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AGAINST PARTICLE/FIELD DUALITY: ASYMPTOTIC PARTICLE STATES AND INTERPOLATING FIELDS IN INTERACTING QFT (OR: WHO'S AFRAID OF HAAG'S THEOREM?)

ABSTRACT. This essay touches on a number of topics in philosophy of quantum field theory from the point of view of the LSZ asymptotic approach to scattering theory. First, particle/field duality is seen to be a property of free field theory and not of interacting QFT. Second, it is demonstrated how LSZ side-steps the implications of Haag's theorem. Finally, a recent argument due to Redhead (1995), Malament (1996) and Arageorgis (1995) against the concept of localized particle states is addressed. Briefly, the argument observes that the Reeh–Schlieder theorem entails that correlations between spacelike separated vacuum expectation values of local field operators are always present, and this, according to the above authors, dictates against the notion of a localized particle state. I claim that this moral is excessive and that a coherent notion of localized particles is given by the LSZ approach. The underlying moral to be drawn from this analysis is that questions concerning the ontology of interacting QFT cannot be appropriately addressed if one restricts oneself to the free theory.

0. INTRODUCTION[1]

Quantum field theory (QFT) is arguably the best confirmed theory known to physics. (Un)fortunately, it is beset with interpretational difficulties even more perplexing than those of its cousin non-relativistic quantum mechanics. Issues such as renormalizability, localizability, the notions of particle and field, the nature of spacetime, and the natures of theory reduction and unification are not only of interest to philosophers of QFT, but are also at the crux of the central problem facing contemporary theoretical physics: that of consistently unifying general relativity and quantum theory. In this essay I shall focus specifically on interpretational issues arising in interacting QFT surrounding the notions of particle and field.

I indicate first and foremost how particle/field duality is not a viable ontology for interacting QFT. In Section 1, I indicate how the duality thesis (briefly, to every field there corresponds a particle and vice versa) is motivated by the equivalence of the "particle" and "field" approaches to the canonical quantization of free fields in Minkowski spacetime, and by the complementarity of the "particle" and "field" operators in the resulting

Erkenntnis **53:** 375–406, 2000.

Fock space representation. In Sections 2 and 3, I indicate how interactions are described by means of covariant perturbation theory, and how the notion of an asymptotic particle state arises in this context. I also review the conceptual difficulty for interacting QFT raised by Haag's Theorem. In Section 4, I indicate first how the LSZ formalism solves this problem, and second how it provides us with an improved definition of an asymptotic particle state and also with the notion of an interpolating field. In Sections 5 and 6, I indicate how the notions of asymptotic particle state and interpolating field dictate against the particle/field duality thesis. I demonstrate (a) For every asymptotic particle state, there corresponds an indefinite number of interpolating fields; and (b) There are fields that admit no asymptotic particle states. In Section 7, I draw some morals this analysis has for particle interpretations of QFT, indicating in particular why we need not be afraid of the Reeh–Schlieder theorem in this endeavor. Finally, in Section 8 I consider some objections to my use of asymptotic particle states and interpolating fields in the duality thesis in particular, and in particle and field interpretations of interacting QFT in general.

1. THE DUALITY THESIS

In the following, I shall use neutral scalar field theory as a simple example. The extension to fields with arbitrary spin follows naturally.

In most expositions, one is presented with two equivalent ways of constructing a local quantum field theory in Minkowski spacetime. The first starts with Wigner's definition of single-particle states as irreducible representations of the Poincaré group IO(1, 3). A Fock space $\mathcal{F}$ is then constructed, raising and lowering operators $a^\dagger(\mathbf{p})$, $a(\mathbf{p})$, are introduced, and position-dependent local field operators $\hat{\varphi}(x)$ are obtained as their Fourier transforms (where the hat is used here solely to distinguish the quantum case from the classical case). The alternative approach is to start with the theory of a classical field, postulating the standard canonical commutation relations (ccr) for the field variables and their conjugate momenta, and then identifying the Fourier expansion coefficients of the fields as raising and lowering operators on a Fock space. After this is done, one finds that both Fock spaces are in fact identical. Schematically,

(I) IO(1, 3) $\rightarrow$ "particles" $\rightarrow$ $\mathcal{F}$ $\rightarrow$ $a^\dagger(\mathbf{p}), a(\mathbf{p}) \xrightarrow[\text{F.T.}]{} \hat{\varphi}(x)$ (local quantum field)

(II) $\varphi(x)$ (classical field) $\xrightarrow[\text{ccr}]{} \hat{\varphi}(x) \xrightarrow[\text{F.T.}]{} a^\dagger(\mathbf{p}), a(\mathbf{p})$ $\rightarrow$ $\mathcal{F}$ $\rightarrow$ "particles"

Some expositions construe the equivalence of the two approaches as indicating the dual natures of the "particle" picture (Approach I) and the field picture (Approach II) (see, e.g., Teller 1995, 113; Peskin and Schroeder 1995, 26; Baez, et al. 1992, 58–59). This duality is further motivated by the complementarity in Fock space of the particle number operator $N = \int d\tau_p a^\dagger(\mathbf{p})a(\mathbf{p})$, and the field operator, $\varphi(x) = \int d\tau_p[a(\mathbf{p})+a^\dagger(\mathbf{p})]$. Some take this to indicate a particle/field duality analogous to particle/wave duality in non-relativistic quantum mechanics. Dirac is often cited as giving this view legitimacy:

> ...the dynamical system consisting of an assembly of similar bosons is equivalent to the dynamical system consisting of a set of oscillators – the two systems are just the same system looked at from two different points of view .... We have here one of the most fundamental results of quantum mechanics, which enables a unification of the wave and corpuscular theories of light to be effected. (Dirac 1947, 229)

These remarks lend themselves to two types of duality thesis. A strong version claims that particle and field representations are dual in the sense of being underdetermined by the theory. Either one is adequate in descriptions of physical phenomena. This version can immediately be put to rest, for there are types of physical phenomena that do not admit this democracy of representation (optical phenomena requiring descriptions in terms of coherent states, for example). A weaker notion of duality is encapsulated in the following thesis:

Duality Thesis: To every particle, there corresponds a field; and, conversely, to every field, there corresponds a particle.

Such a thesis poses the question, What constitutes a particle/field? Naively, taking a cue from free field theory, we can phrase the duality thesis in terms of elementary particles and fields, where an elementary particle is an irreducible representation of IO(1, 3), and an elementary field for a theory T is a local operator-valued distribution on the appropriate Hilbert space that appears explicitly in T's Lagrangian.[2] For instance, in free Dirac–Maxwell theory, the Lagrangian density is given by $\mathcal{L} = -\frac{1}{4}F^2_{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$, where ψ is the elementary field of the electron. The associated elementary particle can be represented by the irreducible representation of IO(1, 3) for which $s = \frac{1}{2}$ and $p^2 = m_e^2$. However, this picture breaks down once interactions are introduced. Due to the persistence of interactions in field theory, single-particle states are hard to isolate and can no longer be represented explicitly in terms of the mass-energy spectrum of the theory they appear in. For instance, QED electrons are off-shell ($p^2 \neq m^2$) due to self-interactions, and quarks and $W^\pm$, Z

bosons do not appear in the spectrums of QCD and electro-weak theory, respectively (due, respectively, to confinement and instability). Furthermore, the use of effective Lagrangians problematizes viewing fields that appear in the Lagrangian as elementary. For instance, low-energy pion-nucleon scattering can be described by means of an effective Lagrangian in which appear pion and nucleon fields (Weinberg 1996, Chapter 19.5). The presence of ghost fields in the Lagrangians of non-Abelian gauge field theories also problematizes this view.

Instead of taking cues from free field theory in addressing such ontological questions, I suggest looking at how the interacting theory is actually formulated. I shall look specifically at the LSZ formalism[3] and extract from it notions of particle and field that are appropriate in the interacting context. This is done in Section 4. I first motivate LSZ by considering the standard perturbative approach to scattering theory, emphasizing the problems due to the persistence of interactions and their relation to Haag's theorem. With the appropriate notions of particle and field in hand, I then demonstrate in Sections 5 and 6 that the Duality Thesis is wrong, not because the notions of particle and field no longer make sense in interacting field theory; but because the notions that do make sense dictate against the thesis.

2. ASYMPTOTIC PARTICLE STATES AND THE S-MATRIX

At a purely qualitative level, scattering processes occur when some number, say n, of particles, traveling freely a short time in the past (effectively at $t = -\infty$ for elementary particle time scales) collide with each other and then separate. A short time after the collision ($t = +\infty$), the system is in a superposition of free states, each of which describes a possible end result of the collision. The probability amplitudes for these results are given by the S-matrix, and it is these amplitudes that are what are actually measured in scattering experiments in the forms of scattering cross-sections and decay rates.

In a bit more detail, the state of the system before and after the scattering event has occurred is represented by a multiparticle "in" (resp. "out") state $|\alpha\rangle_{\text{as}} = |\mathbf{p}_1 \ldots \mathbf{p}_n\rangle_{\text{as}}$, where each particle is labeled by its momentum p (ignoring spin and additional quantum numbers for simplicity), and "as" (i.e., "asymptotic") denotes "in" or "out". To localize such states, one may construct them out of single-particle wave-packet states of the form $|\tilde{\mathbf{p}}\rangle_{\text{as}} = \int \mathrm{d}\tau_p g(\mathbf{p})|\mathbf{p}\rangle_{\text{as}}$, where $|\mathbf{p}\rangle_{\text{as}}$ is required to become a free, single-particle state at asymptotic times, and the effect of integrating over the Gaussian function $g(\mathbf{p})$ is to localize the state as a wave-packet. Expanding the states $|\alpha\rangle_{\text{as}}$ in such a localized superposition and time-evolving them by means of a Hamiltonian H, one obtains (schematically),

$$(2.1)\quad e^{-iHt}\int d\alpha\, g(\alpha)|\alpha\rangle_{\text{as}} = \int d\alpha\, e^{-iE_\alpha t} g(\alpha)|\alpha\rangle_{\text{as}}$$

(where $d\alpha$ denotes $d\tau_{p_1}\ldots d\tau_{p_n}$, etc.). To describe the scattering interaction, H is split into a "free" part H_0 and an interaction part V: $H = H_0 + V$. This is done in such a way that the eigenstates $|\alpha\rangle$ of H_0 have the same eigenvalues as the in/out states: $H|\alpha\rangle_{\text{as}} = E_\alpha|\alpha\rangle_{\text{as}}$, and $H_0|\alpha\rangle = E_\alpha|\alpha\rangle$.[4] The requirement that the localized in/out states are asymptotically free can then be written, using (2.1), as

$$(2.2)\quad e^{-iHt}\int d\alpha\, g(\alpha)|\alpha\rangle_{\text{in/out}} \underset{t\to\mp\infty}{\longrightarrow} e^{-iH_0 t}\int d\alpha\, g(\alpha)|\alpha\rangle,$$

or schematically as $|\alpha\rangle_{\text{in/out}} = \Omega(\mp\infty)|\alpha\rangle$, where $\Omega(t) \equiv e^{iHt}e^{-iH_0 t}$. We will see later that this definition of asymptotic particle states is flawed insofar as the Møller operators $\Omega(t)$ are ill-defined. For the moment, however, I shall proceed in accord with the standard theory.

