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WEINBERG ON QFT: DEMONSTRATIVE INDUCTION AND UNDERDETERMINATION*

ABSTRACT. In this essay I examine a recent argument by Steven Weinberg that seeks to establish local quantum field theory as the only type of quantum theory in accord with the relevent evidence and satisfying two basic physical principles. I reconstruct the argument as a demonstrative induction and indicate it's role as a foil to the underdetermination argument in the debate over scientific realism.

INTRODUCTION[1]

Much ink has been spilt on the underdetermination thesis in the debate over scientific realism. In general, anti-realists argue that the theoretical claims of any given theory are underdetermined by evidence, hence there are no grounds for belief in them. Recently, Norton (1993; 1994) has claimed that underdetermination in practice rarely occurs, and explains this by observing that grounds for belief in a theory can be established by means other than simple hypothetico-deductive inference. In particular, he describes how demonstrative induction was used to establish belief in Planck's quantum hypothesis.[2] In this essay, I look at another example of demonstrative induction; namely, Weinberg's (1995; 1997) argument that seeks to establish local quantum field theory as the only type of quantum theory that is in accord with the relevant evidence and satisfies the general principles of Lorentz Invariance and Cluster Decomposition.

The essay is divided into two parts. In Part I, I present a version of the underdetermination argument and indicate how demonstrative induction can serve as a foil. I then present Weinberg's argument schematically and consider the extent to which it can be considered a demonstrative induction. In Part II, I present an exposition of the technical details involved in the argument.

Synthese **117:** 1–30, 1999.

PART I

1. *Underdetermination and Scientific Realism*

Following Earman (1993) (see, also, Horwich 1982b), I take scientific realism to be composed of two parts: a semantic component and an epistemic component. The semantic component characterizes the realist's desire to read scientific theories literally. It maintains that the theoretical claims of certain theories have referential status. The epistemic component characterizes the realist's contention that there can be good reason to believe the theoretical claims of certain theories. The underdetermination argument attempts to demonstrate that semantic realism undermines epistemic realism. In general, it takes the following form, where (ER) and (SR) are Epistemic Realism and Semantic Realism respectively and (EI) denotes what I shall refer to as the Epistemic Indistinguishability thesis:

(ER) Belief in some class C of theories is justified.

(EI) For any theory $T \in C$, there is a theory $T' \in C$ such that,

- (i) Any reason to believe T is a reason to believe T' and vice versa;
- (ii) If T' and T are read literally, they make contradictory claims.

(SR) For all $T \in C$, T is to be read literally.

$\therefore$ $((SR) \wedge (EI)) \Rightarrow \sim(ER)$.

(The assumption driving the conditional in the conclusion evidently is that if belief in T entails belief in $\sim T$, then we should refrain from belief in T.) The options for the epistemic realist are then:

(a) Reject (SR).
(b) Reject (EI). There are two ways to do this.

- (i) Reject (EIii) by claiming that a literal construal of T and T' need not entail a contradiction.
- (ii) Reject (Eli) by claiming that, for two theories T and T', there are always reasons to prefer one theory over the other.

In this essay, I consider how demonstrative induction can serve as a basis for Option (bii).[3] To do so, it is first necessary to make a further disticntion between two variants of Option (bii), a non-empirical version and an empirical version. The former looks to such non-empirical traits as simplicity, explanatory power, unifying power, etc., as ways of characterizing the epistemically privileged class C of theories appearing in (ER). Fine (1986) and Kukla (1994b) have charged that forms of inference based on such traits beg the question for the realist insofar as there is no justification for such forms that will satisfy a die-hard anti-realist who licenses warrant only for inferences based on empirical data. For such an anti-realist, the epistemic indistinguishability thesis becomes an empirical indistinguishability thesis and a distinction between empirical claims and theoretical claims must be made.[4] One way to effectively engage the anti-realist then is to adopt her criterion of warrant and then demonstrate that inferences based on this criterion, in some cases, yield unique theories. Specifically, I shall take the view that the underdetermination argument in its empirical indistinguishability guise is motivated by a hypothetico-deductive approach to confirmation wherein belief in a theory is conditioned solely by its empirical consequences. The claim then is that, in actual practice, other forms of inference based on empirical data play roles in determining theory from evidence.[5] In this essay, I look at a particular example of one such form; namely, a demonstrative induction that seeks to establish local quantum field theory as the unique theory inferred from evidence obtained from scattering experiments in conjunction with two basic physical principles.

