3/4/2015  Chapter 7  - Inductors + Capacitors

- Capacitors C + Inductors L

- L's & C’s can store & deliver finite amounts of energy.

More into the realm of active vs. passive elements.

- Active elements supply energy and can supply an average power greater than 0 over an infinite time interval.

\[ C \text{ is defined by a V-I relationship} \]

\[ \frac{dQ}{dt} = \frac{\Delta Q}{\Delta t} = \frac{dV}{dt} = \frac{d}{dt} \left( \frac{\Delta Q}{\Delta t} \right) \]

\[ \text{Capacitance} = \frac{\text{charge}}{\text{voltage}} \]

\[ C = \frac{Q}{V} \]

\[ \text{Capacitance is an Ampere-second per Volt, or Coulomb per Volt} \]

\[ \text{which is equivalent to Farad (F)} \]

\[ \text{Capacitance = Farad} \]

\[ \frac{d}{dt} \text{ charge on plate as time} \]

\[ \frac{\text{charge}}{\text{area}} \]

\[ \frac{\text{charge}}{\text{time}} \]

\[ \text{two parallel conducting plates} \]

\[ \text{plates have area} A \text{ separated by insulator} \]

\[ \text{conductors} w/ \varepsilon = \text{permittivity} \]

\[ \text{separated by distance} d \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \]

\[ \text{C of several hundred } \mu\text{F is very large} \]

\[ \text{Example 7.1} \]

\[ d = C \frac{dv}{dt} \]

\[ \text{a) } V = 8 \times 5 \frac{dv}{dt} = 0 \text{ A} \]

\[ \text{b) First wave is a sine, derivative is a cosine,} \]

\[ \text{multiplied by } 2 \text{ w/ some frequency.} \]

\[ \text{Practice 7.1} \]

\[ \text{Determine } I \text{ if } V = 2e^{-5t} \text{ V w/ } C = 5 \mu\text{F} \]

\[ I = C \frac{dv}{dt} = 5 \times 10^{-3} \times 2e^{-5t} \]

\[ = 5 \times 5 \times 10^{-3} \times 2e^{-5t} \]

\[ = 50e^{-5t} \text{ mA} = \text{ib(c)} \]

\[ \text{ampere} \]

\[ \text{milliampere} \]
\[ V(t) = \int \frac{dv}{dt} = \int v(dt) = \frac{1}{2} v(t^2) \]

Integrate both sides between \( t = 0 \) and \( t = t' \):

\[
\int_{v(0)}^{v(t')} \frac{dv}{dt} = \int_{0}^{t'} v(t') \, dt'
\]

\[
[v(t') - v(0)] = \frac{1}{2} \int_{0}^{t'} v(t') \, dt'
\]

\[
v(t') = \frac{1}{2} \int_{0}^{t'} v(t') \, dt' + v(0) \quad \text{or} \quad v(t') = \frac{1}{2} \int_{0}^{t'} v(t') \, dt' + k
\]

In many cases, \( v(0) \) is not discernible and usually:

\[
t \
\begin{array}{c}
t \\
\rightarrow -\infty
\end{array}
\]

such that:

\[
v(t) = \frac{1}{2} \int_{-\infty}^{t} v(t') \, dt'
\]

Again, \( q(t) = \int v(t) \, dt \)

**Example 7.2**

\[
solve \quad q(t) = \text{constant}
\]

\[
\begin{array}{ll}
\text{Interval (a)} & v(t) = \begin{cases} 
0 & 0 \leq t \leq 3s \\
2000 & 3s \leq t \leq 5s
\end{cases}
\end{array}
\]

\[
\begin{array}{ll}
\text{Interval (b)} & v(t) = 0 \\
\end{array}
\]

\[
\begin{array}{ll}
\text{Interval (c)} & v(t) = 4000 \quad t \leq 3s
\end{array}
\]

\[
\text{Find } q(t): \quad \frac{1}{2} \int_{0}^{3s} 2000 \, dt = \frac{1}{2} \times 20 \times 10^{-3} \times 3
\]

\[
= V(t) = 4000 \quad 0 \leq t \leq 3s
\]

\[
\text{Interval (c)}: \quad v(t) = v(3s) = 4000 \times 3s = 12000 \quad V = 2 \times 3s = 6
\]

\[
\therefore v(t) \text{ at } t = \infty \approx 6 \times 10^{-8} = 8V
\]
Energy Storage: \( p = \dot{v} \cdot \dot{w} = Cv \frac{dv}{dt} \Rightarrow p(t) = Cv(t) \)