Elements of the S-matrix $S_{\beta\alpha}$ are probability amplitudes for transitions between in- and out-states:

$$(2.3)\quad \begin{aligned} S_{\beta\alpha} &\equiv {}_{\text{out}}\langle\beta|\alpha\rangle_{\text{in}} = \langle\beta|\Omega^\dagger(+\infty)\Omega(-\infty)|\alpha\rangle \\ &\equiv \langle\beta|U(+\infty, -\infty)|\alpha\rangle, \end{aligned}$$

where $U(t, t_0) \equiv \Omega^\dagger(t)\Omega(t_0)$ is referred to as the evolution operator. For small interactions V, it is given by the power series expansion,

$$(2.4)\quad \begin{aligned} U(t, t_0) = \sum_{n=0}^{\infty}((-i)^n/n!)\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \ldots \\ \times \int_{t_0}^{t} dt_n T\{V_I(t_1)V_I(t_2)\ldots V_I(t_n)\}, \end{aligned}$$

where $V_I(t) \equiv e^{iH_0 t}Ve^{-iH_0 t}$, and the time-order operator T orders the $V_I(t)$ terms by increasing time t. The S-matrix operator S is now identified with $U(+\infty, -\infty)$ and represented schematically by $S = T\{\exp(-i\int_{-\infty}^{\infty} dt\, V_I(t))\}$. In order to calculate S-matrix elements (2.3), an explicit form for the in/out states remains to be had. The appropriate form and its relation to the states $|\alpha\rangle$ will be obtained in Section 4.

3. COVARIANT PERTURBATION THEORY AND HAAG'S THEOREM

In Section 4, I shall develop the LSZ formalism. This allows S-matrix elements (2.3) to be calculated from time-ordered vacuum expectation values of interacting local quantum fields, $\tau(x_1, \ldots, x_n) = \langle\Omega|T\{\phi_H(x_1)\ldots\phi_H(x_n)\}|\Omega\rangle$, referred to hereafter as τ-functions. Such τ-functions involve interacting Heisenberg fields $\phi_H(x)$ and the interacting vacuum state $|\Omega\rangle$. One would like to express them in terms of the free fields and vacuum state that appear in the non-interacting theory, since these are easily manipulated. This is accomplished by the Gell-Mann/Low "magic formula". After a brief aside in Section 3.1, I present a development of the magic formula in Section 3.2 and indicate how it runs afoul of Haag's theorem in Section 3.3. This will motivate the move to the LSZ formalism and its attendant notions of asymptotic particle state and interpolating field.

3.1. The Persistence of Interactions

It turns out that the 2-point interacting τ-function $\langle\Omega|T\{\phi_H(x)\phi_H(y)\}|\Omega\rangle$ has a non-perturbative representation in terms of free field elements. It is instructive to run over the basic notions involved, as they provide a concrete manifestation of an essential feature of interacting QFT; namely, the persistence of interactions. This feature is at the heart of the major conceptual difficulties normally associated with the theory, including renormalizability, Haag's Theorem and the Reeh–Schlieder Theorem. Non-perturbative techniques will also come in handy in clarifying the definition of an asymptotic particle state in Section 4.2 below.

Recall that, in the free theory, Heisenberg field solutions to the Klein–Gordon equation are given by,[5]

$$\text{(3.1)} \quad \varphi_{H_f}(x) = e^{iH_f t}\varphi(0,\mathbf{x})e^{-H_f t} = \int \mathrm{d}\tau_p[a(\mathbf{p})e^{-ip\cdot x} + a^\dagger(\mathbf{p})e^{ip\cdot x}].$$

These act on the free vacuum $|0\rangle$, yielding, for instance, the single-particle plane-wave function, $\langle 0|\varphi_{H_f}(x)|\mathbf{p}\rangle = e^{-ix\cdot p}$. The time-ordered vacuum expectation value of two such fields is the free Feynman propagator $-i\langle 0|T\{\varphi_{H_f}(x)\varphi_{H_f}(y)\}|0\rangle \equiv \Delta_F(x-y;m^2)$ which gives the amplitude for a particle of mass m alone in the universe to propagate from x to y. In momentum space, this is given by,

$$\text{(3.2)} \quad -i\int d^4x e^{ip\cdot(x-y)}\langle 0|T\{\varphi_{H_f}(x)\varphi_{H_f}(y)\}|0\rangle$$

$$= \Delta_F(p) = \frac{1}{p^2 - m^2 + i\varepsilon}.$$

The position of the pole, $p^2 = m^2$, in $\Delta_F(p)$ gives the particle's mass. Note also that the residue of the pole is unity.

In the interacting theory, the momentum space interacting Feynman propagator $\tilde{\Delta}_F(p)$ can be written as,[6]

$$\text{(3.3)} \quad -i \int d^4 x e^{ip\cdot x} \langle \Omega | T\{\phi_H(x)\phi_H(0)\} | \Omega \rangle$$

$$= \frac{Z}{p^2 - m_{\text{phy}}^2 + i\varepsilon} + \int_{M_n^2}^{\infty} \mathrm{d}\mu^2 \rho(\mu^2) \frac{1}{p^2 - \mu^2 + i\varepsilon},$$

where $Z = |\langle \Omega | \phi_H(0) | \mathbf{p} \rangle|^2$ is the field strength renormalization constant. This should be compared with the free field case (3.2). The first term on the right of (3.3) describes the contribution to $\tilde{\Delta}_F(p)$ from the single particle state $|\mathbf{p}\rangle$ with physical mass $p^2 = m_{\text{phy}}^2$. The second term describes additional contributions from the continuum of multiparticle states. (3.3) essentially is a sum of propagation amplitudes for states created from the vacuum by $\phi_H(0)$. It differs from the free theory (3.2) by the presence of Z (which is unity in the free theory) and the multiparticle contribution term. The existence of this term is a manifestation of the persistence of interactions in field theory; the fact that interactions can never be turned off. This entails not only that the field source interacts with itself (hence the mass renormalization of footnote 4), but that it interacts with everything else in the universe. The persistence of interactions is at the heart of the foundational issues surrounding renormalization, Haag's Theorem and the Reeh–Schlieder Theorem. More will be said on the latter two issues below.

The constant Z is the residue of the pole in the single particle contribution to the 2-point τ-function. The renormalized interacting field is given by $\phi_r(x) = \frac{1}{\sqrt{Z}}\phi_H(x)$ so that the residue of the pole of the single particle propagator for $\phi_r(x)$ is unity, in keeping with the free theory. In general, an interacting field is renormalized so that the single-particle contribution to its propagator has the same behavior near its pole as the propagator of a free field (i.e., the position of the pole is the physical mass m_{phy}^2 and the residue is unity).

For higher order τ-functions, unfortunately, non-perturbative techniques become intractable, and one is forced to resort to perturbation theory, to which I now turn.

3.2. Covariant Perturbation Theory and the Magic Formula

Recall that the task is to express an interacting τ-function in terms of free τ-functions; once this is done, S-matrix elements can then be calculated in terms of free τ-functions. To begin, we require that the ground state $|\Omega\rangle$ of H, and the ground state $|0\rangle$ of H_0, satisfy $H|\Omega\rangle = E_0|\Omega\rangle$ and $H_0|0\rangle = 0$.[7] From solutions $\varphi(t, \mathbf{x})$ of the free theory, one defines *Heisenberg* fields, $\phi_H(x) = e^{iHt}\varphi(0, \mathbf{x})e^{-iHt}$; and *interaction* fields, $\phi_I(x) = e^{iH_0t}\varphi(0, \mathbf{x})e^{-iH_0t}$. Both coincide with free fields at the reference time $t = 0$. The interaction fields are given by (3.1) (insofar as H_0 is naively taken to be the free field Hamiltonian H_f). The Heisenberg fields are then given by,

$$\phi_H(x) = e^{iHt}e^{-iH_0t}\phi_I(x)e^{iH_0t}e^{-iHt} \equiv U^\dagger(t)\phi_I(x)U(t), \tag{3.4}$$

where $U(t)$ satisfies $U(t)U^\dagger(t) = 1$ and $U(t)U^\dagger(t_0) = U(t, t_0)$. $U(t, t_0)$ has already appeared in Section 2 as the evolution operator between in/out states (where $\Omega(t) = U^\dagger(t)$).

It is now possible to transform an interacting τ-function into a free τ-function. Consider the n-point function $\langle\Omega|T\{\phi_H(x_1)\ldots\phi_H(x_n)\}|\Omega\rangle$. Using (3.4), the Heisenberg fields can be replaced with interaction fields, yielding,

$$\langle\Omega|U^\dagger(\tau)T\{\phi_I(x_1)\ldots\phi_I(x_n) \times \exp(-i\textstyle\int_{-\tau}^{\tau}\mathrm{d}t\,V_I(t))\}U(-\tau)|\Omega\rangle. \tag{3.5}$$

The interacting ground state $|\Omega\rangle$ can be written in terms of the free ground state $|0\rangle$ via:

$$|\Omega\rangle = \lim_{\tau\to\infty}(e^{-iE_0(t_0-(-\tau)}\langle\Omega|0\rangle)^{-1}U(t_0, -\tau)|0\rangle \tag{3.6}$$

(for details see, e.g., Peskin and Schroeder 1995, 86). After a bit of algebra one obtains the Gell-Mann/Low "magic" formula for covariant perturbation theory:[8]

$$\langle\Omega|T\{\phi_H(x_1)\ldots\phi_H(x_n)\}|\Omega\rangle = \lim_{\tau\to\infty}\frac{\langle 0|T\{\phi_I(x_1)\ldots\phi_I(x_n)\exp(-i\int_{-\tau}^{\tau}\mathrm{d}t\,V_I(t))\}|0\rangle}{\langle 0|T\{\exp(-i\int_{-\tau}^{\tau}\mathrm{d}t\,V_I(t))\}|0\rangle}. \tag{3.7}$$

The calculation of interacting τ-functions has now been reduced to the calculation of free τ-functions. The latter are easily calculated using the form (3.1). (Wick's Theorem relating time-ordered products to normal-ordered products and contractions of fields, is employed at this stage to simplify the calculations.)

3.3. *Haag's Theorem*

I now consider the ramifications Haag's theorem has for the perturbative expansion (3.7). After Arageorgis (1995, 119), the former amounts to two results:[9]

1. If two pure ground states are not equal, then they generate unitarily inequivalent irreducible representations.
2. If two local quantum fields are unitarily equivalent at any given time, then both fields are free if one of them is free.