2. *Weinberg's Demonstrative Induction*

Weinberg claims the following:

> ... quantum field theory is the way it is because (aside from theories like string theory that have an infinite number of particle types) it is the only way to reconcile the principles of quantum mechanics (including the cluster decomposition property) with those of special relativity. (1995, xxi)

To see how this can be construed as a demonstrative induction, consider the general form of the latter (referred to hereafter as DI) as given in Norton (1994, 11):

(1) Premises of lesser generality.
(2) Premises of greater generality.

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(3) Conclusion of intermediate generality.

The distinguishing characteristics of this form are (a) contrary to its name, it is a deductive argument: The conclusion is meant to follow deductively from the premises, and (b) as a consequence, the inductive risk involved in making the inference is placed squarely in the premises. This latter characteristic should be compared with hypothetico-deductive forms of inference in which the inductive risk is placed on the rule of inference itself. (In other words, the risk of using a demonstrative induction lies in accepting the premises; whereas the risk of using a hypothetico-deductive inference lies in the form of the inference itself.)[6]

Weinberg's argument is based on three general physical principles that govern the way descriptions of physical processes are constructed. These are associated in turn with quantum mechanics, special relativity and the locality constraint referred to as cluster decomposition.

I. (*Quantum Mechanics*) First recall that the state of a physical system as described by quantum mechanics is completely (up to arbitrary phase) specified by a vector in a Hilbert space $\mathcal{H}$. Physically measurable quantities are probabilities for experimental outcomes and are represented by the squared amplitudes of the overlap of state vectors.[7] As a result of the complex/linear properties of Hilbert spaces, such amplitudes can be linearly superposed and transform under probability-preserving unitary transformations.

II. (*Special Relativity*) Second, special relativity requires that descriptions of physical processes satisfy Lorentz invariance; i.e., they remain invariant under transformations of the inhomogeneous Lorentz (alternatively, Poincaré) group IO(3, 1). This in turn is based on the assumptions that spacetime is isotropic and homogeneous and its symmetries, at the local level, are generated by IO(3, 1).

III. (*Cluster Decomposition*) Finally, a description of a physical process isolated in a laboratory setting should be independent of the complete state of the world outside the laboratory.

Since the empirical evidence for QFT comes in the form of scattering experiments, the above principles can be collapsed into 2 conditions on the quantum mechanical characterization of scattering given by the *S*-matrix. Following the DI schema above, Weinberg's argument then takes the general form:

(1) Empirical evidence for QFT.
(2) A physically satisfactory *S*-matrix satisfies the principles of Lorentz Invariance (LI) and Cluster Decomposition (CD).

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(3) Local Quantum Field Theory.

Three clarificatory remarks are in order concerning this reconstruction.

First, by field theory, Weinberg means the Fock space representation built from Wigner's analysis of single-particle states as irreducible representations of the Poincaré group. Specifically, for the purposes of Weinberg's argument, the distinguishing characteristic of the application of such a theory to descriptions of physical processes is the presence of a Hamiltonian density constructed out of local field operators. In slightly more detail, in most expositions one is presented with two equivalent ways of constructing a local quantum field theory in Minkowski spacetime.[8] The first starts with Wigner's definition of single-particle states as irreducible representations of the Poincaré group. A Fock space $\mathcal{F}$ is then constructed, raising and lowering operators $a^{\dagger}(q)$, $a(q)$ are introduced, and position-dependent local field operators $\hat{\psi}(x)$ are obtained as their Fourier transforms (where the hat is used here solely to distinguish the quantum case from the classical case). The alternative is to start with the theory of a classical field, postulate the standard canonical commutation relations (ccr) for the field variables and their conjugate momenta, and then identify the Fourier expansion coefficients of the fields as raising and lowering operators on a Fock space. Schematically,[9]

(I) IO(3, 1) $\rightarrow$ "particles" $\rightarrow \mathcal{F} \rightarrow a^{\dagger}(q),\, a(q) \underset{\text{F.T.}}{\rightarrow} \hat{\psi}(x)$ (local quantum field)

(II) $\varphi(x)$ (classical field) $\underset{\text{ccr}}{\rightarrow} \hat{\psi}(x) \underset{\text{F.T.}}{\rightarrow} a^{\dagger}(q),\, a(q) \rightarrow \mathcal{F} \rightarrow$ "particles"

The second clarificatory remark concerns the basic source of Premise (1) in the above reconstruction of Weinberg's argument. This comes from scattering processes in which some number of particles, traveling freely a short time in the past (effectively at $t = -\infty$ for elementary particle time scales) collide with each other and then separate. According to the principles of quantum mechanics, a short time after the collision (effectively at $t = +\infty$), the system is in a superposition of free states, each of which describes a possible end result of the collision. The probability amplitudes for these results are given by the S-matrix. In effect, then, to describe scattering events in terms of the physical principles of quantum mechanics requires introduction of the S-matrix.

