Lecture Notes (04/02/2015)

Review of Natural Response (Transient Response) \(\Rightarrow\) eventually convert all stored energy into heat.

Source - Free RL Circuit
\[
\dot{i}(t) = i(0^-) - \frac{E}{L} t = I_0 e^{-\frac{t}{\tau}}
\]
- Time constant \(\tau = \frac{L}{R}\)
- \(\tau\) \(\uparrow\) \(\Rightarrow\) decay slower
- \(\tau\) \(\downarrow\) \(\Rightarrow\) decay factor

Source - Free RC Circuit
\[
V(t) = V_0 e^{-\frac{t}{\tau}} = V_0 e^{-\frac{t}{RC}}
\]
- \(\tau = RC\)

Review of Problem with Switch (find \(t > 0\))

\(\Delta\) \(i_L\), \(V_C\) can not change instantaneously \(\Rightarrow\) find other values in circuit
\(\Leftrightarrow\) \(i_L(0^-) = i_L(0^+)\)
\(V_C(0^-) = V_C(0^+)\)

\(\Delta\) How to find \(\tau\)

\[
\tau = \frac{L}{R_{eq}} \quad \text{or} \quad \tau = R_{eq} C
\]
(for \(t > 0\))
- \(R_{eq} = R_1 + R_2 = 500 \Omega\)
- \(R_{eq} = \frac{40}{40 + 10} = 30 \Omega\)
- \(L_{eq} = 5 + 10 = 15\)

Review of calculating energy
\(\triangleleft\) Energy stored in capacitor / inductor \(\Delta W_C(t) = \frac{1}{2} C (V_C(t) - V_C(0)) + \Delta W_L(t)\)
\(\triangleleft\) Energy turned into heat in resistor \(\Delta W_R = \int_0^t p_{avg} dt = \frac{1}{2} R I^2 t\)
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Force Response (steady-state)

Driven RL Circuits

Using KVL

\[ V_o u(t) - iR - L \frac{di}{dt} = 0 \]

\[ \Rightarrow \dot{i} = \left( \frac{V_o}{R} - \frac{V_o}{R} e^{-\frac{Rt}{L}} \right) u(t) \]

\[ = \frac{V_o}{R} \left( 1 - e^{-\frac{Rt}{L}} \right) u(t) \]

\[ i(0^+) = \left( \frac{1}{1000} \right) = 12 \text{ mA} \]

\[ i(t-3) = \dot{i}(0^+) \left( 1 - e^{-\frac{R(t-3)}{L}} \right) u(t-3) \]

\[ = (12 - 12 e^{-\frac{1000(t-3)}{3000}}) u(t-3) \text{ mA} \]

\* The circuit will eventually assume the force response

\[ \text{Transient period: initial} \Rightarrow \text{final} \]
\[
i = i_n + i_f \\
i_n = A e^{-\frac{R}{L} t} \quad (A \neq i(0)) \\
i_f = \frac{V_0}{R} \\
i = Ae^{-\frac{R}{L} t} + \frac{V_0}{R} \\
\text{plug in the initial condition} \\
A + \frac{V_0}{R} = 0 \\
i(t) = \frac{V_0}{R} \left(1 - e^{-\frac{R}{L} t}\right)
\]

**Problem 2.**

**EX 2.**

\[56V \quad \begin{array}{c}
\downarrow \\
520 \Omega \\
220 \Omega \\
100 \Omega \\
\downarrow \\
\hline
\end{array} \quad \text{\(v(t)\)}
\]

1. Find \(L\)
\[L = \frac{L}{R_{eq}} = \frac{3}{6/2} = 2\Omega
\]

2. Find \(i_n\)
\[i_n = ke^{-\frac{t}{L}}
\]

3. Find \(i_f\)
\[i_f = i(\infty) = \frac{100}{2} = 50A \\
i = i_n + i_f = 50 + ke^{-\frac{t}{L}}
\]

4. Find initial value \((i_n(0) = i_n(0^+))\)
\[i_n(0) = i_n(0^+) = \frac{50}{2} = 25 \\
25 = 50 + k \\
k = -25
\]
Procedures of finding complete response

<1> Draw Circuit for $t < 0 \ (0^-) \ , \ t > 0 \ (0^+)$

<2> Find $T$ (zero output source (short voltage source, open current source))
   - Find equivalent $L_{eq} \ \text{Req} \ \text{C}_{eq}$
   - $T = \frac{L_{eq}}{\text{Req}}$ or $T = \text{Req} \ \text{C}_{eq}$

<3> Find $i(0^-)/v_c(0^-)$, use dc analysis (short inductor, open capacitor)

<4> Find $i(\infty)/v_c(\infty)$, use dc analysis (short inductor, open capacitor)

<5> Find $i(10^+)/v_c(10^+)$, use $i(10^-) = i(0^+)$, $v_c(10^-) = v_c(0^+)$

<6> Find complete response: $i(t) = i(\infty) + [i(0^+) - i(\infty)] e^{-\frac{t}{T}}$

Note: If you are asked to find $i(t) \ \text{or} \ v(t)$, etc. Use basic circuit analysis & property of capacitor & inductor to find from (in $i \rightarrow i(t)$ etc.)