sigma_x \sigma_y (1 - r^2)^{1/2}} \exp \left[ -\frac{1}{2(1 - r^2)} \left( \frac{(x_1 - \bar{x})^2}{\sigma_x^2} + \frac{(x_2 - \bar{x})^2}{\sigma_y^2} - 2r(x_1 - \bar{x})(x_2 - \bar{x}) \right) \right]$$

(2)

where $x_1 = A(x_1)$, $x_2 = A(x_2)$, $\bar{x} = \text{average of } A(x)$, $\sigma_x = \text{variance of } A(x)$, $\sigma_y = \text{variance of } A(x_2)$, and $r = r_{sa}(A(x_1), A(x_2))$, $-1 \leq r_{sa}(A(x_1), A(x_2)) \leq 1$.

The conditional probability density function of $x_2$ given the value of $x_1$ is [11,15]

$$f_{x_2|x_1}(x_2|x_1) = \frac{1}{\sqrt{2\pi(1 - r^2)}} \exp \left[ -\frac{1}{2(1 - r^2)} \left( \frac{(x_2 - \bar{x})^2}{\sigma_x^2} \right) \right]$$

(3)

Thus, the conditional distribution of $A(x_2)$ given the value of $A(x_1)$ can be calculated by assuming a normal distribution with a conditional mean

$$\bar{A}(x_2|x_1) = \bar{A}(x_1) + r_{sa}(A(x_1), A(x_2)) (A(x_2) - \bar{A}(x_2))$$

(4a)

and a conditional variance

$$\sigma^2_{A(x_2|x_1)} = (1 - r_{sa}^2)(A(x_2) - \bar{A}(x_2))^2$$

(4b)

When random variables $x$ and $y$ are in decibels, then the density functions in equations (2) and (3) describe a log-normal distribution. The autocorrelation coefficients are computed with data values in decibels since a joint log-normal distribution is assumed.

Spatial Correlation

Inspection of the profiles and intuition about the physical causes of multipath propagation lead us to believe that multipath component amplitudes are correlated over small distances. Over large distances, it is reasonable to assume signal strengths are uncorrelated. This has been shown for urban mobile radio channels [10,12,13,17]. A valid model must be able to generate profiles which produce the same correlation statistics as measured data. Also, the distance at which multipath signal amplitudes become uncorrelated is important in the analysis of antenna diversity.

We assume that multipath amplitudes are jointly log-normally distributed over local areas. The log-normal distribution was shown to be a good model for individual multipath signal levels over local areas in Section II. If the amplitude of a multipath signal which occurs at a particular excess delay at one location is known, then the conditional amplitude of the multipath signal amplitude at a distance $\Delta x$ away with the same excess delay is log-normally distributed with mean and variance as derived from (4a) and (4b), where

$$A(x + \Delta x; t) | A(x_1; t) = \bar{A}(x + \Delta x; t) + \sigma_{A(x + \Delta x; t)} (A(x_1; t) - \bar{A}(x_1; t))$$

(5a)

and

$$A(x; t) = \text{multipath signal amplitude at location } x \text{ and excess delay } t$$

(5b)

$$A(x + \Delta x; t) = \text{conditional spatial average of multipath signal amplitude at location } x \text{ and excess delay } t$$

The estimate of $\hat{r}_{sa}(x; t)$ for each local area is obtained from [15,16,18]

$$\hat{r}_{sa}(x_1; x_2; t) = \frac{\text{E}[A(x_1; t) - A(x_1; t) (A(x_2; t) - A(x_2; t))]}{\sqrt{\text{E}[(A(x_1; t) - A(x_1; t))^2] \text{E}[(A(x_2; t) - A(x_2; t))^2]}}$$

(6)

Assuming spatial wide-sense stationarity over small distances and particular excess delay times,

$$A(x_1; t) = A(x_2; t) = A(x; t)$$

(7a)

and

$$\hat{r}_{sa}(x_1; x_2; t) = \hat{r}_{sa}(x_1; x_2; t) = \hat{r}_{sa}(x_1; x_1; t) = \hat{r}_{sa}(x_1; t)$$

(7b)

The autocorrelation coefficient reduces to

$$\hat{r}_{sa}(x_1; t) = \frac{\text{E}[A(x_1; t) - A(x_1; t) (A(x_1 + \Delta x; t) - A(x_1; t))]}{\text{E}[A(x_1; t) - A(x_1; t)]^2}$$

(8)

This is defined as the autocorrelation coefficient with respect to distance for a constant excess delay within a local area. Typical second order statistics are computed by averaging the local area statistics over the measurement ensemble.

