Supernode: when a voltage source is common to two nodes (non-reference, independent & dependent)

Supermesh: when a current source is common to two meshes

\[ V_1 = 3V \]

At the 2-3 supernode
\[ 4 = \frac{V_2 - V_1}{3} + \frac{V_3 - V_4}{4} + \frac{V_5 - 0}{2} \]

At node 4
\[ 0 = \frac{V_2 - V_1}{3} + \frac{V_4 - 0}{3} + \frac{V_4 - V_3}{4} \]

By inspection, \( i_4 = 2A \) (not common to two meshes)

HWS Problem 4_45

Current sources are common to two meshes

Supermesh for a large loop:
\[ -I = 2i_1 + 3i_2 + 7(i_2-i_4) + 8(i_1-i_4) \]
\[ i_2 - i_1 = 1 \]
\[ i_1 - i_3 = 3 \]

We don't know the voltage across the current source.

We cannot write KVL equations.
Superposition: 1) in a linear circuit, the response (voltage/current), can be obtained by adding the response caused by separate independent source acting alone.

2) Look at each independent source one at a time. With the other independent source "turned off" or "zeroed out".

3) The dependent sources are active.

Example: Practice 5.2 P29

Use nodal analysis:

\[
\begin{align*}
Z &= \frac{V_1'}{1} + \frac{V_1' - V_2'}{15} \\
4i' &= \frac{V_1'}{5} + \frac{V_2' - V_1'}{15} \\
V_1'' &= \frac{3}{7} \left( \frac{V_1''}{5} \right) + \frac{V_2'' - V_1''}{15} \\
i'' &= \frac{V_2''}{5} \\
V_1 &= V_1' + V_1'' \\
V_2 &= V_2' + V_2''
\end{align*}
\]
Source Transformation

Equivalent Practical source.

\[ R_s = R_p \]

\[ V_s = R_p I_s = R_s I_s \]

\[ \text{Direction: } \rightarrow + \]

\[ \text{eq. P13.8 Practice 5.4} \]

\[ R_L \text{ load} \]

\[ V = 12 \times \frac{1}{2} = 6V \]

\[ 10 + 6 - 4 = 12V \]

\[ 10 + 3 + 3 = 16.5V \]

\[ \text{Voltage source in series: } V_s = V_1 + V_2 + V_3 \]

\[ \text{Current source in parallel: } I_s = I_1 + I_2 + I_3 \]

\[ \text{Goal: end up with either all current source/voltage source} \]

\[ \text{The resistor value does not change during source transformation} \]

\[ \text{but is not the same resistor} \]

\[ \text{If voltage/current associated with controlling variable of dependent source} \]

\[ \text{should not be included in source transformation} \]

\[ \text{If the voltage/current is of interest, should not be included} \]
Thevenin/Norton equivalent circuit I

- Simplify the analysis of linear circuit (replace a complex network)

How to get V_{TH}, I_{TH}, R_{TH}?

1. Use source transformation. (when there is no dependent source)

   \[
   \begin{align*}
   &\text{Example:} \\
   &\begin{array}{c}
   12V \\
   3 \quad \frac{1}{3} \\
   3 \quad \text{RL}
   \end{array} \\
   &\Rightarrow \begin{array}{c}
   3A \\
   3 \quad \frac{1}{3} \\
   3 \quad \text{RL}
   \end{array} \\
   &\Rightarrow \begin{array}{c}
   36V \\
   3 \quad \frac{1}{3} \\
   3 \quad \text{RL}
   \end{array}
   \end{align*}
   \]

2. Thevenin Theorem
   - Find V_{OC}, (disconnect RL)
   - Find R_{TH} in current (zero out independent source)

   \[
   \begin{align*}
   &\text{Example:} \\
   &\begin{array}{c}
   12V \\
   3 \quad \frac{1}{3} \\
   3 \quad \text{RL}
   \end{array} \\
   &\Rightarrow \text{V}_{OC} = 12 \times \frac{6}{6+6} = 6V \\
   &\Rightarrow R_{TH} = 6||6 + 3 = 6\Omega
   \end{align*}
   \]

3. Norton's Theorem
   - Find I_{SC} (short RL)
   - Find R_{TH} (zero out independent source)

   \[
   \begin{align*}
   &\text{Example:} \\
   &\begin{array}{c}
   12V \\
   3 \quad \frac{1}{3} \\
   3 \quad \text{RL}
   \end{array} \\
   &\Rightarrow I_{SC} = \frac{12}{3\times6+6} \times \frac{6}{6+3} = 1A \\
   &\Rightarrow R_{TH} = 6||6+3 = 6\Omega
   \end{align*}
   \]
4. Thevenin / Norton theorem II

Find \( V_{oc}, I_{sc} \Rightarrow R_{th} \)

\( \text{(dependent source & independent source)} \)

\[ V_X = V_{oc} \]

\[ -4 + 2x\left(\frac{V_X}{4}\right) + 3 \times 0 + V_X = 0 \]

\[ \Rightarrow V_X = V_{oc} = 8 \text{ V} \]

\[ I_{sc} = \frac{4}{2+3} = 0.8 \text{ A} \]

\[ R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{8 \text{ V}}{0.8 \text{ A}} = 10 \Omega \]

Answer:

\[ \Rightarrow 8 \text{ V} \]

5. Add Test Source (only dependent source)

\[ \text{Ex. 3. P49. Ex. 5.10} \]

No independent source, both \( V_{oc}, I_{sc} = 0 \)

\[ \text{Add 1 A test source. } \Rightarrow R_{th} = \frac{V_{test}}{1} \]

\[ \therefore i = -1 \]

\[ \text{Nodal analysis: } i = \frac{V_{test} - 1.5 \Omega}{3} + \frac{V_{test}}{2} \]

\[ V_{test} = 6 \text{ V} \]

\[ R_{th} = 6.5 \Omega \]
Maximum Power Transfer

\[ P_{\text{max}} = \frac{V_s^2}{4R_s} = \frac{V_{\text{TH}}^2}{4R_{\text{TH}}} \]

Δ-Y Conversion

\[ P_A = \frac{P_1 R_2 + P_2 R_3 + P_3 R_1}{R_2} \quad \text{if } R_1 = R_2 = R_3 = R \]
\[ P_A = 3R \]
\[ R_A = 3R_Y \]

\[ P_B = \frac{P_1 R_2 + P_2 R_3 + P_3 R_1}{R_3} \]

\[ P_C = \frac{P_1 R_2 + P_2 R_3 + P_3 R_1}{R_1} \]

\[ R_1 = \frac{P_A R_B}{R_A + R_B + P_C} \quad \text{if } P_A = P_B = P_C = R \]
\[ R_A = \frac{R}{3} \]
\[ R_Y = \frac{1}{3} R_A \]

Ex1: \[ P_{156} \quad \text{Ex 5.12} \]

\[ R = 60 + 20 + 20 + 30 \]
\[ = 135 \Omega \]