**Units**

**Base (or fundamental) units:** units of mass, length, time, temperature, current, light intensity. All other units can be expressed as combinations of these fundamental units. For example, force (e.g. Newtons) is equal to mass length/time² (e.g. kg m/s²).

**Derived or compound units:** Units such as Newtons (N) that are combinations of fundamental units. Sometimes, these combinations are given their own names (example N, erg, watt).

**Systems of units:** Main systems of units in use are the International System of Units (SI), the centimeter-gram-second system of units (cgs), and the American engineering system of units (AES). The fundamental units in these systems are as follows:

<table>
<thead>
<tr>
<th></th>
<th>SI</th>
<th>cgs</th>
<th>AES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>kg</td>
<td>g</td>
<td>lb m</td>
</tr>
<tr>
<td>Length</td>
<td>m</td>
<td>cm</td>
<td>ft</td>
</tr>
<tr>
<td>Time</td>
<td>s</td>
<td>s</td>
<td>s</td>
</tr>
</tbody>
</table>

Often, prefixes are attached to a unit to indicate scale or magnitude.

- peta \(10^{15}\)
- tera \(10^{12}\)
- giga \(10^9\)
- mega \(10^6\)
- kilo \(10^3\)
- deka \(10^1\)
- deci \(10^{-1}\)
- centi \(10^{-2}\)
- milli \(10^{-3}\)
- micro \(10^{-6}\)
- nano \(10^{-9}\)
- pico \(10^{-12}\)
- femto \(10^{-15}\)
- atto \(10^{-18}\)

For example: 1 kilometer = 1000 meters, 1 microliter = \(10^{-6}\) liters.

**Mathematical manipulation of dimensioned quantities**

1. Two quantities can only be added or subtracted if they have the same units. This also means that equations must be dimensionally homogeneous – that is, all additive terms in an equation must have the same units. (see Example 2.6-1)
2). If multiplying or dividing two quantities, the units do not have to be the same. The units from the quantities being multiplied or divided likewise multiply or divide. Thus

- 1 m divided by 2 s yields 0.5 m/s.
- 1 kg multiplied by 9.8 m/s² yields 9.8 kg m/s².

If two quantities being divided have the same units, or two quantities being multiplied have reciprocal (e.g. m and m⁻¹) units, the units cancel and the result will be **dimensionless**. As you’ll see later on, dimensionless quantities play an important role in chemical engineering as they simplify calculations and even help minimize the number of measurements that need to be taken to characterize the behavior of a chemical or biological system. They are used extensively in the scale-up of processes and in similitude analysis.

3). Exponents, transcendental functions (i.e. trigonometric, logarithmic, and exponential functions), and arguments of transcendental functions are dimensionless (recall: what is an argument to a function?).

4). The conversion of units from one to another (e.g. meters to inches, pounds mass (lbm) to kilograms, seconds to minutes) is accomplished through the use of conversion factors. Our text lists the most common conversion factors on the inside of the front cover. (see Example 2.3-1)

**Force and Weight**

Newton’s 2nd law of motion states that force \( F \) on an object equals the object’s mass \( m \) times its acceleration \( a \). This statement is the definition of force \( F \):

\[
F = ma/g_c
\]  
(1)

For equation 1 to be dimensionally correct, the conversion factor \( g_c \) MUST be included. In SI units \( g_c = 1 \) kg m/s²·N, in the cgs system \( g_c = 1 \) g cm/s²·dyne, and in AES \( g_c = 32.174 \) lbm ft/s²·lbf.

The weight of an object is the force on it due to gravity,

\[
W = mg/g_c
\]  
(2)

where \( g \) is the gravitational acceleration. Although \( g \) varies with the distance and the mass of the two objects interacting, on earth’s surface it can be taken as constant to a very good approximation. At sea level and 45° latitude the value of \( g \) is \( g = 9.8066 \) m/s² (SI) = 32.174 ft/s² (AES).
**Numerical Representation**

**Scientific Notation:** The expression of a number as the product of another number and a power of 10. For example,

\[ 132,576 = 1.32576 \times 10^5 \]

Above, the number 132,576 was rewritten into scientific notation with five decimal places. We could also have written it as, for example, \( 1.33 \times 10^5 \), in which case we would have lost three **significant figures**.

**Significant figures:** The number of digits from the first nonzero digit on the left of a number to either:

1). if there is a decimal point, the last digit on the right of the number
2). if there is not a decimal point, the last nonzero digit of the number

How many significant digits are there in the following numbers?

- \( 1.334 \times 10^3 \)
- -100
- -100.
- 143.540

For measured or for calculated quantities, the number of significant figures indicates precision. The last (i.e. rightmost) significant digit of a number could be off by as much as half a unit for that decimal position.

When a number is altered by mathematical operations, the number of significant digits in it may change. If two (or more) numbers are multiplied or divided, the number of significant digits in the result is equal to the lowest number of significant digits in the values being multiplied or divided. When two (or more) numbers are added or subtracted, the decimal position of the last (i.e. rightmost) significant digit in the result is determined as follows. The values being added or subtracted are compared, and the one whose last significant digit is in the leftmost decimal position is determined. The decimal position of the last significant digit in the result is the same as this position.

**Examples:**

- \( 5.0 + 7.33 = ? \)
- \( 1.0001 - 1.000 = ? \)
- \( 1.0001 - 1.0000 = ? \)
- \( 573.5/254 = ? \)
- \( 33.5*2.5 = ? \)

**Rounding off:** the convention in our text is to round to the nearest integer in the preceding decimal position (e.g. \( 1.23 \rightarrow 1.2 \); \( 76.57 \rightarrow 76.6 \)). If the number being rounded off is a 5, the convention is to make the last digit of the rounded-off number even.