Result (1) indicates that the overlap $\langle 0|\Omega\rangle$ of the free ground state and the interacting ground state is incompatible with the existence of the unitary operator $U(t, t_0)$. If we allow that $\langle 0|\Omega\rangle \neq 0$, then, by Result (1), the interaction picture defined by (3.4) does not in fact exist. On the other hand, if we allow (3.4), then by the contrapositive of (1), the ground states must be equal, hence there is no overlap: $\langle 0|\Omega\rangle = 0$. This entails that the Gell-Mann/Low magic formula (3.7) contains cancellations of infinity.

There is another place where infinities occur in the magic formula due to Haag's theorem. Note that the interacting fields $\phi_H(x)$ in (3.7) should be replaced with renormalized fields $\phi_r(x) = \frac{1}{\sqrt{Z}}\phi_H(x)$, as indicated in Section 3.1. However, by Haag's theorem, $Z = |\langle\Omega|\phi_H(0)|\mathbf{p}\rangle|^2 = 0$, since the overlap $\langle\Omega|\phi_H(0)|\mathbf{p}\rangle$ must be zero (being the overlap of two elements, $\langle\Omega|\phi_H(0)$ and $|\mathbf{p}\rangle$, of different Hilbert spaces). Hence the renormalized fields $\phi_r(x) = \frac{1}{\sqrt{Z}}\phi_H(x)$ are singular.

Haag's theorem presents us with two types of problem. One focuses on concerns over mathematical consistency in dealing with infinities. The other involves concerns over conceptual coherence and can be identified in particular with the apparent incoherence of using the interaction picture in a situation in which its use dictates its non-existence. The first type of problem I believe is not too interesting. Renormalization techniques are a dime a dozen. In particular, the infinite phase factor relating $|\Omega\rangle$ to $|0\rangle$ in (3.6) was canceled at the expense of introducing the divergent term in the denominator of (3.7). This latter represents vacuum-to-vacuum transitions, or bubble graphs, in the language of Feynman diagrams; and these "cancel" similar bubble graphs generated in the numerator. The upshot is that, heuristically, in the perturbative expansion, only non-bubble graphs need to be calculated. Also, the infinities associated with the integrals defining the mass and field strength renormalization constants, Z and δm^2, can be handled by imposing an ultraviolet cut-off, or by using one of several other regularization techniques (such as Pauli-Villars, dimensional regularization, etc.). I submit therefore that if Haag's theorem is indeed a foundational problem for interacting QFT, it must be in the second conceptual sense. I shall now demonstrate that it is this second type of conceptual problem that the LSZ formalism explicitly addresses.

4. LSZ FORMALISM

The LSZ formalism allows S-matrix elements to be calculated from interacting τ-functions. It is based on a weak convergence condition that relates the free dynamics to the interacting dynamics asymptotically, instead of perturbatively. In Section 4.1, I describe this condition and how it avoids Haag's theorem. In Section 4.2, I indicate the nature of the resulting asymptotic particle states and how they may be considered free for all practical purposes. Finally, in Section 4.3, I describe the LSZ reduction formula which provides the means by which the S-matrix can be effectively calculated and indicates the function of interpolating fields.

4.1. The LSZ Asymptotic Condition and Haag's Theorem

The LSZ formalism replaces the interaction fields $\phi_I(x)$ of Section 3.2 with asymptotic fields $\phi_{\text{as}}(x)$. The latter are assumed to be free fields, hence can be decomposed into the Fourier form (3.1) with asymptotic raising/lowering operators $a_{\text{as}}(\mathbf{p})$, $a^\dagger_{\text{as}}(\mathbf{p})$ replacing $a(\mathbf{p})$, $a^\dagger(\mathbf{p})$. These are given explicitly by inverting (3.1):

$$(4.1)\quad a^\dagger_{\text{as}}(\mathbf{p}) = -i\int d^3x e^{-ip\cdot x} \overleftrightarrow{\partial}_0 \phi_{\text{as}}(x),$$

$$a_{\text{as}}(\mathbf{p}) = i\int d^3x e^{ip\cdot x} \overleftrightarrow{\partial}_0 \phi_{\text{as}}(x),$$

where $A\overleftrightarrow{\partial}_0 B = A\partial_0 B - B\partial_0 A$. The associated Hilbert spaces will be denoted $\mathcal{H}_{\text{in}}$ and $\mathcal{H}_{\text{out}}$ with Lorentz-invariant vacuum states $|\Omega\rangle_{\text{in}}$ and $|\Omega\rangle_{\text{out}}$.

The problem posed by Haag's theorem is how to relate the asymptotic fields $\phi_{\text{as}}(x)$ to the interacting Heisenberg fields $\phi_H(x)$ in a way consistent with the assumption that the former are governed by the free dynamics. It turns out that a strong convergence requirement of the form,

$$(4.2)\quad \phi_{\text{in}}(x) \underset{t\to-\infty}{\longleftarrow} \frac{1}{\sqrt{Z}}\phi_H(x) \underset{t\to+\infty}{\longrightarrow} \phi_{\text{out}}(x),$$

will not work (see below); this essentially reiterates the relation (3.4). The LSZ weak convergence asymptotic condition modifies this by requiring simply that matrix elements converge in the limit:

$$\text{(4.3)} \quad \lim_{t\to\pm\infty} i\int d^3x f^*(x)\overleftrightarrow{\partial}_0\langle\beta|\tfrac{1}{\sqrt{Z}}\phi_H(x)|\alpha\rangle = \langle\beta|a_{\text{out/in}}[f]|\alpha\rangle,$$

$$\lim_{t\to\pm\infty} -i\int d^3x f(x)\overleftrightarrow{\partial}_0\langle\beta|\tfrac{1}{\sqrt{Z}}\phi_H(x)|\alpha\rangle = \langle\beta|a^\dagger_{\text{out/in}}[f]|\alpha\rangle,$$

where $|\alpha\rangle$, $|\beta\rangle$ are arbitrary elements of the Hilbert space $\mathcal{H}$ of interacting states.[10] Equation (4.3) can be taken as a definition of the asymptotic raising/lowering operators, $a_{\text{as}}[f]$, $a^\dagger_{\text{as}}[f]$, in terms of the limits of interacting raising/lowering operators that act on $\mathcal{H}$.[11] Rigorous proofs exist that show that the limits in (4.3) are well-defined. These assume the existence of a mass gap and asymptotic completeness: $\mathcal{H} = \mathcal{H}_{\text{out}} = \mathcal{H}_{\text{in}}$.[12] For the purposes of this essay, I shall be more concerned with (a) how (4.3) avoids Haag's theorem and (b) how the particle states arising from the fields defined in (4.3) are free for all practical purposes.

To see how (a) comes about, the following theorem is useful.

THEOREM 1. $|\Omega\rangle_{\text{in}} = |\Omega\rangle = |\Omega\rangle_{\text{out}}$ (up to phase). The ground states of $\mathcal{H}$, $\mathcal{H}_{\text{in}}$, $\mathcal{H}_{\text{out}}$ are identical up to phase.

Proof. Use (4.3) and asymptotic completeness to show that $\langle\beta|a_{\text{as}}[f]|\Omega\rangle = 0$, for all $\langle\beta|$ in $\mathcal{H}$.

Thus the antecedent condition of Result (1) of Section 3.3 is avoided.

Condition (4.3) avoids Result (2) of Section 3.3 in the following sense. Strong convergence (4.2) implies the equality $\lim_{t\to\mp\infty}\frac{1}{Z}\langle\Omega|\phi_H(x)\phi_H(y)|\Omega\rangle = {}_{\text{as}}\langle\Omega|\phi_{\text{as}}(x)\phi_{\text{as}}(y)|\Omega\rangle_{\text{as}}$, and this indicates that $\phi_H(x)$ is free (footnote 9), which contradicts the assumption that it is an interacting field. This equality does not hold for weak convergence (4.3). For a complete set of states $|n\rangle$ and the weak convergence condition $\lim_{t\to\mp\infty}\frac{1}{\sqrt{Z}}\langle\Omega|\phi_H(x)|n\rangle = \langle\Omega|\phi_{\text{as}}(x)|n\rangle$, we have,

$$\begin{aligned}
&\langle\Omega|\phi_{\text{as}}(x)\phi_{\text{as}}(y)|\Omega\rangle \\
&= \sum_n \lim_{t\to\mp\infty}\frac{1}{Z}\langle\Omega|\phi_H(x)|n\rangle\langle n|\phi_H(y)|\Omega\rangle \\
&\neq \lim_{t\to\mp\infty}\sum_n \frac{1}{Z}\langle\Omega|\phi_H(x)|n\rangle\langle n|\phi_H(y)|\Omega\rangle \\
&= \lim_{t\to\mp\infty}\frac{1}{Z}\langle\Omega|\phi_H(x)\phi_H(y)|\Omega\rangle.
\end{aligned}$$

Hence no contradiction arises. The crucial observation here is that, in general, the sum of a limit is not equal to the limit of the sum.

The right-hand side of the magic formula (3.7) can now be reformulated, replacing the free fields $\phi_I(x)$ and the free vacuum $|0\rangle$ with asymptotic fields $\phi_{\text{as}}(x)$ and the interacting vacuum $|\Omega\rangle$. The calculation of scattering amplitudes still involves expanding (3.7) in a possibly divergent power series; however I argued above that these are calculational problems that can be handled by renormalization techniques. The conceptual problem indicated by Haag's theorem, on the other hand, is no longer present, having been addressed by the introduction of asymptotic fields defined by the LSZ weak convergence condition (4.3).

4.2. Asymptotic Particle States

In this subsection, I demonstrate that the ever-present contribution from the multiparticle continuum can be neglected for asymptotic particle states. Hence they are, for all practical purposes, free states.

DEFINITION 1. A localized asymptotic single-particle state $|\tilde{\mathbf{p}}\rangle_{\text{as}}$ is given by,

$$(4.4)\qquad |\tilde{\mathbf{p}}\rangle_{\text{as}} = a^{\dagger}_{\text{as}}[f_{\mathbf{p}}]|\Omega\rangle,$$

where $a^{\dagger}_{\text{as}}[f_{\mathbf{p}}]$ is defined by (4.3).

THEOREM 2. The asymptotic single-particle state $|\mathbf{p}\rangle_{\text{as}}$ is free for physically meaningful time scales.