The third clarificatory remark concerns the two general principles asserted in Premise (2). First, an operator Q on a Hilbert space is Lorentz invariant[10] just when it commutes with the unitary operator representations $U(\Lambda, a)$ of the Poincaré group: $U(\Lambda, a)QU^{\dagger}(\Lambda, a) = Q$. Second, Cluster Decomposition in brief requires that scattering experiments in regions of spacetime separated by great spatial distances do not interfere. This imposes a factorization condition on the S-matrix describing such processes.

Weinberg's strong claim then is that the local field theory formalism obtained via Approach I in general, and in particular, the presence of a Hamiltonian density constructed out of local field operators, is the only way to guarantee that the S-matrix satisfies the general principles of Lorentz Invariance and Cluster Decomposition. To reiterate, local quantum field theory is the only way to reconcile the principles of quantum mechanics with special relativity and Cluster Decomposition. In slightly more technical detail, he demonstrates the following: Given the Dyson expansion of the S-matrix,

(A) The following conditions are sufficient for the S-matrix to be Lorentz invariant:

  (i) The interaction Hamiltonian density $\mathcal{H}_{\text{int}}(x)$ of the theory is a Lorentz scalar;
  (ii) $[\mathcal{H}_{\text{int}}(x), \mathcal{H}_{\text{int}}(x')] = 0$, for spacelike $(x - x')$.

(B) The following condition is sufficient for the S- matrix to satisfy Cluster Decomposition:

  (i) The full Hamiltonian H is a sum of products of raising and lowering operators $a^\dagger(q)$, $a(q)$ with coefficients that are smooth functions of the momenta apart from a single 3-momentum delta function factor.

(C) A sufficient condition for the compatibility of (A) and (B) is the following:

  (i) $\mathcal{H}_{\text{int}}(x)$ is a sum of products of local quantum fields $\psi(x)$; i.e., fields that satisfy $[\psi(x), \psi(x')] = 0$, for spacelike $(x-x')$, and are linear in $a^\dagger(q)$, $a(q)$, with coefficients that are smooth functions of momenta.

In Part II, I flesh out the technical details involved in Approach I and how the inference is made from Premises (1) and (2) to local QFT by means of the claims (A), (B) and (C). In the next section, I consider the extent to which the argument can be considered a valid demonstrative induction.

3. *QFT and Underdetermination*

In this section, I first consider the feasibility of the premises in the reconstruction of Weinberg's argument in Section 2. Next I consider the extent to which the argument can be considered a demonstrative induction.

(1) A demonstrative induction is only as good as its premises. I take the nature of Premise (1) to be uncontroversial. For inductive skeptics who think otherwise, I offer little solace beyond the qualification that demonstrative inductions of this type are meant to be effective against the anti-realist who employs the empirical indistinguishability version of the underdetermination argument as reconstructed in Section 1. Such an anti-realist is willing to concede belief in empirical claims but balks when it comes to theoretical claims. I refer to footnote 4 for skeptics who claim that the distinction between empirical and theoretical claims on which this response is based is problematic. (I claim, at the least, that anti-realists who wish to employ the underdetermination argument must at some point make such a distinction.)

I claim further that the principles of quantum mechanics given in Section 2 are unavoidable insofar as (i) they are fundamental to the manner in which quantum mechanical descriptions of physical phenomena are constructed, and (ii) such descriptions are highly confirmed. (By "highly confirmed" I mean they satisfy whatever criteria are judged necessary and sufficient to warrant belief by the anti-realist who adopts the empirical indistinguishability thesis.) The same holds for the principle of Lorentz Invariance. This is a basic tenet of special relativity and, insofar as the latter is highly confirmed, LI is unavoidable in descriptions of physical phenomena.

