14. Quantum Particles: Identity and Individuality

I. Approaches to the Notion of Individuality

1. Property-Based "Bundle" View
   • An individual = a bundle of properties.
   • So: Properties individuate objects. No two individuals can be completely indiscernible.

   **Principle of the Identity of Indiscernibles:**
   If two objects are indiscernible, then they are identical.

2. Haecceitism
   • Every individual possesses a "primitive thisness" (haecceity) that makes it unique.
   • Self-identity formulation: Every individual is identical to itself.

What about quantum objects? Can they be considered individuals?
II. Fermions and Bosons

Electrons are characterized by:

- Energy $n$ 
  $n = 1, 2, ...$
- Orbital angular momentum $\ell$ 
  $\ell = 0, 1, 2, ... (n - 1)$
- Z-component of orbital angular momentum $m_\ell$ 
  $m_\ell = -\ell, ... 0, ..., \ell$
- Spin $m_s$ 
  $m_s = -1/2, +1/2$

So: The state of an electron is characterized by four values ($n, \ell, m_\ell, m_s$).

Pauli Exclusion Principle (1925):
No two electrons can be in the same state; i.e., no two electrons can have all the same values of ($n, \ell, m_\ell, m_s$).
### Energy shells

<table>
<thead>
<tr>
<th>Z</th>
<th>Element</th>
<th>( n ):</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H hydrogen</td>
<td>( \ell ):</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>He helium</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Li lithium</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Be beryllium</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>B boron</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>C carbon</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>N nitrogen</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>O oxygen</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>F fluorine</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Ne neon</td>
<td>2</td>
<td>2</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Orbital properties

- **Energy shells**
  - K shell \(( n = 1 )\)
  - L shell \(( n = 2 )\)
  - M shell \(( n = 3 )\)
  - N shell \(( n = 4 )\)
  - etc.

- **Orbitals**
  - s orbital \(( \ell = 0 )\)
  - p orbital \(( \ell = 1 )\)
  - d orbital \(( \ell = 2 )\)
  - f orbital \(( \ell = 3 )\)
  - etc.

### Example:
The 3 electrons in a lithium atom are characterized by:
\((1, 0, 0, +1/2), (1, 0, 0, −1/2), (2, 0, 0, +1/2)\).

- **Now:** Consider spin property \( m_s \).
- Electrons have "Spin-1/2". This is a 2-valued spin property (Hardness, Color, etc.). It's values are represented by \( m_s = +1/2, −1/2 \).
There are other more complex multi-valued spin properties.

**Two basic types:** "half-integer-spin properties" and "integer-spin properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>number of $m_s$ values</th>
<th>$m_s$ values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin-1/2</td>
<td>two</td>
<td>$-1/2, +1/2$</td>
</tr>
<tr>
<td>Spin-3/2</td>
<td>four</td>
<td>$-3/2, -1/2, +1/2, +3/2$.</td>
</tr>
<tr>
<td>Spin-5/2</td>
<td>six</td>
<td>$-5/2, -3/2, -1/2, +1/2, +3/2, +5/2$</td>
</tr>
</tbody>
</table>

**Experimentally:** Matter consists of spin-1/2 particles (electrons, quarks, neutrinos), and the forces (EM, strong, weak) consist of spin-1 particles (photons, gluons, $W^\pm, Z$).

**Theoretically:** Gravitational force is mediated by spin-2 particle (graviton) and mass is mediated by spin-0 particle (Higgs).
Experimentally:

- A multi-particle system cannot have both half-integer-spin and integer-spin.

- Both half-integer-spin and integer-spin multi-particle states are *Permutation Invariant*: Exchanging single-particle states results in a multi-particle state that is physically indistinguishable from the original unpermuted multi-particle state.


- Integer-spin multi-particle states do not obey such an Exclusion Principle: There can be integer-spin multi-particle states that consist of two or more identical single-particle states.
**Group Rules for Half-Integer-Spin Particles: Fermi-Dirac (FD) Statistics.**

(a) Multi-particle states are invariant under particle permutations. *(Permutation Invariance.)*

(b) Two or more particles with same (non-spaiotemporal) properties *cannot* be in the same state. *(Generalized Exclusion Principle.)*

**Group Rules for Integer-Spin Particles: Bose-Einstein (BE) Statistics.**

(a) Multi-particle states are invariant under particle permutations. *(Permutation Invariance.)*

(b) Two or more particles with same (non-spaiotemporal) properties *can* be in the same state. *(No Exclusion Principle.)*

**Def:** A *fermion* is a particle that obeys FD Statistics. A *boson* is a particle that obeys BE Statistics.

**Bosonic 2-particle states:**

\[
\sqrt{\frac{1}{2}} \left\{ |\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle, \ |\phi\rangle|\phi\rangle, \ |\psi\rangle|\psi\rangle \right\}
\]

