Two passive circuit elements (store & deliver finite amounts of energy)

Capacitor
\[
\int v(t)\,dt = i(t)
\]

Inductor
\[
\int i(t)\,dt = v(t)
\]

U-I Relationship
\[
i(t) = C \frac{dv(t)}{dt}
\]
\[
v(t) = \frac{1}{C} \int_{t_0}^{t} i(t')\,dt' + v(t_0)
\]

Power
\[
p = vi = v \frac{dv}{dt}
\]

Energy
\[
w_e(t) - w_e(t_0) = \frac{1}{2} C \{v(t)^2 - v(t_0)^2\}
\]
\[
w_e(t) = \frac{1}{2} C v^2
\]

in series
\[
C_{eq} = C_1 + C_2 + \ldots + C_n
\]

in parallel
\[
C_{eq} = \frac{1}{1/C_1 + 1/C_2 + \ldots + 1/C_n}
\]

Derivative & Integral
\[
\frac{d}{dt}(c) = 0
\]
\[
\frac{d}{dt}(t) = 1
\]
\[
\frac{d}{dt}(t^n) = n t^{n-1}
\]
\[
\frac{d}{dt}(e^x) = e^x
\]
\[
\frac{d}{dt}(\sin x) = \cos x
\]
\[
\frac{d}{dt}(\cos x) = -\sin x
\]
\[
(fg)' = fg' + fg' \\
\text{(product rule)}
\]
\[
\frac{d}{dt}(te^t) = e^t + te^t
\]
\[
\frac{d}{dt}((t-t)e^{-t}) = te^{-t}
\]

Capacitor & Inductor for DC circuits

If DC \( v(t) = \text{const} \)
\[
i(t) = C \frac{dv(t)}{dt} = 0
\]

If DC \( i(t) = \text{const} \)
\[
v(t) = L \frac{di(t)}{dt} = 0
\]

open circuit

Short circuit
Chapter 8. Basic RL & RC Circuits

8.1 Source-Free RL Circuit

Solution of differential equation

\[ -V_R + V_L = iR + V_L = iR + L \frac{di}{dt} = 0 \]

\[ \Rightarrow \frac{di}{dt} + \frac{R}{L} i = 0 \]

Initial condition: \( i(0) = I_0 \)

(1) Direct Approach (separate variables & integrating)

\[ \frac{di}{i} = -\frac{R}{L} dt \]

\[ \int_{i_0}^{i(t)} \frac{di'}{i'} = \int_0^t -\frac{R}{L} dt' \]

\[ i(i) |_{i_0} = -\frac{R}{L} (t-0) \]

\[ i(t) = I_0 e^{-\frac{R}{L} t} \rightarrow \text{(check } i(0) = I_0) \]

\[ \text{satisfy differential equation} \]

\[ \begin{align*}
R &= 200 \Omega \\
L &= 50 \text{ mH}
\end{align*} \]

\[ i_L(t) = 2 \ e^{-\frac{200}{500} t} \]
2. An Alternative Approach

\[ \int \frac{di}{i} = - \int \frac{R}{L} \, dt + k \]

initial condition

\[ \ln I_0 = k \]

\[ i(t) = I_0 e^{-\frac{R}{L} t} \]

3. A more general solution approach (intuition/experience)

Assume a general solution with unknown constant

\[ i(t) = Ae^{st} \]

Substitute the assumed solution into differential equation

\[ (s_1 + \frac{R}{L}) Ae^{st} = 0 \]

\[ s_1 + \frac{R}{L} = 0 \]

\[ s_1 = -\frac{R}{L} \]

\[ i(0) = A = I_0 \]

Final form of solution

\[ i(t) = I_0 e^{-\frac{R}{L} t} \]

4. Characteristic Equation

\[ s_1 + \frac{R}{L} = 0 \]

if \( a \frac{df}{dt} + bf = 0 \)

\( \Rightarrow (as + bf) = 0 \)

\( \Rightarrow as + bf = 0 \)

characteristic equation \( \Rightarrow \) signal root \( s = -\frac{b}{a} \)

\( \Rightarrow \) solution \( f = Ae^{-\frac{b}{a} t} \)
Example 8.2 \( P_{dc} \)

\[
\begin{align*}
V(0, t) & \rightarrow \text{open} \\
\text{Switch closed} & \quad \text{Connect for a long time (b)} \\
\text{Switch open} & \quad (c)
\end{align*}
\]

(b) \[ \begin{align*}
\text{DC} \\
- V + 10 i(t) + 5 \frac{di(t)}{dt} &= 0 \\
\Rightarrow \quad i(t) &= \frac{-V}{10} \\
V(t) &= 40 \times (-2.4) = -96 V
\end{align*} \]

\[ i_L(10) = \frac{-96}{10} = -9 A \]

(c) \[ \begin{align*}
\text{Dc} \\
-50 i(t) + 5 \frac{di(t)}{dt} &= 0 \\
\Rightarrow \quad i_L(t) &= I_0 e^{-\frac{R}{L} t} \\
\Rightarrow \quad V(t) &= 2.4 e^{-\frac{R}{L} t}
\end{align*} \]

Energy

Power

\[ P_R = i^2 R = I_0^2 R e^{-2\frac{R}{L} t} \]

\[ W_R = \int_0^\infty P_R \, dt = \frac{1}{2} R \int_0^\infty e^{-2\frac{R}{L} t} \, dt = \frac{1}{4} L I_0^2 \]
\[ i(t) = I_0 \ e^{-\frac{t}{\tau}} = I_0 \ e^{-\frac{4}{\tau}} \]

\[ \frac{d}{dt} \left( \frac{i}{I_0} \right) \bigg|_{t=0} = \frac{-k}{\tau} \]

\( \left( \frac{k}{L} \right) \tau = 1 \Rightarrow \tau = \frac{L}{k} \) (5) \( \leftarrow \) time constant

- Time for \( \frac{i}{I_0} \) drop from unity to zero
- Constant rate of decay

\[ t = \tau \quad \frac{i}{I_0} = 0.368 \]

\[ t = 0 \quad \frac{i(0)}{I_0} = 1 \Rightarrow \frac{i}{I_0} = 1 \]