Only locations where multipath components exist are used in the autocorrelation coefficient estimate computation. This means that the correlation of multipath amplitudes is conditioned upon simultaneous path existence. Empirical data show that over small scale distances of 1 meter, multipath components at particular excess delays exist over the local area, although they may undergo fading [1,3]. In cases where no multipath components exist at a particular excess delay, the autocorrelation coefficient estimate for the particular excess delay interval is not included in the estimate of the average local autocorrelation coefficient function (ACCF). With only nineteen profiles, the maximum number of data pairs to average into the autocorrelation coefficient is 19-j for an integer separation of j profiles (Ax = j/4). The question then becomes: how many data pairs are necessary to obtain an accurate estimate of the true autocorrelation coefficient function? Data are presented for separations of up to ten profiles (2.5 Ax), but it must be kept in mind that our estimate of the ACCF is more accurate at small separations than larger ones. In addition to the fewer number of data pairs to average, the assumption of wide sense stationarity over distance becomes invalid. We will show for local areas in different factories and different T-R separations, the autocorrelation coefficient function can vary widely.

Our estimate of the autocorrelation coefficient for various separations and excess delays varies widely [9]. This is partially due to the small amount of data we have to compute the second order statistics over local areas. It also suggests that the spatial ACCF may be a random process for small changes in receiver location, in which case the channel is not wide sense stationary, even over small distances. Based on our data, autocorrelation coefficient functions at different measurement locations appear very different. This indicates that over large distances, the channel is not wide-sense stationary.
and that multipath components which exist at particular excess delays exist multipath amplitudes is jointly log-normal over space and excess delay, at those same excess delays over of the correlation coefficient to a statistical impulse response charmel model has been described, using the assumptions that the distribution of multipath profiles, which arrive at particular excess delay times. This is fortunate, since the conditional probability distribution of multipath existence is a power law and have signals which are completely characterized by first and second moments, and are simple to use in a statistical channel impulse response model. It appears that both spatial and temporal correlation may be a random process over local areas. This means that the indoor radio channel is not wide-sense stationary over distance or time. However, it is useful to assume that the channel is wide-sense stationary over local areas in order to compute the second-order (correlation) statistics, and apply them to a statistical model. On the average, multipath amplitudes are correlated over short distances (4/2) and for small excess delay time differences (< 50 ns), but one must remember that the autocorrelation coefficient function of multipath signal amplitudes may assume almost any value. Application of the correlation coefficient to a statistical impulse response channel model has been described, using the assumptions that the distribution of multipath amplitudes is jointly log-normal over space and excess delay, and that multipath components which exist at particular excess delays exist at those same excess delays over a space of 1 meter.

Figure 9. Conditional probability of path occupancy for LOS topographies.

IV. JOINT PROBABILITY OF PATH OCCUPANCY

The temporal autocorrelation coefficient function, described in section III, is defined under the assumption that multipath components exist for both excess delay times \( \tau_1 \) and \( \tau_2 \). Application of the autocorrelation coefficient in the channel model requires knowledge of the conditional probability of path occupancy, where the existence of a multipath signal at excess delay \( \tau_2 \) is conditioned upon the existence of a signal at \( \tau_1 \) (\( \tau_1 < \tau_2 \)).

Figure 9 shows the probability of path occupancy at time \( \tau_2 \) given that a path exists at time \( \tau_1 = 0 \) ns for LOS topographies. It is apparent from the figure that for a low threshold (48 dB below level received at 10 T-R separation), the probability can be modeled as a linear function of time difference \( \tau_2 - \tau_1 \). As \( \tau_2 \) increases, however, the probability approaches an exponential decay [9]. This implies that power delay profiles are not wide-sense stationary for small values of \( \tau_1 < 100 \) ns, since the conditional probability distribution of multipath existence is a function of excess delay. The decay constants [9] are very similar for \( \tau_2 \) greater than 100 ns which suggests that power delay profiles may be temporally wide-sense stationary for \( \tau_1 \) greater than 100 ns. Observations similar to LOS topographies are made for obstructed topographies [9]. Figure 9 suggests that adaptive power control at the receiver or transmitter is desirable to reduce the likelihood of receiving multipath components, as long as sufficient SNR can be maintained for the direct (LOS) signal.

By using antenna diversity and or equalization to improve the instantaneous SNR of the desired signal component, in conjunction with power control to reduce intersymbol interference (ISI) due to multipath, it is likely that increased data rate transmission can be supported by indoor radio channels.

V. CONCLUSION

Although the indoor radio channel cannot be completely characterized by only first and second-order statistics, they provide a more complete model of multipath propagation inside buildings than has been previously available [8]. We have shown that multipath components which arrive at particular excess delays have amplitudes which attenuate according to some mean \( d \) power law and have signals which are log-normally distributed about the mean. Also, we showed that a log-normal distribution is a good model for multipath signal amplitudes over local areas for particular excess delay times. This implies that power delay profiles may be temporally wide-sense stationary for \( \tau_1 \) greater than 100 ns. Observations similar to LOS topographies are made for obstructed topographies [9]. Figure 9 suggests that adaptive power control at the receiver or transmitter is desirable to reduce the likelihood of receiving multipath components, as long as sufficient SNR can be maintained for the direct (LOS) signal.

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VI. REFERENCES