Proof. For the "out" case, operating on the left of (4.4) with $\langle\Omega|\frac{1}{\sqrt{Z}}\phi_H(x')$, one obtains the asymptotic single-particle plane-wave function,[13]

$$(4.5)\qquad \langle\Omega|\frac{1}{\sqrt{Z}}\phi_H(x')|\mathbf{p}\rangle_{\text{out}} = \lim_{t\to\infty} -i\int d^3x e^{-ip\cdot x}\overleftrightarrow{\partial}_0\langle\Omega|$$

$$\times \frac{1}{Z}\phi_H(x')\phi_H(x)|\Omega\rangle$$

$$= e^{-ip'x'} + \lim_{t\to\infty}\int_{M_n^2}^{\infty} \mathrm{d}\mu^2\rho(\mu^2)$$

$$\times \frac{E_\mu(\mathbf{p}') + p_0'}{2E_\mu(\mathbf{p}')}e^{-E_\mu(\mathbf{p}')(t-t')}e^{-i\mathbf{p}\cdot\mathbf{x}'+ip_0't},$$

This should be compared with the free theory case $\langle 0|\varphi_{H_f}(x)|\mathbf{p}\rangle = e^{-ix\cdot p}$ (see below (3.1)). The second term on the right of (4.5) is the multiparticle contribution. By the Riemann–Lebesque Lemma,[14] it vanishes if the integrand is a smooth function of μ^2. In general, however, it will not be smooth at the points $\mu^2 = M_n^2$, where M_n^2 is the threshold at which the n-particle state contribution begins. It turns out that the contributions from such multiparticle states in fact are washed out for physically meaningful time scales. It can be shown that, in the vicinity of the threshold mass M_n^2, the multiparticle contribution to (4.5) goes as,

$$(4.6)\quad (M|t-t'|)^{-3/2(n-1)}e^{-iE_{M_n}(\mathbf{p}')(t-t')}$$

where M is some characteristic particle mass scale.[15] For time differences $|t-t'| \gg M^{-1}$, (4.6) is negligible, decreasing as an inverse power. For example, a typical mass is $M \sim 1$ GeV, whence $M^{-1} \sim 10^{-23}$ sec (in "natural" units with $\hbar = 1$). Then for $|t-t'| \sim 10^{-13}$ sec $\gg M^{-1}$, the 2-particle contribution from (4.6) is on the order of 10^{-15}, and the contributions from n-particle states, $n > 2$, will be even smaller. Hence, for such time differences, the asymptotic state $|\mathbf{p}\rangle_{\text{out}}$ is, for all practical purposes, free and satisfies the same normalization conditions as the free single-particle state, ${}_{\text{out}}\langle\mathbf{p}'|\mathbf{p}\rangle_{\text{out}} = (2\pi)^3 2E_{\mathbf{p}}\delta^3(\mathbf{p}'-\mathbf{p})$. A similar analysis holds for the in-state $|\mathbf{p}\rangle_{\text{in}}$ as well.[16]

4.3. *The LSZ Reduction Formula and the Role of Interpolating Fields*

I now indicate how the LSZ weak convergence condition (4.3) allows S-matrix elements to be expressed in terms of interacting τ-functions. Consider the S-matrix element ${}_{\text{out}}\langle\beta|\alpha\rangle_{\text{in}} = {}_{\text{out}}\langle\tilde{\mathbf{p}}_1\ldots\tilde{\mathbf{p}}_n|\tilde{\mathbf{q}}_1\ldots\tilde{\mathbf{q}}_m\rangle_{\text{in}}$ for m in-coming localized particles with ith momentum $\mathbf{q}_i$ and n out-going localized particles with ith momentum $\mathbf{p}_i$. We want to express this in terms of the $(n+m)$ τ-function $\langle\Omega|T\{\phi_H(x_1)\ldots\phi_H(x_{n+m})\}|\Omega\rangle$. The strategy is to extract, out of the in/out multiparticle states, individual particle states $\tilde{\mathbf{p}}_i$, $\tilde{\mathbf{q}}_i$, one at a time, using the asymptotic raising/lowering operators (footnote 10, smeared versions) and then use (4.3) to replace these operators with interacting fields $\phi_H(x)$. This is repeated until the in/out multiparticle states have been reduced to the vacuum and we are left with a τ-function. The end result is the LSZ reduction formula, given here for scalar fields:

$$(4.7)\quad {}_{\text{out}}\langle\tilde{\mathbf{p}}_1\ldots\tilde{\mathbf{p}}_n|\tilde{\mathbf{q}}_1\ldots\tilde{\mathbf{q}}_m\rangle_{\text{in}}$$
$$= (i/\sqrt{Z})^{m+n}\int d^4x_1\ldots d^4y_m\, f_{\mathbf{q}_1}(y_1)\ldots f_{\mathbf{q}_n}(y_m)\overrightarrow{K}_{y_1} - \overrightarrow{K}_{y_m}$$
$$\times\langle\Omega|T\{\phi_H(x_1)\ldots\phi_H(x_n)\phi_H(y_1)\ldots\phi_H(y_m)\}|\Omega\rangle$$
$$\times\overleftarrow{K}_{x_1}\ldots\overleftarrow{K}_{x_n} f^*_{\mathbf{p}_1}(x_1)\ldots f^*_{\mathbf{p}_n}(x_n),$$

where $K_x = \partial^2_{(x)} + m^2$ is the Klein–Gordon (KG) operator (for details consult Kaku 1993, 141–5).

To put (4.7) into a more suggestive form, one can take the Fourier transform of both sides. This turns the KG operators K_i into factors of the form $(p_i^2 - m_i^2 + i\varepsilon)$. It also introduces a momentum-conserving delta function $\delta^4(p_i - q_i)$ due to the translation-invariance of the τ-function. This serves to force all momenta on-shell ($p^2 = m^2$), and one obtains the form,[17]

$$\text{(4.7}'\text{)}\quad {}_{\text{out}}\langle \mathbf{p}_1 \dots \mathbf{p}_n | \mathbf{q}_1 \dots \mathbf{q}_m \rangle_{\text{in}} = (i/\sqrt{Z})^{m+n}(2\pi)^4 \times\delta^4\left(\sum_m q - \sum_n p\right)\prod_j (q_j^2 - m_{q_j}^2 + i\varepsilon) \times\tau(p_1 \dots p_n, q_1 \dots q_m)\prod_i (p_i^2 - m_{p_i}^2 + i\varepsilon),$$

where $\tau(p_1 \dots p_n, q_1 \dots q_m)$ is the momentum-space τ-function. Isolating it then leads to the behavior,

$$\text{(4.8)}\quad \tau(p_1 \dots p_n, q_1 \dots q_m) \underset{\substack{p_i^2 \to m_{p_i}^2 \\ q_j^2 \to m_{q_j}^2}}{\longrightarrow} \left(\prod_{i=1}^n \frac{-i\sqrt{Z}}{p_i^2 - m_{p_i}^2 + i\varepsilon}\right) \times \left(\prod_{j=1}^m \frac{-i\sqrt{Z}}{q_j^2 - m_{q_j}^2 + i\varepsilon}\right) {}_{\text{out}}\langle \mathbf{p}_1 \dots \mathbf{p}_n | \mathbf{q}_1 \dots \mathbf{q}_m \rangle_{\text{in}}.$$

Thus the S-matrix element ${}_{\text{out}}\langle \mathbf{p}_1 \dots \mathbf{p}_n | \mathbf{q}_1 \dots \mathbf{q}_m \rangle_{\text{in}}$ is the coefficient of the multipole term of the on-shell limit (viz. $p^2, q^2 \to m^2$) of its associated (momentum-space) τ-function. This form of the reduction formula is instructive insofar as it indicates the role of the interacting fields $\phi_H(x)$. These are referred to as interpolating fields insofar as they may be said to interpolate between asymptotic particle states. According to (4.8), each interpolating field $\phi_H(x_i)$ serves to produce a pole at $p_i^2 = m_{p_i}^2$ in the Fourier transform of the τ-function. The position of this pole is the mass of the particle associated with the field. Intuitively, since the form of the pole is just the form of a propagator, an interpolating field serves to produce a propagator for its associated asymptotic particle.

5. NON-UNIQUENESS OF INTERPOLATING FIELDS: PARTICLES WITH NO UNIQUE FIELDS

In this section I indicate how the Duality Thesis fails insofar as the particle content of an interacting theory underdetermines the field content: To any stable particle there corresponds an infinite number of interpolating fields.

In the LSZ reduction formula (4.8), the interpolating fields $\phi_H(x)$ explicitly appear in the Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_H)^2 - \frac{1}{2}m^2\phi_H^2 + \mathcal{L}_I$. Furthermore, they satisfy the condition $\langle\Omega|\phi_H(x)|q\rangle_{\text{as}} \neq 0$, for each asymptotic particle state $|\mathbf{q}\rangle_{\text{as}}$. It turns out that this is a sufficient condition for any local field $O(x)$ to be an interpolating field for $|\mathbf{q}\rangle_{\text{as}}$, regardless of whether $O(x)$ appears in the original Lagrangian, provided only that $O(x)$ transform appropriately under IO(1, 3). This motivates the following definition:

DEFINITION 2. An interpolating field associated with an asymptotic single-particle state $|\mathbf{q}\rangle_{\text{as}}$ is a local quantum field $O(x)$ that transforms irreducibly under IO(1, 3) and satisfies $\langle\Omega|O(x)|\mathbf{q}\rangle_{\text{as}} \neq 0$.

The following theorem establishes this definition:

THEOREM 3. Let $O(x)$ be any local field operator transforming irreducibly under IO(1, 3) with $\langle\Omega|O(x)|\mathbf{q}\rangle_{\text{as}} \neq 0$, where $|\mathbf{q}\rangle_{\text{as}}$ is a stable asymptotic single-particle state. Then $O(x)$ can be used as an interpolating field for $|\mathbf{q}\rangle_{\text{as}}$ in the LSZ reduction formula; i.e., $O(x)$ contributes a pole at $p^2 = m_q^2$ to the $n+1$-point τ-function $\langle\Omega|T\{O(x)A(y_1)\ldots A(y_n)\}|\Omega\rangle$, where $A(y_i)$ local field operators.

Proof. See Appendix for a brief outline. Detailed proofs are given in Weinberg (1995, 428–39), and Nishijima (1969, 332–5).

(Theorem 3 can be qualified even further. It can be shown that the fields $O(x)$ for which $\langle\Omega|O(x)|\mathbf{q}\rangle_{\text{as}} \neq 0$ form an equivalence class under local relativity, where $O(x)$ is local relative to $O'(y)$ just when $[O(x), O'(y)] = 0$, for spacelike $(x - y)$ (see Emch 1972, 293; Haag 1992, 103 and references therein). Such equivalence classes are called Borcher's classes. Hence, to every asymptotic single-particle state $|\mathbf{q}\rangle_{\text{as}}$ there corresponds a Borcher's class of interpolating fields.)