Finally, I have two comments to make concerning Cluster Decomposition (CD) that aim at clarifying its relation to special relativity and its relation to the similar locality constraint of micro-causality (4.1.11 below) for fields. First, note that special relativity is usually associated with two principles: LI (Lorentz Invariance) and NSC ("no superluminal causation"). CD is independent of LI insofar as interaction Hamiltonians can be constructed that are unitary and LI, but do not satisfy CD.[11] Moreover, CD is weaker than NSC insofar as CD is applicable to classical as well as relativistic settings. It is only in the latter context that CD translates into NSC. In this setting, I take it to be uncontroversial in the same sense as LI is above.[12] In this setting, CD serves the same purpose for the *S*-matrix as micro-causality does for fields: both are locality constraints that prohibit causal influences from propagating between space-like separated regions of spacetime. Moreover, micro-causality of fields is a sufficient condition for the S-matrix to satisfy CD. The following proof is given in Brown (1992, 313).

An N-point time-ordered correlation function, or τ-function, is a time-ordered vacuum expectation value of N local fields $\langle T\{\phi(x_1)\cdots\phi(x_N)\}\rangle$ (I consider only the neutral scalar field case for simplicity). The significance of τ-functions for the purposes of this proof is that they can be used to calculate S-matrix elements by means of the Lehmann–Symanzik–Zimmermann (LSZ) reduction formula.[13] Hence, if τ-functions satisfy CD, so does the S-matrix. Consider now an $(N+M)$-point τ-function $\langle T\{\phi(x_1)\cdots\phi(x_N)\phi(y_1)\cdots\phi(y_M)\}\rangle$ where the coordinates $x_1\cdots x_N$ and $y_1\cdots y_M$ are separated by a large spacelike interval R^μ, $R^2>0$. Define new coordinates $\bar{y}_b^\mu$ by $y_b^\mu = R^\mu + \bar{y}_b^\mu$ (so the $\bar{y}_b^\mu$ are close to the x_b). By micro-causality, $[\phi(x_a),\phi(y_b)]=0$, since $(x_a-y_b)^2>0$, $\forall a, b$. Hence the time-ordering factors, and we have:

$$\begin{aligned}(3.1)\quad &\langle T\{\phi(x_1)\ldots\phi(x_N)\phi(\bar{y}_1)\ldots\phi(\bar{y}_M)\}\rangle\\ &= \langle T\{\phi(x_1)\ldots\phi(x_N)\}T\{\phi(\bar{y}_1)\ldots\phi(\bar{y}_M)\}\rangle\\ &= \langle T\{\phi(x_1)\ldots\phi(x_N)\}\, \mathrm{e}^{-\mathrm{i}P_\mu R^\mu}\, T\{\phi(\bar{y}_1)\ldots\phi(\bar{y}_M)\}\rangle,\end{aligned}$$

where in the second line $\phi(y_b) = \mathrm{e}^{-\mathrm{i}P_\mu R^\mu}\phi(\bar{y}_b)\,\mathrm{e}^{\mathrm{i}P_\mu R^\mu}$, corresponding to a translation of the fields $\phi(y_b)$ by R^μ (recalling that translations leave the vacuum invariant). Inserting a complete set of states $\sum|n\rangle\langle n| = 1$, one then obtains,

$$\begin{aligned}(3.2)\quad &= \langle T\{\phi(x_1)\ldots\phi(x_N)\}\rangle\langle T\{\phi(\bar{y}_1)\ldots\phi(\bar{y}_M)\}\rangle\\ &+ \textstyle\sum_{n\neq \mathrm{VAC}} \langle 0|T\{\phi(x_1)\ldots\phi(x_N)\}|n\rangle\, \mathrm{e}^{-\mathrm{i}p_n R^\mu}\\ &\langle n|T\{\phi(\bar{y}_1)\ldots\phi(\bar{y}_M)\}|0\rangle.\end{aligned}$$

For $R^2\to\infty$, the second term in (3.2) vanishes by the Riemann–Lebesque Lemma.[14] One then has

$$\begin{aligned}&\langle T\{\phi(x_1)\ldots\phi(x_N)\phi(y_1)\ldots\phi(y_M)\}\rangle\\ &= \langle T\{\phi(x_1)\ldots\phi(x_N)\}\rangle\langle T\{\phi(y_1)\ldots\phi(y_M)\}\rangle,\end{aligned}$$

which is the desired CD result (where use has been made of the translation invariance of τ-functions to replace the $\bar{y}_b$ coordinates with y_b).