**Fermionic 2-particle state:**

\[
\sqrt{\frac{1}{2}} \left\{ |\phi\rangle|\psi\rangle - |\psi\rangle|\phi\rangle \right\}
\]
III. Classical vs. Quantum Statistics

**Group Rules for Half-Integer-Spin Particles: Fermi-Dirac (FD) Statistics.**

(a) Multi-particle states are invariant under particle permutations.  
*(Permutation Invariance.)*

(b) Two or more particles with same (non-spaciotemporal) properties *cannot* be in the same state. *(Generalized Exclusion Principle).*

**Group Rules for Integer-Spin Particles: Bose-Einstein (BE) Statistics.**

(a) Multi-particle states are invariant under particle permutations.  
*(Permutation Invariance.)*

(b) Two or more particles with same (non-spaciotemporal) properties *can* be in the same state. *(No Exclusion Principle.)*

**Group Rule for Classical Particles: Maxwell-Boltzmann (MB) Statistics.**

(a) Multiparticle states are not invariant under particle permutations.  
*(No Permutation Invariance.)*

(b) Two or more particles with same (non-spaciotemporal) properties *can* be in the same state. *(No Exclusion Principle.)*
Suppose: We have two particles in a 2-particle state composed of two single-particle states $A$, $B$. How can we calculate the probability that one of the particles is in state $A$ and the other is in $B$?

1. Classical particles:

- Use MB statistics: There are 4 possible forms for the 2-particle state. (4 possible ways to distribute two classical particles over two states.)

- Assign each of these possible 2-particle states equal probability of $1/4$ (Principle of Indifference).

- Pr(one particle in $A$ and one particle in $B$) = Pr(state 3) + Pr(state 4) = $1/4 + 1/4 = 1/2$. 

\[
\begin{array}{c|c}
A & B \\
\hline
(1) & \bullet & \blacksquare \\
(2) & \blacksquare & \bullet \\
(3) & \bullet & \blacksquare \\
(4) & \blacksquare & \bullet \\
\end{array}
\]
• Suppose: We have two particles in a 2-particle state composed of two single-particle states $A$, $B$. How can we calculate the probability that one of the particles is in state $A$ and the other is in $B$?

2. Bosons:

- Use BE statistics: There are 3 possible forms for the 2-particle state. (3 possible ways to distribute two bosons over two states.)

- Assign each of these possible 2-particle states equal probability of 1/3 (Principle of Indifference).

- \( \text{Pr(one particle in } A \text{ and one particle in } B) = \text{Pr(state 3)} = 1/3. \)
• **Suppose:** We have two particles in a 2-particle state composed of two single-particle states $A$, $B$. How can we calculate the probability that one of the particles is in state $A$ and the other is in $B$?

3. **Fermions:**

(1) 

\[ \begin{array}{c|c}
A & B \\
\blacklozenge & \blacklozenge \\
\end{array} \]

• Use FD statistics: There is only one possible form for the 2-particle state (due to the Exclusion Principle).

• $\Pr(\text{one particle in } A \text{ and one particle in } B) = \Pr(\text{state 1}) = 1.$
IV. Quantum Individuals

Question: What does Permutation Invariance of quantum states say about the status of quantum particles as individuals?

Initial Response:

• Classical particle are individuals: switching two of them makes a difference.
• Quantum particles are not individuals: switching two of them does not make a difference.

But: Permutation Invariance applies to states. It is a constraint on the possible states that a given multiparticle system can be in. Individuality need not depend on constraints placed on the possible states a system can be in.

Let's consider two approaches to viewing quantum particles as individuals...
1. The Haecceity View of Individuals

Claim: Classical and quantum particles possess "primitive thisness" (haecceities).

The Main Difference:

• Classical haecceities are physically distinguishable: you can tell them apart based on the states they occupy.

• Quantum haecceities are physically indistinguishable: no experiment can distinguish between two quantum haecceities.

2. The "Bundle" view of Individuals

Claim: Classical and quantum particles consist of bundles of properties.

Question: Do classical and quantum particles, so-conceived, satisfy the Principle of the Identity of Indiscernibles (PII)?

PII: If two objects are indiscernible, then they are identical.
Depends on what you mean by indiscernible.

- Suppose two objects are indiscernible just when they have all the same properties.
- Suppose we identify two types of properties:

1. A **monadic property** = a "single-place" property that an object can possess without reference to other objects. (*Example*: mass.)
2. A **relational property** = a "multi-place" property that an object can only possess with respect to one or more other objects. (*Example*: Being taller than.)
**PII.v1:** If two objects agree on all of their monadic and relational properties, then they are identical.