As an example of a well-defined asymptotic particle state with no unique field associated with it, take the pion. It is stable, has well-defined q-numbers, and is given by an irreducible representation of IO(1, 3). Hence the states $|\mathbf{p}_\pi\rangle_{\text{as}}$ are well-defined. By Theorem 3, any local irreducible field

$\Pi(x)$ that satisfies $\langle\Omega|\Pi(x)|\mathbf{p}_\pi\rangle_{\text{as}} \neq 0$ serves as a pion field insofar as it can be used to interpolate between in/out asymptotic pion states.

It is tempting to object at this point by claiming that the duality thesis does not apply to pions since they are composite particles, reducible, under the auspices of QCD, to bound states of quark/anti-quark doublets. My reply is two-fold. First, pion fields have as much right to elementary status as quark fields insofar as pion fields appear in effective Lagrangians for low-energy pion scattering. (One might object here that quark fields are "more elementary" insofar as QCD applies over a much wider range of energies than effective pion theories. However, there is growing consensus that QCD and the Standard Model in general are themselves effective theories that approximate a more "elementary", at present unknown, theory that applies at even larger energy scales. The duality thesis, phrased in terms of elementary particles/fields, is then either vacuously true or must be contextualized to a given theory.) Second, it is problematic to describe pion scattering via QCD, for quark fields $\psi(x)$ are not interpolating fields. There are no particle states $|\mathbf{q}\rangle_{\text{as}}$ such that $\langle\Omega|\psi(x)|\mathbf{q}\rangle_{\text{as}} \neq 0$. I now show how this comes about.

6. QUARK CONFINEMENT: FIELDS WITH NO PARTICLES

In this section I indicate how the Duality Thesis fails insofar as there is good reason to believe that there are fields that have no corresponding particle states. These are the quark fields that appear in the Lagrangian of quantum chromodynamics (QCD). To a good approximation, the potential between two quarks grows linearly with increasing distance. The effect is that quarks cannot exist in asymptotic free single-particle states. This is a non-perturbative prediction of the lattice approximation to QCD.

In brief the amplitude for a quark/anti-quark creation/annihilation event can be identified as the continuum limit of a lattice Wilson loop observable $W(C)$. The latter obeys the Area Law for Wilson loops on a lattice:

$$W(C) = e^{-KA}, \tag{6.1}$$

where A is the area enclosed by the loop C and K is a constant.[18] This Area Law can then be used to determine the form of the interaction potential E_0 between the quark/anti-quark pair. It turns out that the potential is linear in the separation distance R:

$$E_0(R) = KR \tag{6.2}$$

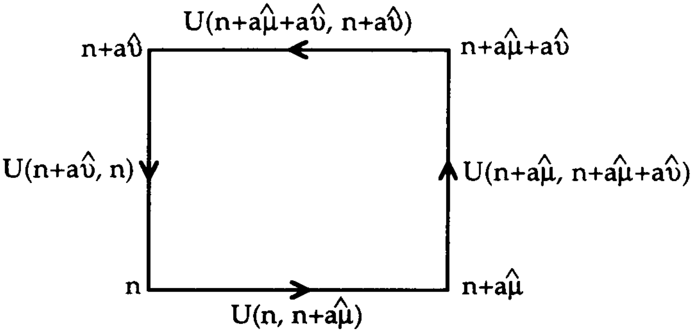

Figure 1. Elementary plaquette p.

Hence confinement results and asymptotic particle states for quarks cannot exist. In the remainder of this section, I shall briefly run through the central ideas. Readers willing to accept the above statements without further comment may skip ahead to Section 7.

Wilson loops are gauge-invariant observables that appear in Yang–Mills theories. In non-Abelian theories like QCD, for which the gauge fields $A_\mu(x)$ do not commute, Wilson loops are given by $W(C) = P\{\exp(ig \oint_C \mathrm{d}x^\mu A_\mu(x))\}$ (where P is the path-order operator). Intuitively, $W(C)$ can be viewed as a phase shift experienced by a source due to motion around a loop C in a background gauge field. When Yang–Mills theory is put on a lattice, Wilson loops are given by the expectation value $W(C) = \langle Tr \prod_n U_n \rangle$, where the loop C has been divided into n discrete links U_n.[19] This expectation value can be calculated using functional integration techniques (Creutz 1983, 36). The result is $W(\partial p) = Z^{-1} \int (\mathrm{d}U) Tr\{U_p\} e^{-S}$, for an elementary loop ∂p (such loops enclose elementary plaquettes p (footnote 19)). Here U_p is the product of the links in ∂p, Z is a normalization constant, and S is the lattice Yang–Mills action.[20] To calculate $W(C)$ for an arbitrary loop C, it turns out that the only contributions to the integral come from plaquettes that fill the area enclosed by C.[21] Thus to first order we have $W(C) = W(\partial P)^{A/a^2}$, where A/a^2 is the number of plaquettes filling the area A enclosed by C (a^2 being the area of each plaquette). The Area Law (6.1) then follows after a bit of algebra.

To make the connection with quarks, consider the loop in Figure 2, call it (R, T). It can be interpreted as representing two static color charges q, q' (a quark/anti-quark pair) created at time $t = 0$ a distance R apart, and subsequently annihilated at time $t = T$. It can be shown that the amplitude for the qq' event just described is the continuum limit of the expectation value given by the lattice Wilson loop $W(R, T)$. The interaction potential

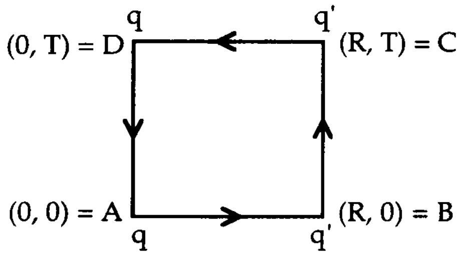

Figure 2. Quark/anti-quark loop.

between the qq' pair can then be determined by means of the Area Law (6.1).

Let $\Psi(t) = \bar{\psi}(0,t)Pe^{ig\int_0^R \mathrm{d}x A_1(x,t)}\psi(R,t)$ be the qq' pair operator.[22] Then the amplitude for the qq' event is given by $\langle 0|\Psi^\dagger(T)\Psi(0)|0\rangle \equiv \Omega(T,R)$. One now contracts the quark/anti-quark fields to produce two propagators of the form $S_F(y-y')$ (via Wick's Theorem). It can be shown that these propagators are proportional to exponential terms involving the time components $A_0(\mathbf{x},t)$ of the gauge fields in the large mass (viz. static) approximation.[23] The amplitude $\Omega(T,R)$ then takes the form $\langle 0|Pe^{ig\int_0^R \mathrm{d}x A_1(x,t)}Pe^{ig\int_0^T \mathrm{d}t A_0(x,t)}Pe^{-ig\int_R^0 \mathrm{d}x A_1(x,t)}Pe^{-ig\int_T^0 \mathrm{d}t A_0(x,t)}|0\rangle$, and this is identifiable as the continuum limit of the lattice expectation value $\langle Tr\{U_{AB}U_{BC}U_{DC}^\dagger U_{AD}^\dagger\}\rangle = W(R,T)$.

The Area Law for $W(R,T)$ can now be used to determine the behavior of the interaction potential between q and q' (see, e.g., Kaku 1993, 513–4). Inserting a complete set of states into $\Omega(T,R)$, one obtains,

$$\sum_n \langle 0|\Psi^\dagger(T)|n\rangle\langle n|\Psi(0)|0\rangle = \sum_n |\langle 0|\Psi^\dagger(0)|n\rangle|^2 e^{-E_n T}.$$

Since $E_n > E_0$ for all n, in the limit $T \to \infty$ (i.e., for large loops (R,T)), the ground state energy E_0 dominates. Hence,

$$\text{(6.3)} \qquad \lim_{T\to\infty} \Omega(T,R) \sim e^{-E_0(R)T}.$$

Comparing (6.3) with the Area Law (6.1), one obtains (6.2).

7. DISCUSSION: THE REHABILITATION OF PARTICLES

Given that the particle/field duality thesis should be abandoned, are we left with anything more to say about what interacting QFT is describing, other than the rather bland statement that it describes *both* asymptotic particle states *and* interpolating fields? I believe that any stronger statement is simply not well-motivated. In particular, I believe that fundamentalism, of either the field or particle type, is not forth-coming. I have claimed that the notion of a particle as a localized asymptotic LSZ state is well-motivated insofar as it addresses the problem at the heart of the interacting theory manifested by Haag's theorem. In this section I suggest that such a notion also addresses a recent critique of the particle interpretation based on the Reeh–Schlieder theorem. This is discussed in Section 7.2 below. Section 7.1 offers brief commentary on the problems raised by Teller and Redhead against a particle interpretation of the LSZ formalism.

7.1. The Redhead/Teller Objection

Teller (1995, 123) charges that the LSZ formalism restricts what we can identify as particles to asymptotic times, "...thereby significantly limiting the interpretation of the [interacting] theory in terms of [particles]" (his "quanta"). (See, also, Redhead 1988, 21, for a similar assessment.) This charge is based on the fact that, while an occupation number operator can be constructed for the asymptotic in/out particle states, no such operator can be constructed for the interacting states. This is due to the fact that the un-renormalized interacting fields $\phi_H(x)$ do not satisfy the canonical commutation relations. To see this, note that the interacting Feynman propagator in position space is given by $\tilde{\Delta}_F(x-y) = -i\langle\Omega|[\phi_H(x), \phi_H(y)]|\Omega\rangle$, and the time derivative of the free propagator is $\partial_t i\Delta_F(x-y) = i\delta^3(\mathbf{x}-\mathbf{y})$. Taking the time derivative of the Fourier transform of (3.3) then yields,

$$(7.1) \quad [\pi_H(x), \phi_H(y)] = i\delta^3(\mathbf{x}-\mathbf{y})\left(Z + \int_{M_n^2}^{\infty} \mathrm{d}\mu^2 \rho(\mu^2)\right),$$

where $\pi_H(x) = \partial_t\phi_H(x)$. For the free field case, $Z = 1$, and the multiparticle contribution is zero. As indicated in Section 3.1, the interacting fields $\phi_H(x)$ should be replaced with renormalized fields $\phi_r(x) = \frac{1}{\sqrt{Z}}\phi_H(x)$. However, as (7.1) explicitly shows, this absorption of Z into the interacting field only reproduces the correct single-particle contribution to the commutator. It is the existence of multiparticle contributions that prevents (7.1) from having the canonical form. This consequently prevents the renormalized interacting fields from commuting with the free Hamiltonian. Thus an occupation number operator cannot be constructed for the $\phi_H(x)$, which leads Redhead and Teller to conclude that they cannot be given a particle interpretation