- e.g. \( 1.35 \rightarrow 1.4 \)
- \( 1.25 \rightarrow 1.2 \)
A Few Definitions from Statistics

Imagine you made a series of \( n \) measurements: \( x_1, x_2, x_3 \) through \( x_n \). That is, you have a set \( \{x_i\} \) of \( n \) values \( (i = 1,2,\ldots,n) \). Ideally, all measurements would be the same, since you measured the same property under the same set of conditions. Perhaps you measured the weight of a box, for example. But because all measurements have some degree of uncertainty associated with them, the measured values will in general differ. Then the following statistical properties can be defined to describe the set of measured values.

Sample (arithmetic) mean \( \bar{x} \) of \( \{x_i\} \):
\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \quad (3)
\]

Range \( R \) of \( \{x_i\} \):
\[
R = \max\{x_i\} - \min\{x_i\} \quad (4)
\]

Variance \( s_x^2 \) of \( \{x_i\} \):
\[
s_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \quad (5)
\]

Standard deviation \( s_x \) of \( \{x_i\} \):
\[
s_x = \sqrt{s_x^2} \quad (6)
\]
(see Example 2.5-2).

Estimation of Data

*Given:* a set of data that provides the value of a dependent variable \( y \) as a function of the independent variable \( x \) at several discrete points \((x,y)\) (note: \( y \) could also be a function of more than one independent variable). We can then define:

**Interpolation:** estimation of \( y \) for points that lie within the range of \( x \).

**Extrapolation:** estimation of \( y \) for points that lie outside the range of \( x \).

**Two-point linear interpolation:** choose the two points, \((x_1,y_1)\) and \((x_2,y_2)\), that neighbor the point of interest \( x \) and use *linear* interpolation to determine \( y \) at \( x \):
\[ y = y_1 + \frac{x - x_1}{x_2 - x_1} (y_2 - y_1) \]  

(7)

**Fitting of a straight line:** IF the \( y \)-\( x \) dependence makes it appropriate to do so, a straight line can be fitted to the data. The line can be fitted by inspection or, more precisely, by the method of least squares (see Appendix A.1). The equation of the line will be of the form \( y = ax + b \) (\( a \) = slope, \( b \) = intercept) and, once determined, can be subsequently used to interpolate or extrapolate the value of \( y \) for any \( x \).

**Fitting of nonlinear data:** In general, the dependence of \( y \) on \( x \) will not be linear. In such situations, one can try to recast the \( y \)-\( x \) data so as to linearize it; that is, to plot some function \( f(y) \) against some function \( g(x) \) for which the dependence is linear; i.e. \( f = ag + b \) holds. (see Example 2.7-2)

More generally, it may be necessary to fit a second or higher degree polynomial (note: a line is a polynomial of degree 1), or some other function, to the data to adequately capture the dependence of \( y \) on \( x \).

**Review of exponential, power law, and logarithmic functions**

The exponential function of \( x \) is written: \( e^x \)  
{or, equivalently, \( \exp(x) \) }

The natural logarithm (ln) is defined as the inverse of the exponential function,

\[ \ln(e^x) = x \quad e^{\ln x} = x \]

*Given:* \( y = ae^{bx} \)

\( \ln y = ? \)

How could we linearize the equation \( y = ae^{bx} \)?

*Recall:* the logarithm of a number \( N \) in a specific “base” \( B \) is the power \( P \) that \( B \) has to be raised to in order to yield \( N \). That is, \( N = B^P \) where \( P \) is the logarithm of \( N \), in base \( B \). The base in the natural logarithm is the number \( e = 2.7182818 \ldots \) What is the natural logarithm (ln) of \( e^2 = 7.38905609 \ldots \)?

The other common logarithm in use is \( \log_{10} \), where the base is 10. What is the \( \log_{10} \) of 1000? Of 0.0001? What is the inverse function of \( \log_{10} \)?

What is \( \log_2 8 \)?
Power Law function - y depends on x raised to a power: \( y = ax^b \) (8)

\[ \ln y = ? \]

How could we linearize the equation \( y = ax^b \)?

A few comments on graphing techniques.

On a **linear** (our text refers to this as **rectangular**) plot both the y- and x-axes have a linear scale. Linear axes use equidistant ticks, with each tick corresponding to the same change in the magnitude of the value being plotted.

On a **semilog** plot, the y-axis has a logarithmic scale and the x-axis has a linear scale. On the logarithmic y-axis, the ticks are not spaced in an equidistant fashion. Rather, the spacing of the y-ticks is scaled such that plotting \( y \) results in the axis being **linear** in the logarithm of \( y \). Thus, the y-axis is linear in \( \log_B y \), while logarithmic in \( y \) (this is confusing and bears some thought). Note that to achieve the same plot as a semilog y-x plot, you could have plotted \( \log_B y \) vs \( x \) on a linear plot. Finally, if a semilog plot of \( y \) vs \( x \) yields a line, it means that \( y \) depends exponentially on \( x \), i.e. \( y = a e^{bx} \).

On a **log-log** plot, both the y and x axes are logarithmic. Thus, they are **linear** in \( \log_B y \) and \( \log_B x \), respectively. If a log-log plot of \( y \) vs \( x \) yields a line, it means that \( y \) depends on \( x \) according to a power law, i.e. \( y = ax^b \).

Note that if you plot \( \log y \), instead of \( y \), on a logarithmic axis, the axis will be linear in \( \log(\log y) \), NOT \( \log y \).