Schematically, the above demonstrates: (micro-causality for fields) $\Rightarrow$ (CD of S-matrix). Recall that Weinberg's argument runs in the opposite direction. According to his strong claim, we have,

(A) (LI of S-matrix) $\Rightarrow$ (micro-causality for $\mathcal{H}_{\mathrm{int}}(x)$);

(B) (CD of S-matrix) $\Rightarrow$ ($a^\dagger(q), a(q)$ decomposition of $\mathcal{H}_{\mathrm{int}}(x)$);

and hence,

(C) (LI & CD of S-matrix) $\Rightarrow$ (field decomposition of $\mathscr{H}_{\text{int}}(x)$ with micro-causality for fields).

It appears that whether one prioritizes micro-causality for fields or cluster decomposition for the S-matrix depends on what one takes to be the fundamental observables of the theory. For Weinberg, the fundamental observable is the S-matrix. Indeed, here is Weinberg's take on the priority of micro-causality for fields:

> Such considerations of causality are plausible for the electromagnetic field, any one of whose components may be measured at a given spacetime point .... However, we will be dealing here with fields like the Dirac field of the electron that do not seem in any sense measurable. The point of view taken here is that [the micro-causality condition] is needed for the Lorentz invariance of the S-matrix, without any ancillary assumptions about measurability or causality. (1995, 198)

(Note that, technically, a field at a point is not an observable. Fields are operator-valued distributions defined as smeared averages over arbitrarily small regions of spacetime. The only modification to the micro-causality constraint is that it should take the form $[\psi_m^\dagger[f], \psi_n[h]]_\pm = [\psi_m[f], \psi_n^\dagger[h]]_\pm = 0$, when the supports of the test functions f and h are spacelike separated.)

This emphasis on the fundamentality of the S-matrix also influences Weinberg's attitude toward other approaches to quantization. Consider the canonical field approach (Approach II of Section 2) which begins with the theory of a classical field given either by a Lagrangian or a Hamiltonian. Given that fields are the primary observables of the theory, this approach seems sensible. However, it's not clear how such an approach guarantees a physically satisfactory S-matrix. There are Hamiltonians that are manifestly non-Lorentz covariant, yet yield perfectly acceptable Lorentz invariant S-matrices. On the other hand, given a scalar Lagrangian, Noether's theorem guarantees Lorentz Invariance of the S-matrix. However, unitarity of the S-matrix is now obscure. (The unitarity constraint on the S-matrix was glossed over in the above. Generally, in a scattering process, something has to happen; i.e., the squares of the amplitudes of all the scattering probabilities (the elements of the S-matrix) must sum to one. This requires that the S-matrix operator be unitary. This is guaranteed by the hermiticity of the Hamiltonian in the Dyson expansion.)

(2) To what extent can Weinberg's argument be considered a demonstrative induction? What he demonstrates, Claims (A)–(C) of Section 2, can be represented schematically by,

$$\text{(local QFT)} \Rightarrow \text{(LI and CD of } S\text{-matrix)},$$

where, for "local QFT", read "$\mathscr{H}_{\text{int}}(x)$ is a sum of products of local fields $\psi(x)$ which are linear in $a^\dagger(q)$, $a(q)$ with coefficients that are smooth functions of momenta". In various places, however, he makes the stronger claim,

$$\text{(LI and CD of } S\text{-matrix)} \Rightarrow \text{(local QFT).}$$

Indeed, this is what is needed for the reconstructed demonstrative induction of Section 2 to be valid. That he is not ultimately making this stronger claim is evident in Weinberg (1997, 7–8) where he lists four objections to it:

(i) The argument assumes perturbation theory and it is generally thought that power expansions such as (4.2.7) diverge at high orders for theories like QED and QCD (see, e.g., Kaku 1993, 451, for a discussion).

(ii) The requirement that the Hamiltonian density be a local Lorentz scalar is not necessary for Lorentz invariance of the S-matrix. For instance, in Coulomb gauge, the QED Hamiltonian density contains a non-local, non-Lorentz covariant Coulomb interaction term which serves to cancel a similar non-covariant term in the photon propagator. In general, non-covariant terms in the propagator arise for any interaction involving a vector field (see, e.g., Weinberg 1995, 278). Gauge theories are of this type and canonical quantization becomes problematic as a result, especially in the non-Abelian case (quantization via the functional integral approach is the standard procedure in these cases).