I suggest that a particle interpretation should not be dependent on the existence of a (free field) occupation number operator. To require otherwise seems to me to be placing undue emphasis on the free theory. Due to the persistence of interactions, any notion of particle derived from the free theory will be hard to extend to the interacting theory. Instead, I propose using the definition (4.4), that was motivated directly by how the interacting theory is formulated, as the basis for a particle interpretation. In particular, I suggest that a "particle" be considered a system that minimally possesses an asymptotic state (i.e., a system that is free for all practical purposes at asymptotic times). Whether or not such a system has a corresponding number occupation operator, I would claim, is irrelevant. Under a literal construal of the LSZ description of scattering experiments, there are two types of system that we might consider to be particles: "asymptotic" particles defined directly by (4.4), and "interacting" particles. These latter may be defined in analogy with (4.4) by,

$$(7.2) \quad |\tilde{\mathbf{p}}\rangle_{\text{int}} = a^{\dagger}_{\text{int}}[f_{\mathbf{p}}, t]|\Omega\rangle,$$

where $a^{\dagger}_{\text{int}}[f_{\mathbf{p}}, t]$ is an "interacting" raising operator (see footnote 11). I suggest that both types of system can legitimately be called particles in so far as both types have well-defined asymptotic states ((4.4) is by definition an asymptotic state; (7.2) has (4.4) as an asymptotic state). It turns out that "asymptotic" particles so-defined also possess number occupation operators whereas "interacting" particles do not. My point is that this deficiency should not prevent us from interpreting the latter as particles. Furthermore, I suggest viewing both types of system not as distinct types of particle; but rather, as different states in which a particle can be found; viz. an asymptotic (free for all practical purposes) state, and an interacting state. To avoid confusion, I shall use the term "LSZ particle" to refer to such a system capable of possessing both an asymptotic state (given by (4.4)) and an interacting state (given, schematically, by (7.2)). The main claim of this subsection, then, is that a viable particle interpretation of interacting QFT can be had, based on the notion of an LSZ particle (by this I do not mean to say that only LSZ particles appear in the ontology of QFT; I also claim that fields appear as well).

7.2. The Reeh–Schlieder Objection: Localizability and the Vacuum

Recently Redhead (1995), Malament (1996), and Arageorgis (1995) have developed an argument against the notion of particle in interacting field theory. In brief, they represent a particle by a projection operator P onto the appropriate single-particle subspace of $\mathcal{H}$. (Think of P as the outcome of a particle detection measurement.) After Arageorgis (1995, 334), P must satisfy the following constraints:

(1) $\langle\Omega|P|\Omega\rangle = 0$,

(2) $\langle\Psi|P|\Psi\rangle = 1$, for some $\Psi \in \mathcal{H}$.

These are interpreted as stating that the vacuum is empty and the particle exists in some state Ψ, respectively. By the Reeh–Schlieder Theorem,[24] if P is a local observable (viz. an element of a von Neumann algebra $\mathcal{R}(O)$ for O a bounded open region of spacetime), then (1) implies $P = 0$, hence (1) and (2) are inconsistent. Moreover, the assumption that P is a local observable can be relaxed, and we can require only that,

(3) $[A, P] = 0$, for all local observables $A \in \mathcal{R}(O')$ and some bounded open region O'.[25]

It can then be shown that (3) is inconsistent with (1) and (2). This is taken to imply that the notion of particle given by (1) and (2) comes with it a radical holism, given by the denial of (3), for the region O' may be spacelike related to the region in which P finds support. Malament concludes,

> To whatever extent we have evidence that [Nature] does not allow such correlations, we have evidence that quantum mechanical phenomena must ultimately be given a field-theoretic interpretation. (1996, 2)

The moral that Redhead draws is slightly stronger:

> [Particle states] are an idealization which leads to a plethora of misunderstandings about what is going on in quantum field theory. The theory is about fields and their local excitations. That is all there is to it. (1995, 135)

I want to suggest that this moral is a bit excessive insofar as an LSZ particle is localizable for all practical purposes (FAPP-localizable, hereafter) in the asymptotic regime; i.e., an LSZ particle satisfies conditions (1)–(3) above in the asymptotic regime, for all practical purposes. As discussed in Section 4.2, for time scales on the order of 10^{-13} sec, the asymptotic localized single-particle state $|\tilde{\mathbf{p}}\rangle_{\text{as}}$ satisfies,

(1′) $\langle\Omega|\tilde{\mathbf{p}}\rangle_{\text{as}} = 0$,

(2′) ${}_{\text{as}}\langle\tilde{\mathbf{p}}'|\tilde{\mathbf{p}}\rangle_{\text{as}} = (2\pi)^3 2E_{\mathbf{p}}\delta^3(\mathbf{p}' - \mathbf{p})$.

Furthermore,

(3′) $[\phi_{\text{as}}[f], \phi_{\text{as}}[g]] \to 0$, as the distance between the supports of f and g goes to infinity.

For a proof of (3′) see Haag (1992, 85). Strictly speaking, (3′) requires that the asymptotic smeared fields $\phi_{\mathrm{as}}[f]$ be "almost local" operators; i.e., the functions f and g decrease rapidly at infinity. But this is what was assumed in the construction of the asymptotic states $|\tilde{\mathbf{p}}\rangle_{\mathrm{as}}$.

I submit that anything more than FAPP-localizability in the asymptotic regime is too much to expect for the interacting theory. There are two possible objections to this:

(1) FAPP-localizability is not good enough for the notion of a particle;

(2) Even if it is good enough, the interacting states of LSZ particles do not possess it.

(1) objects to treating LSZ particles as localized in the asymptotic regime in so far as they still possess finite exponential "tails", regardless of however fast these tend to zero, that may span spacelike separated regions, and this possibility should not be countenanced under any reasonable definition of localizability. My response is that such tails are a consequence of the persistence of interactions. To require no tails is in essence to require no interactions. This seems to me to hold the free theory as a paradigm from which to draw interpretive conclusions. It certainly is possible to claim that, by definition, the notion of particle only makes sense in the free theory. What seems to me to be a more interesting project is to see how much of the particle concept can be retained in the interacting theory while minimizing damage to intuitions about localizability. This response holds as well for objection (2) above. The intuition that localizability is something we would expect of a particle regardless of the state it might find itself in just fails for the interacting theory. This does not mean that we have to give up the notion of particle completely. I suggest that LSZ particles provide a half-way house that effectively bridges the conceptual gap between the free theory and the interacting theory.

It might further be objected that FAPP-localizability is a property that philosophers of physics should shy away from. FAPP-localized particles may be justified for the practicing physicist, but their application to foundational issues may be questionable. In particular, the notion of FAPP-localized particles runs the risk of being labeled ad hoc, appealed to simply to avoid the consequences of the Reeh–Schlieder Theorem. But this charge is easily defused. Specifically, to the extent that we are concerned with the conceptual difficulty posed by Haag's Theorem, we should be willing to adopt the notion particle as defined by the LSZ weak convergence limit (which, itself, is certainly not ad hoc: it has been rigorously proven to exist). It then turns out that in doing so we are able to avoid a further conceptual difficulty posed by the Reeh–Schlieder Theorem. (One might argue that both difficulties stem from the same source; namely, the persistence of interactions in interacting QFT; hence, they are not completely independent of one another. My hunch is that they are independent enough on whatever criterion of independence one might adopt to explicate the notion of ad hocness.)

8. SUMMARY AND FURTHER DISCUSSION

In this essay, I have argued first and foremost that the particle/field duality thesis cannot be applied to interacting quantum field theory: any attempt to give it a precise formulation in the interacting context fails. I took the thesis to be the general claim that to every field there corresponds a unique particle, and vice versa; and then considered various ways by which the notions of field and particle could be cashed out. I argued that a literal reading of fields as those objects appearing in the Lagrangian of one's theory is problematic. I further argued that the standard Wigner group-theoretic definition of particle is problematic in the interacting theory context. I then considered a notion of particle obtained from the LSZ formulation of interacting QFT; namely, that based on the notion of an asymptotic particle state. I argued that there is a very good reason for adopting this formalism; namely, that by adopting it, one is able to avoid the conceptual difficulty posed by Haag's Theorem. I then demonstrated that, given such a notion of particle, and the corresponding notion of interpolating field, the duality thesis is wrong in so far as,

(a) to every asymptotic particle state there corresponds an indefinite number of interpolating fields;

and

(b) there are fields with no corresponding asymptotic particle states.

Finally, I argued that the LSZ notion of asymptotic particle state helps to address the locality problem raised by the Reeh–Schlieder Theorem for particle interpretations of interacting QFT. In particular, if we allow "particles" to be "LSZ particles" which may exist in asymptotic states and interacting states, then locality can be retained in FAPP-form for asymptotic states, and this is the best one can expect given the nature of the interacting theory. In this last section, I shall consider a few potential objections to claims (a) and (b) (I thank an anonymous referee for raising these concerns).

8.1. *Objections to (a)*

One might wish to read "field" in the duality thesis as "physical field", and then reject claim (a) in so far as interpolating fields are not physical fields; rather, they are formal artifacts; surplus structure of the formalism. Perhaps the duality thesis survives in some form in which a unique set of "field facts", which underlie the indefinite number of possible interpolating fields of a given asymptotic particle state, are uniquely correlated to that state (in particular, one might associate the essential structure underlying the notion of a field with a Borscher's equivalence class of interpolating fields (see Section 5)). I would agree that this is one way of making objection (a) to the duality thesis disappear. However, I would add that it assumes a particular interpretational stance with respect to interacting QFT, one that I shall now attempt to make explicit.

Certainly part of being a realist with respect to QFT is to read the theory literally.[26] A realist takes the theoretical claims that QFT makes about fields and particles at their face value: "[Theoretical claims] are not to be understood either as mere assertions of verifiability, as covert, complex reports on observation, or as meaningless devices for the systematization of data" (Horwich 1982, 182). Part of the task facing the realist then is to decide just how to read QFT literally. In particular, how should a semantic realist approach the duality thesis? It seems to me that a semantic anti-realist will claim that the thesis just contends that "field facts" are uniquely correlated with "particle facts" and be content with leaving it at that. The semantic realist, on the other hand, wants to know just what a "field fact" in the theory amounts to. My concern with the duality thesis is a concern with how to read it through semantic realist's eyes. I argued above in Section 1 that a literal construal of a field as a particular mathematical object appearing in the Lagrangian of one's theory is problematic for use in the duality thesis. Moreover, even if we grant that such objections are not worrisome for a semantic realist, and that by "physical field" we mean "mathematical field appearing in a theory's Lagrangian", there is still a problem. Note that interpolating fields may occur in the Lagrangian of one's theory. The electron field $\psi(x)$ that occurs in the Dirac–Maxwell Lagrangian (Section 1) is a perfectly acceptable interpolating field in so far as it can be used to interpolate between asymptotic electron states in the LSZ formalism. What Theorem 3 of Section 5 demonstrates is that it is not unique in this ability. The point here is that, if one is motivated to consider the fields that appear in Lagrangians as “physical” fields to which the duality thesis applies, then one should also allow that the duality thesis applies to interpolating fields as well. Objection (a) then shows that a duality thesis, so-informed, is incorrect. (Again, this is not to say that some form of duality thesis under which “field facts” are uniquely correlated with “particle facts” is unobtainable. My specific claim is just that the duality thesis fails, not only for standard semantic realist construals of “field” and “particle”, but also for, what I consider to be, more well-informed construals (viz., those informed by the LSZ formalism).)

