Solution to HWS 5

4.11

Since we have \( |E_{\text{tot}}(d)| = \frac{E_0 d_0 \sqrt{2-2 \cos \theta_0}}{d} \)

from the equation we can clearly figure out that

when \( \theta_0 = 2n\pi \), where \( n \) is an arbitrary integer

\[ |E_{\text{tot}}(d)| = \left| \frac{E_0 d_0 \sqrt{2-2 \cos \theta_0}}{d} \right| = 0 \] (Notice here nulls mean the signal vanishes)

and recall the equation

\[ \theta_0 = \frac{2\pi}{\lambda} \frac{2h\text{thr}}{d} = 2n\pi, \text{ thus} \]

\[ d = \frac{2h\text{thr}}{n\lambda} \]

4.12

For 4.47 we have

\[ |E_{\text{tot}}(d)| = \frac{E_0 d_0 \sqrt{2-2 \cos \theta_0}}{d}, \text{ and} \]

\[ P_Y(d) = \frac{|E_{\text{tot}}(d)|^2 A_e}{120\pi}, \text{ while we also have} \]

\[ P_Y(d) = \frac{E_0^2 A_e}{120\pi} = \frac{P_t G_t G_r \lambda^2}{(4\pi d_0)^2} \implies E_0^2 = \frac{P_t G_t G_r \lambda^2}{4\pi d_0^2 A_e} \text{ while } G_t = G_r = 1 \]

thus

\[ P_Y(d) = \frac{E_0^2 d_0^2}{120\pi} (2-2 \cos \theta_0) A_e = \frac{32 P_t G_t \lambda^2}{16\pi d_0^2 A_e} (2-2 \cos \theta_0) \]

For \( f = 1800 \text{ MHz} \), \( \lambda = \frac{c}{f} = 0.167 \text{ m} \) thus \( \theta_0 = \frac{2\pi}{\lambda} \frac{2h\text{thr}}{d} \approx \frac{9029.75}{d} \)

For 4.52

\[ P_Y(d) = \frac{P_t G_t G_r h^2}{d^2} = \frac{14400}{d^2} P_t \]

The comparison will be given in Matlab code.
4.13

This question is similar to 4.12.

For 4.47

\[ P_r(d) = \frac{|E_{0} \cdot (zd)|^2 \cdot A_e}{12 \pi^2} \]

\[ E_{0} = \frac{P_0 \lambda}{4 \pi d^2} \cdot A_e \]

\[ P_r(d) = \frac{P_0 \lambda^2}{16 \pi^3} \left( 2 + 2 \cos \theta \right), \quad \theta \approx \frac{90 \pm 75}{2} \]

For 4.52

\[ P_r(d) = P_t G_t G_r \frac{h_1^2 h_2^2}{d^4} = \frac{14400}{d^4} \]

The comparison will be given in Matlab code.

4.19

From the figure we have \( h = 400 \text{ m}, \ h_1 = 60 \text{ m}, \ h_2 = 5 \text{ m} \), thus

\[ \tan \beta = \frac{400 - 60}{3000} \approx 0.1133 \]

\[ \tan \gamma = \frac{400 - 5}{2000} \approx 0.1975 \]

\[ \beta = \arctan 0.1133 \approx 0.1129 \text{ rad}, \quad \gamma = \arctan 0.1975 \approx 0.1950 \text{ rad} \]

\[ \lambda = \beta + \gamma \approx 0.308 \text{ rad} \]

Then

\[ v = v = \lambda \frac{dA}{dA} \approx 0.308 \sqrt{\frac{2 \times 6 \times 10^{38}}{500}} \approx 26.14 \]

Since \( v > 2.4 \), by use eqn 4.51.e

\[ R_l \cdot d_l (\text{dB}) = 20 \log \left( \frac{0.225}{v} \right) \approx -41.30 \text{ dB} \]

\[ P_l d (\text{dB}) = -R_l d (\text{dB}) = 41.30 \text{ dB} \]

The free-space received power should be

\[ P_{\text{free}}(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2} = \frac{10 \cdot 10 \cdot 2 \cdot \left( \frac{1}{15} \right)^2}{(4\pi)^2 \cdot (5000)^2 \cdot 1.25} \approx 4.50 \times 10^{-9} \text{ W} \approx 53.46 \text{ dBm} \]

Actual power should be,

\[ P_r(d) = P_{\text{free}}(d) + R_l d (\text{dB}) = -53.46 - 41.30 \text{ dB} = -94.76 \text{ dBm} \]

The effect of diffraction will introduce additional path loss.
Transmitter power is assumed 1W
Transmitter power is assumed 1W

- $p$ (in dBm)
- $d$

- Red line: Horizontal polarization exact value
- Blue line: Approximation