**PII.v2:** If two objects agree on all of their monadic and relational properties, excluding spatiotemporal properties, then they are identical.

**PII.v3:** If two objects agree on all of their monadic properties, then they are identical.

**Now Claim:**
- Classical particles can violate **PII.v2** and **PII.v3**. There can be two classical particles that have the same monadic properties and relational properties, excluding spatiotemporal ones.
- But classical particles satisfy **PII.v1**. They can always be discerned by their positions in space (and time).
- Quantum particles can violate **PII.v2**, **PII.v3**, and **PII.v1**. *(Assumption: quantum particles don't always have well-defined positions!)*
- *So:* Either quantum particles are not individuals, or they must possess haecceities.
But: Do these versions of the PII exhaust all the possible ways two objects can be discerned from each other?

Counterexample
Consider 2 identical iron spheres one mile apart in an otherwise empty universe.

- Spheres agree on all monadic and relational properties, excluding spatiotemporal ones.
- If spatiotemporal properties are relational with respect to other objects, then they will agree on these. (e.g., They both stand in the spatial relation of being 1 mile from the other.)
- **But**: If spatiotemporal properties are relational with respect to absolute spacetime, then whether or not the spheres are identical will depend on the global topology of the spacetime.

```
\begin{center}
\begin{tikzpicture}
\draw (0,0) circle (1.5cm);
\draw[dashed] (0,0) ellipse (1.5cm);
\draw[->] (0,0) -- (1.5,0);
\draw[->] (0,0) -- (-1.5,0);
\node at (0,0) {1 mile};
\end{tikzpicture}
\end{center}
```

*closed global topology*

```
\begin{center}
\begin{tikzpicture}
\draw (0,0) circle (1.5cm);
\draw[->] (0,0) -- (1,0.5);
\draw[->] (0,0) -- (1,-0.5);
\node at (0,0) {1 mile};
\end{tikzpicture}
\end{center}
```

*open global topology*

- So the spheres violate \textit{PII}.v1, \textit{PII}.v2, and \textit{PII}.v3. But intuitively they are distinct: there are \textit{two} of them.

- The spheres are individuated by \textit{number alone} ("\textit{solo numero}").

- To indicate this, use an \textit{irreflexive} two-place relational property: a two-place relational property that relates one sphere with the other, but does not relate either of the spheres with itself.
• **Example:** "Being 1 mile apart from." This is a two-place relation that either sphere has with the other, but *neither has with itself*.

• Suggests the following ways two objects can be discernible:

(a) Two objects are *absolutely discernible* if they differ in a monadic property.
(b) Two objects are *relatively discernible* if they differ in a relational property.
(c) Two objects are *weakly discernible* if they differ in an irreflexive relational property.

**Putative examples of weakly discernible objects:**

• The two iron spheres.
• The points in a Euclidean space.
• Right and left hands in an empty universe.
**Claim:** Fermions are (minimally) *weakly* indiscernible.

- Due to the Exclusion Principle, two fermions will always be discernible; minimally by their values of spin.
- **Now:** Consider a 2-fermion state that is maximally symmetric; *i.e.*, in which the two fermions agree to the maximal extent on all their properties:

\[
\sqrt{\frac{1}{2}} \left\{ |\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right\}
\]

- **Suppose:** Both fermions have exactly the same mass, charge, and all other intrinsic properties.
- **Note:** This state is *spherically symmetric* (a spatial 2-particle property that the 2-fermion state as a whole possesses), so each fermion has exactly the same spatiotemporal properties and relations.
- **But:** There is an irreflexive relation that holds between them: "having opposite direction of each component of spin to".
- **So:** The fermions satisfy the PII, appropriately construed. Thus, *even under the Bundle View, fermions can be considered individuals.*
• **But:** Bosons are *not* constrained by the Exclusion Principle. Hence they can be in states in which they are not discernible, even weakly.

• **Example:** $|\phi\rangle_1|\phi\rangle_2$, where both bosons have all the same properties.

• **So:** Bosons do *not* satisfy the PII.

• Thus they cannot be considered individuals under the Bundle View.

**Potential problem**

• The irreflexive relational property "having opposite direction of each component of spin to" is supposed to hold for both fermions in the state

$$\sqrt{\frac{1}{2}} \left\{ |\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right\}$$

• This suggests that both fermions have definite values of spin when they are in this state.

• But this is a matter of interpretation! Under a literal interpretation, we cannot say that either fermion has a definite value of spin in this state!

• **General Moral:** Whether or not quantum particles are individuals depends not only on the theory of individuality you adopt, but also on the interpretation of QM you adopt.