8.2. *Objections to (b)*

An epistemic realist might question claim (b). One might argue that evidence for quark confinement (and thus evidence for treating quarks as fields with no corresponding particles) is not on par with evidence for treating quarks as particles. Evidence of this latter type arguably comes from experiments involving deep inelastic scattering in which high energy particles scatter off of the constituents of nucleons in a manner that implies that these constituents behave like free point-like particles. Such experiments originally contributed to the acceptance of the QCD theory of quark interactions. They also established that QCD is characterized by asymptotic freedom. This is a property unique to non-Abelian gauge theories like QCD, which entails that, at high energies and short distances, the coupling constant of the theory goes to zero. In the QCD context, this means that the strong color force experienced by quarks weakens as the distance of quark separation decreases. In the limit when two quarks, considered as point-particles, coincide in spacetime, the coupling they experience due to the strong color force is zero, and they can effectively be treated as free point-particles. Conversely, at low energies and large distances, the coupling grows linearly and confinement results. Hence perhaps the notion of an LSZ particle is not adequate, since it does not allow us to treat quarks as particles.[27] In particular, the property of possessing an asymptotic state may be inadequate to the particle concept.

I have two responses to this objection. First, it is not that apparent how to flesh out the intuition that quarks are particles if it is motivated by deep inelastic scattering experiments. Asymptotic freedom dictates that quarks behave as free point particles in the limit of large energies/small distances. In effect, to treat two quarks as free particles, they have to coincide at the same point in spacetime. Such entities do not seem very much like particles. In any event, I would again stress that my primary claim is that, under standard semantic realist interpretations of interacting QFT, the duality thesis cannot be maintained. This is not to say that one cannot treat quarks as particles. It is just to say that one cannot maintain a dual particle/field interpretation of interacting QFT that applies at once to field theories characterized by asymptotic freedom, like QCD, as well as to field theories that are not so characterized (like quantum electrodynamics). My claim is that a non-dual particle/field interpretation of interacting QFT is viable based on the notions of LSZ particle and interpolating field. Under this interpretation, quarks cannot be viewed as particles, but must be viewed as fields.

9. CONCLUSION

The persistence of interactions indicates that particle states in the interacting theory will always possess non-vanishing exponential tails, even at asymptotic times, and the Reeh–Schlieder theorem indicates that such tails may span spacelike separated regions in spacetime. However, these facts alone do not dictate against a coherent notion of particle, as I have attempted to demonstrate. Such a coherent notion of particle may be derived from the asymptotic particle states that appear in the LSZ formalism of interacting field theory. I have argued that there is a very good reason for adopting this notion, in so far as the LSZ formalism solves the conceptual problem generated by Haag's theorem.

More generally, I have argued that particle/field duality is incoherent in interacting field theory in so far as, given notions of particle and field motivated by the interacting theory (as opposed to the free theory), (1) for every particle there exists an infinite number of corresponding fields, and (2) there are fields with no corresponding particle states.

Finally, I conclude that, if we are to take interacting field theory seriously (and not view it as a stop-gap temporary fix en route to a more well-behaved theory), then we should look to it, as opposed to free field theory, to inform us as to what the world would be like if it were true.

APPENDIX

Theorem 3, Section 5. Let $O(x)$ be any local Heisenberg field operator transforming irreducibly under IO(1, 3) with $\langle\Omega|O(x)|\mathbf{q}\rangle_{\mathrm{as}} \neq 0$, where $|\mathbf{q}\rangle_{\mathrm{as}}$ is a stable asymptotic single-particle state. Then $O(x)$ contributes a pole at $p^2 = m_q^2$ to the $n+1$-point τ-function $\langle\Omega|T\{O(x)A(y_1)\ldots A(y_n)\}|\Omega\rangle$, where $A(y_i)$ are local Heisenberg field operators.

Proof. In momentum space $\tau(p_1, q_1 \dots q_n) = \int d^4x d^4y_1 \dots d^4y_n e^{i(p\cdot x - q_1\cdot y_1 - \dots - q_n\cdot y_n)} \langle\Omega|T\{O(x)A(y_1)\dots A(y_n)\}|\Omega\rangle$. Now insert a complete set of states $1 = |\Omega\rangle\langle\Omega| + \int \mathrm{d}\tau_q |\mathbf{q}\rangle_{\mathrm{as\ as}}\langle\mathbf{q}| + \sum$ (multiparticle states) and consider the single-particle overlap term with the time ordering $x^0 > \max(y_i^0)$:

$$\text{(A.1)} \quad \int d^4x \dots d^4y_n e^{i(p\cdot x - q_1\cdot y_1 - \dots - q_n\cdot y_n)} \theta(x^0 - \max(y_i^0)) \times \int \mathrm{d}\tau_q \langle\Omega|O(x)|\mathbf{q}\rangle_{\mathrm{as\ as}}\langle\mathbf{q}|T\{A(y_1)\dots A(y_n)\}|\Omega\rangle.$$

The term $\langle\Omega|O(x)|\mathbf{q}\rangle_{\mathrm{as}}$ can be written as $\langle\Omega|e^{iP\cdot x}O(0)e^{-iP\cdot x}|\mathbf{q}\rangle_{\mathrm{as}} = e^{-iq\cdot x}\langle\Omega|O(0)|\mathbf{q}\rangle_{\mathrm{as}}$, and the θ-function is given by $\theta(x) = \int \mathrm{d}\omega e^{i\omega x}\frac{1}{2\pi i(\omega - i\varepsilon)}$. Equation (A.1) then becomes,

$$\text{(A.2)} \quad \int d^4y_1 \dots d^4y_n e^{i(q_1\cdot y_1 - \dots - q_n\cdot y_n)} \int \mathrm{d}\tau_q \times \int \mathrm{d}\omega \frac{1}{2\pi i(\omega - i\varepsilon)} \int d^4x e^{i(p-q)\cdot x} e^{i\omega(x^0 - \max(y_i^0))} \times \langle\Omega|O(x)|\mathbf{q}\rangle_{\mathrm{as\ as}}\langle\mathbf{q}|T\{A(y_1)\dots A(y_n)\}|\Omega\rangle.$$

The $\int d^4x$ integral can now be done, yielding a factor of $(2\pi)^4\delta^3(\mathbf{p} - \mathbf{q})\delta(p_0 - E_{\mathbf{q}} + \omega)$, where $E_{\mathbf{q}} = q^0 = \sqrt{\mathbf{q} + m^2}$. Hence we have,

$$\text{(A.3)} \quad \int d^4y_1 \dots d^4y_n e^{i(q_1\cdot y_1 - \dots - q_n\cdot y_n)} \int \mathrm{d}\tau_q (2\pi)^3\delta^2(\mathbf{p} - \mathbf{q}) \times \int \mathrm{d}\omega \frac{\delta(p_0 - E_q + \omega)}{i(\omega - i\varepsilon)} e^{-i\omega\max(y_i^0))} \times \langle\Omega|O(x)|\mathbf{q}\rangle_{\mathrm{as\ as}}\langle\mathbf{q}|T\{A(y_1)\dots A(y_n)\}|\Omega\rangle$$

$$= \int d^4y_1 \dots d^4y_n e^{i(q_1\cdot y_1 - \dots - q_n\cdot y_n)} (2E_p)^{-1} \frac{e^{-i(E_p - p_0)\max(y_i^0)}}{i(E_p - p_0 - i\varepsilon)} \times \langle\Omega|O(x)|\mathbf{q}\rangle_{\mathrm{as}}\langle\mathbf{q}|T\{A(y_1)\dots A(y_n)\}|\Omega\rangle.$$

Now note that

$$\frac{1}{E_p - p_0 - i\varepsilon} = \frac{E_p + p_0}{E_p^2 - p_0^2 - i\varepsilon} \xrightarrow[p_0 \to E_p]{}$$

$$\frac{2E_p}{\mathbf{p}^2 + m_q^2 - p_0^2 - i\varepsilon} = \frac{-2E_p}{p^2 - m_q^2 + i\varepsilon}.$$

Hence

$$\tau(p_1, q_1 \ldots q_n) \underset{p^2 \to m_q^2}{\longrightarrow} \frac{i}{p^2 - m_q^2} \langle \Omega | O(x) | \mathbf{q} \rangle_{\text{as}}$$

$$\times \int d^4 y_1 \ldots d^4 y_n e^{i(q_1 \cdot y_1 - \cdots - q_n \cdot y_n)} {}_{\text{as}}\langle \mathbf{q} | T\{A(y_1) \ldots A(y_n)\} | \Omega \rangle.$$

The proof is complete since this is the behavior advertised by the theorem: the field $O(x)$ has been “extracted” from the τ-function and, in doing so, produces a momentum space Feynman propagator $\frac{i}{p^2 - m_1^2}$; i.e., $O(x)$ produces a pole at $p^2 = m_q^2$.

NOTES

1 In this essay, 4-vectors x are given by $(x^0, -\mathbf{x})$, bold-face denotes 3-vectors, and the Lorentz covariant measure is denoted by $d\tau_p \equiv (2\pi)^{-3} d^3\mathbf{p}/2E_p$, where $E_\mathbf{p} = p^0 = \sqrt{\mathbf{p}^2 + m^2}$ and $p^0 > 0$. Single-particle states are thus normalized according to $\langle \mathbf{p} | \mathbf{p}' \rangle = (2\pi)^3 2E_\mathbf{p} \delta^3(\mathbf{p} - \mathbf{p}')$.

2 An irreducible representation of the Poincaré group may appear too abstract a notion to define a particle. The intuition motivating such a definition is based on two essential properties such irreducible representations possess that we minimally associate with the notion of a particle. First, they are uniquely labeled by 2 parameters associated with mass and spin (or helicity for the massless case). Second, they are invariant under IO(1, 3) transformations, hence they conform to our intuitions concerning the continuity of particle identity through spacetime.

3 Developed originally in H. Lehmann, K Symanzik and W. Zimmermann: 1957, ‘On the Formulation of Quantized Field Theories II’, *Nuovo Cimento* **6**, 319.

4 In general, an interaction will shift the mass, so the spectrums of H and H_0 will not be identical. To mask this effect, a modified split $H = H_0' + V'$ can be made where $V' = V - \Delta$, $H_0' = H_0 + \Delta$, and Δ is the energy difference corresponding to the mass shift. For instance, if the mass in H is m_B^2 (the bare mass) and the mass shift due to interaction is δm^2, then, under the modified split, the mass occurring in H_0' is $m_B^2 + \delta m^2 \equiv m_{\text{phy}}^2$. This is the physical mass that is actually measured. Note, too, that H_0 need not be identical to the free Hamiltonian. The split is made so that the interaction V is weak compared to H_0, allowing H_0 to be treated as the zeroth-order approximation to H.