(iii) String theory is a counterexample to the strong claim. String theories are conformally invariant field theories in 4 or more spacetime dimensions for which S-matrices that satisfy LI and CD can be constructed. They differ from local QFT's insofar as, when formulated via functional integration in phase space, the "paths" integrated over are 2 dimensional world sheets as opposed to 1 dimensional particle trajectories. For local QFT's, a method exists to second quantize the purely classical sum over paths, resulting in a quantum theory. For string theories, no such method of second quantization at present exists. This, however, does not reflect on the ability of such theories to produce S-matrices that are LI and CD.

(iv) Finally, as indicated above, the argument assumes the fundamentality of the S-matrix. This assumes 3 types of idealization. First, it requires that spacetime is sufficiently flat. Second, it requires that asymptotic particle states are well-defined; i.e., it assumes no interactions occur effectively for all times before and after the scattering event. Third, it ignores possible effects due to quantum gravity at sufficiently small spacetime scales.

To get around these objections, Weinberg (1997, 8) essentially weakens the strong claim by replacing “local QFT” with “effective field theory (EFT)”. The amended argument now is, schematically,

> (LI and CD of S-matrix at low energies) $\Rightarrow$ (local QFT at low energies).

In words, any theory that looks Lorentz invariant and satisfies Cluster Decomposition at low energies and large distances will look like a local quantum field theory at low energies and large distances. Or, in other words,

> (LI and CD of S-matrix) $\Rightarrow$ (EFT).

The idea behind effective field theory is to write down the most general Lagrangian consistent with the symmetries of the physical system to be described and treat the resulting field theory as valid only within a given energy range. The most blatant consequence of this is that it avoids the constraint of renormalizability: By writing down the most general Lagrangian for the system, an infinite number of counterterms from the infinite number of interactions allowed by symmetries becomes available to cancel all divergences in the perturbative expansion. Restriction to the low energy/large distance sector of the theory then allows terms of higher orders to be disregarded. Hence, Lagrangians that are not renormalizable (in the usual power-counting sense) can now be considered viable. (Note that there is non-trivial content to the effective field theory programme insofar as it does have predictive power; it is not just an ad hoc heuristic that avoids the problems of renormalization. For instance, it provides useful perturbation expansions for low-energy pions and low-energy gravitons. For the former, see Weinberg (1996, Chapter 19.5); for the latter, Donoghue (1995).

This move does indeed counter Objections (i), and (iv) by definition. Objection (iii) is also avoided, as string theory produces local QFT at low energies and large distances. Furthermore, at present, there are no other counterexamples in the running. What still remains slightly problematic is Objection (ii). Perhaps it can be addressed simply by translating “local QFT” as,

> local QFT′: The Lagrangian density $\mathcal{L}[\psi_i]$ is a Lorentz scalar functional of local fields ψ_i which are linear in $a^\dagger(q), a(q)$ with coefficients that are smooth functions of momenta.[15]

As indicated in 1) above, this is sufficient for Lorentz invariance of the S-matrix. It is also necessary and sufficient for Cluster Decomposition. If the Lagrangian density is a sum of products of local fields obeying micro-causality, then the S-matrix will satisfy CD, as indicated by the proof in 1) above. Conversely, if the S-matrix satisfies CD, then the interpolating fields that appear in the LSZ τ-function formulation will satisfy micro-causality;[16] and one such particular set of interpolating fields are those that appear in the Lagrangian density. What remains to be shown is that local QFT$'$ is necessary for Lorentz invariance of the S-matrix, and it is this condition to which objection (iii) applies. Schematically, we now have, in the first instance,

$$\text{(LI and CD of } S\text{-matrix)} \Rightarrow \text{(local QFT}'\text{)},$$

with Objections (i), (iii) and (iv) still in force. In the amended second instance,

$$\text{(LI and CD of } S\text{-matrix)} \Rightarrow \text{(EFT)},$$

where the low-energy, large-distance sector is taken to be local QFT$'$. This takes care of Objection (ii), and Weinberg's demonstrative induction succeeds for effective field theory.[17] Note, however, that using local QFT$'$ in the argument spoils Weinberg's desire to avoid assumptions concerning causality and measurability for fields insofar as micro-causality for fields now cannot be seen to follow from the LI constraint on the S-matrix.