Change in Energy over time: \( \int_{t_0}^{t} p(t) \, dt = \int_{t_0}^{t} v \cdot \dot{w} \, dt \)

\( \frac{dE}{dt} = \int_{t_0}^{t} v \cdot \dot{w} \, dt \)

\( \begin{align*}
\frac{dE}{dt} &= \frac{1}{2} \left[ \left( \frac{v(t)}{t} \right)^2 - \left( \frac{v(t_0)}{t} \right)^2 \right] \\
&= \frac{1}{2} \left[ \frac{v(t)}{t} - \frac{v(t_0)}{t} \right]^2
\end{align*} \)

If energy \( e(t) = 0 \) then we have

\[ W(t) = \frac{1}{2} Cv^2 \]

**Ideal Capacitor**

- No current if voltage is not changing with time.
- Open circuit to DC.
- Finite amount of energy can be stored across the capacitor (current zero, voltage constant).
- Capacitor cannot change voltage instantaneously, but current through can.
- Storage of energy does not dissipate energy.

**Inductor**

L is also defined by a voltage - current relationship

\[ v = L \frac{di}{dt} + v_i = \text{H} \]

Inductor is modelled by winding a wire into a coil where inductance is determined by:

\[ L = \frac{\pi N^2 A s}{\mu_0} \]

- \( N \): number of complete turns of wire
- \( A \): cross-section area
- \( s \): the axial length of the helix
- \( \mu_0 \): magnetic permeability, \( \mu_0 = 4\pi \times 10^{-7} \text{H/m} \)
Voltage across an inductor is related to the rate of change of the current through it.

No voltage across an inductor yields a short circuit when current is constant.

Instantaneous changes in voltage is possible, but not current through an inductor.

Example 7.4 \[ V = \frac{L}{dC} \]

\[ \int_{dC}^{dC'} = \frac{1}{L} \int_{0}^{c} \frac{dC}{dC} dC' \]

\[ i(t) = \frac{1}{L} \int_{0}^{c} v(t) dt + v(t_0) \]

\[ i(t) = \frac{1}{L} \int_{0}^{c} v(t) dt + i(t_0) \]

Example 7.4 \[ V(t) = V_{-oo} = 0, \quad i(t) = \frac{1}{L} \int_{0}^{c} v(t) dt \]

\[ i(t_0) = \frac{1}{L} \int_{t_0}^{c} (\cos 5t dt + v(t_0)) \]

\[ = \frac{1}{2} \left[ \frac{v(t_0)}{L} \right] + \frac{1}{2} (\cos 5t_0 + v(t_0)) \]

\[ = 0.1 \sin 5t - 0.2 \sin 5t_0 + i(t_0) \]

Plug in \[ -\frac{V}{2} \] for \[ t_0 = \left( \frac{V}{5} \right) \]

\[ i(t_0) = 1A \]
\[ \phi(t) = \text{Ode } \sin 5t - 0.4 \text{ sin } 5t \] 

\[ = 0.6 \text{ sin } 5t + 1 \text{ = ac } t \]

**Energy Storage**

\[ p = VI = LI \frac{\text{d}i}{\text{d}t} \]

Looking for energy we get:

\[ P \text{ dt} = L \int i^2 \text{dt} \]

\[ = \text{wL}(t) - \text{wL}(t_0) = \frac{1}{2} L \left[ (\text{L}(t))^2 - (\text{L}(t_0))^2 \right] \]

\[ = \frac{1}{2} \text{L}(t) \left[ (\text{L}(t))^2 - (\text{L}(t_0))^2 \right] + \text{wL}(t_0) \]

If energy is 0 at time 0, \[ \text{wL}(t) = \frac{1}{2} 4L^2 \]

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**Ideal Inductor**

- No voltage across inductor if current is not changing w/ time
  
  (Short-circuit to DC)

- Finite amount of energy can be stored in inductor, even if voltage across is zero

- Current cannot change instantaneously, voltage can

- Inductors do not dissipate energy, they only store it.

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**Series & Parallel**

- L's in series act like R's in series
- L's in parallel act like R's in parallel
- C's in series act like R's in parallel
- C's in parallel act like R's in series