5 Here and below free fields $\varphi(x)$ are distinguished from interacting fields $\phi(x)$. In what follows, fields are defined at a point. When it becomes crucial to the exposition, they will appear properly smeared with appropriate test functions. Finally, H_f denotes the (truly) free Hamiltonian, which H_0, in general, denotes the zeroth-order approximation to the interacting Hamiltonian H.

[6] This is the Källén–Lehmann spectral representation (for details see Peskin and Schroeder 1995, 211–6). The derivation is non-perturbative, requiring only Lorentz invariance and unitarity of the 2-*pt* function. The particular form (3.3) also assumes the existence of a mass gap between the vacuum and the lowest energy state. The spectral function $\rho(\mu^2)$ is given by $\rho(\mu^2) = \sum_\alpha (2\pi)\delta(\mu^2 - m_\alpha^2)|\langle\Omega|\phi_H(0)|\alpha\rangle|^2$, where the sum is over all states. The lower bound of integration M_n^2 is the threshold mass at which the n-particle contribution begins, for $n \geq 2$.

[7] The physical vacuum $|\Omega\rangle$ is the state of lowest energy; the bare vacuum $|0\rangle$ is the "no-particle" state.

[8] Given originally in Gell-Mann and Low (1951), 'Bound States in Quantum Field Theory', *Physical Review* **84**, 350. For further discussion consult Haag (1992, 67–71).

[9] This follows the account given in Emch (1972, 247–53). See also Haag (1992, 55–7), and Streater and Wightman (1979, 165–6). In rough outline, the proof of (2) in the latter is based on two subsidiary results. First, given two irreducible fields $\phi_2(x)$, $\phi_1(x)$ defined on Hilbert spaces $\mathcal{H}_1$, $\mathcal{H}_2$ and transforming under irreducible representations $U_1(\Lambda, a)$, $U_2(\Lambda, a)$, of IO(1, 3) with unique invariant vacuum states $|0\rangle_1$, $|0\rangle_2$, and related by a unitary transformation V at some time t according to $\Phi_2(t, x) = V\phi_1(t, \mathbf{x})V^{-1}$, one can prove that the ground states are related by $c|0\rangle_2 = V|0\rangle_1$, where c is a complex constant with modulus 1. This implies that the vacuum expectation values of the fields at a given time t are equal: ${}_1\langle 0|\phi_1(x_1)\ldots\phi_1(x_n)|0\rangle_1 = {}_2\langle 0|\phi_2(x_1)\ldots\phi_2(x_n)|0\rangle_2$. The second result shows that, if $\phi(x)$ is a scalar field for which the vacuum is cyclic, and $\langle 0|\phi(x)\phi(y)|0\rangle = i\Delta^+(x - y; m^2)$, $m > 0$, then $\phi(x)$ is a free field of mass m. Result (2) then follows: If $\phi_1(x)$ is a free field of mass $m > 0$ related to another field $\phi_2(x)$ by a unitary transformation $\phi_2(x) = V\phi_1(x)V^{-1}$, and if $\phi_1(x)$ and $\phi_2(x)$ transform under irreducible representations of IO(1, 3), then $\phi_2(x)$ is a free field of mass m.

[10] In (4.3) the smeared versions of (4.1) have been used:

$$(4.1') \qquad a^\dagger_{\text{as}}[f_p] = -i\int d^3x f_{\mathbf{p}}(x) \overleftrightarrow{\partial}_0 \phi_{\text{as}}(x),$$

$$a_{\text{as}}[f_p] = i\int d^3x f^*_{\mathbf{p}}(x) \overleftrightarrow{\partial}_0 \phi_{\text{as}}(x),$$

where $f_{\mathbf{p}}(x)$ is a normalized positive frequency wave packet solution of the Klein–Gordon equation.

[11] In analogy with (4.1′), footnote 10, the objects appearing within the limits on the LHS of (4.3) can be identified as interacting raising and lowering operators $a_{\text{int}}[f, t]$, $a^\dagger_{\text{int}}[f, t]$. They are not, however, operator-valued distributions insofar as they depend on time ($\phi_H(x)$ is not a solution of the Klein–Gordon equation). This can be fixed by smearing them with smooth functions of time.

[12] See Haag (1992, 88–92) and references therein.

[13] Where $E_\mu(\mathbf{p}') = \sqrt{\mathbf{p}'^2 + \mu^2}$, $p'_0 = \sqrt{\mathbf{p}'^2 + m^2_{\text{phy}}}$ and a form of the spectral representation (footnote 6) has been used.

[14] Briefly, if $f(\omega)$ is a smooth function which vanishes as $\omega \to \pm\infty$, then its Fourier transform vanishes in the limit $t \to \infty$: $\int_{-\infty}^{+\infty} d\omega f(\omega)e^{-\omega t} \xrightarrow[t\to\infty]{} 0$.

[15] Here I rely on the proof in Brown (1992, 293). Near $\mu^2 = M_n^2$, the spectral function behaves like $\rho(\mu^2) \sim (\mu^2 - M_n^2)^{1/2(3n-5)}$. Expanding $E_\mu(\mathbf{p}')$ about M_n^2 and substituting into the multiparticle contribution to (4.5), one obtains, after integration, the behavior (4.6).

16 For localized asymptotic single-particle states $|\tilde{\mathbf{p}}\rangle_{\text{as}}$, the same analysis holds since the wave-packet functions $f_{\mathbf{p}}(x)$ decrease rapidly outside the velocity cone (see Haag 1992, 89).

17 Technically, this requires taking the limit in which all wave-packet Gaussian functions $g(\mathbf{p}_i)$, $g(\mathbf{q}_i)$, tend to delta functions. (4.7′) has a nice graphical representation. In momentum space, a τ-function can be represented by a Feynman graph consisting of a Blob with various legs attached to it, each leg representing a propagator $\frac{1}{p_i^2 - m_{p_i}^2 + i\varepsilon}$ for a given in/out particle. In general, such particles will be off-shell. The corresponding S-matrix is obtained by forcing all momenta on-shell and then amputating external legs via the factors $(p_i^2 - m_{p_i}^2 + i\varepsilon)$.

18 $K = -(1/a^2)\ln W(\partial p)$, where $W(\partial p)$ is the Wilson loop observable for the boundary loop ∂p surrounding the elementary plaquette p with edges of length a (see below for details).

19 For an $SU(N)$ gauge theory, each link U_n is an $N \times N$ matrix representation of $SU(N)$ given explicitly by $U_n = \exp\left(iag\lambda^\alpha A_\mu^\alpha\left(n + \frac{a\hat{\mu}}{2}\right)\right)$. Such links form the edges (of length a) of a 4-dim Euclidean lattice that models spacetime. The 2-dim faces of cells in the lattice are referred to as plaquettes and denoted by p. See Figure 1.

20 Given by $S = \frac{N}{g^2}\sum_p\left(1 - \frac{1}{N}Tr\{U_p\}\right)$, where the p-sum is taken over all oriented plaquettes p in the lattice. One can show that, in the continuum limit $a \to 0$, this form produces the standard Yang–Mills action with coupling constant g (see, e.g., Creutz 1983, 35–6).

21 This "tiling" is due essentially to the properties $\int(\mathrm{d}U)U_{ij} = 0$, and $\int(\mathrm{d}U)U_{ij}U_{kl}^\dagger = (1/N)\delta_{jk}\delta_{il}$, where the link indices label lattice vertices. These indicate that $W(C)$ is non-zero only when either a link in C is paired with the same oppositely oriented link in a plaquette in the term e^{-S}, or when this occurs for plaquette links in the term e^{-S}. Hence in the sum over all plaquettes p in the lattice, only those that fill the area enclosed by C contribute.

22 $\Psi(t)$ acts on the vacuum to produce a qq' pair a distance R apart at time t. The $\psi/\bar{\psi}$-terms are quark/anti-quark fields and the exponential factor is required for gauge invariance. Only 1 spatial dimension is considered.

23 $S_F(y - y')$ is the Green's function for the QCD operator $(i\gamma_\mu D_\mu - m)$. For large m, $(i\gamma_\mu D_\mu - m) \sim (i\gamma^0 D_0 - m)$. The exponential form of the propagator is derived by solving $(i\gamma^0 D_0 - m)S_F(y - y') = \delta^4(y - y')$.

24 See, e.g., Haag (1992, 101), Streater and Wightman (1979, 138), Emch (1972, 290). Briefly, it states that the vacuum Ω is cyclic for the von Neumann algebra $\mathcal{R}(O)$ of any bounded open region O of spacetime; i.e., for all $\Psi \in \mathcal{H}$, there are $A_n \in \mathcal{R}(O)$ such that $\Psi = \sum_n A_n\Omega$. Furthermore, if the spacelike complement of O is non empty, Ω is separating for $\mathcal{R}(O)$; i.e., for all $A \in \mathcal{R}(O)$, if $A\Omega = 0$, then $A = 0$.

25 This is a necessary condition for the independence of the amplitudes of P and A: $\langle AP\rangle = \langle A\rangle\langle P\rangle$. Since the status of P is being left open, we do not stipulate that O' be spacelike to the regions of support for P.

26 After Earman (1993) and Horwich (1982), I take scientific realism to be composed of two components: a semantic component and an epistemic component. The semantic component characterizes the realist's desire to read the theoretical claims of certain theories literally. The epistemic component characterizes the realist's contention that there can be good reasons to believe the theoretical claims of certain theories. (These definitions are

admittedly a bit vague. For instance, if some sub-set of the theoretical claims of a theory T are of the form "Objects of type X exist", then epistemic realism with respect to T would seem to entail semantic realism with respect to T. The distinction between the components, as I see it, is meant to follow the distinction between the two separate enterprises of interpretation and confirmation. For any theory T, the former enterprise endeavors to give an account of what the world would be like if T were true. The latter enterprise endeavors to give an account of the conditions under which we are justified in believing T's claims. The scientific realist must be able to give accounts of both endeavors.)

[27] I don't think that, if correct, this claim can be used as evidence that the LSZ formalism fails to successfully avoid the problem associated with Haag's Theorem. Haag's Theorem specifically demonstrates that the free dynamics cannot be connected to the interacting dynamics by means of the strong convergence criterion (4.2). The LSZ formalism allows the free dynamics to be connected to the interacting dynamics by means of the weak convergence criterion (4.3). The LSZ notion of asymptotic particle state (4.4) is defined in terms of this weak convergence criterion but is independent of the latter's use in avoiding Haag's Theorem. One need not talk about asymptotic particles at all in the LSZ context if one so desires; rather, one can talk only in terms of the convergence properties of matrix elements of fields (4.3).

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