What, then, is the upshot of Weinberg's demonstrative induction? In the first instance, it demonstrates that emphasis on the S-matrix as the fundamental observable describing the behavior of matter at very short distance scales places severe constraints on the form of any theory that attempts to describe such phenomena. We've seen that these constraints are not so severe as to pick local quantum field theory as the only possible description. But they come close. Furthermore, if the programme of effective field theory is adopted, the range of possibilities is restricted even further. Given the fundamentality of local fields along with the S-matrix, a full-fledged demonstrative induction for effective field theory can be constructed. It should finally be noted that these observations certainly do not contribute to a realist interpretation of field theory;[18] however, they do mitigate against an anti-realist interpretation by at least foiling the underdetermination argument: They make it clear that coming up with a theory as descriptively accurate as local quantum field theory is not as easy a matter as the underdetermination argument would have us believe.

PART II

4. *Weinberg's Demonstrative Induction: Technical Details*

In this last section, I provide the technical details behind Weinberg's argument. To set the terminology, in Sections 4.1 and 4.2, I recall some facts governing the standard method of constructing a Fock space of free multiparticle states and the derivation of the Dyson expansion of the S-matrix in terms of transitions between asymptotic multiparticle states. This sets the stage for Weinberg's argument which is presented in Sections 4.3–4.5. The following is based for the most part on Weinberg's own treatment in Chapters 2–4 of his (1995) which may be consulted for further details.

4.1. *Local Field Theory from Poincaré Invariant Quantum Theory*

Wigner's Theorem tells us that proper orthochronous Poincaré transformations are given by unitary operators $U(\Lambda, a)$ on a Hilbert space.[19] The initial task in constructing a local field theory is to determine the states on which these operators act and the transformation rules they obey under this action. Given these states, one can then construct a Fock space, which is the appropriate Hilbert space for field theory.

Recall that single particle states are uniquely labeled by irreducible representations of the Poincaré group ISO(3, 1). Specifically, each state is uniquely labeled by the eigenvalues of the Casimir operators of ISO(3, 1), of which there are two: $P_\mu P^\mu$ with eigenvalues $-m^2$, and $W_\mu W^\mu$ where $W_\mu = -1/2\epsilon_{\mu\upsilon\rho\sigma} J^{\upsilon\rho} P^\sigma$, with eigenvalues $-m^2\sigma(\sigma+1)$ (where the $J^{\upsilon\rho}$ are the generators of homogeneous Lorentz transformations). These serve to label a given irreducible representation by means of its mass m and its spin σ (or helicity for the case $m = 0$). A single particle state may thus be represented by the Hilbert space vector $|p, \sigma\rangle$, where p and σ denote its momentum and spin, respectively. Under finite translations and homogeneous Lorentz transformations, these states transform according to,

$$\text{(4.1.1)}\quad U(1, a)|p, \sigma\rangle = \mathrm{e}^{-iP\cdot a}|p, \sigma\rangle = \mathrm{e}^{-\mathrm{i}p\cdot a}|p, \sigma\rangle.$$

$$\text{(4.1.2)}\quad U(\Lambda)|p, \sigma\rangle = \sqrt{(\Lambda p)^0/p^0}\sum_{\sigma'} D_{\sigma\sigma'}(W(\Lambda, P))|\Lambda p, \sigma'\rangle,$$

respectively, where the Wigner rotation matrix $W(\Lambda, p) = L^{-1}(\Lambda p)\Lambda L(p)$ belongs to the subgroup of the homogeneous Lorentz group that consists of transformations W^μ_υ that leave p^μ invariant: $W^\mu_\upsilon p^\upsilon = p^\mu$. This defines the so-called "little group" of p^μ with matrix representations given by the matrices $D_{\sigma'\sigma}$.

Multiparticle states can be constructed by taking the tensor product of the appropriate number of 1-particle states. A multiparticle state in which the number of particles varies is then obtained as an element of a Fock space $\mathcal{F}$, constructed by taking the direct sum of the tensor products of the appropriate 1-particle Hilbert spaces $\mathcal{H}$: $\mathcal{F} = \mathbb{C} \oplus \mathcal{H} \oplus (\mathcal{H} \otimes \mathcal{H}) \oplus (\mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H}) \oplus \cdots$, where the tensor product $\otimes$ is totally anti-symmetric/symmetric, depending on whether the 1-particle states describe fermions/bosons. The states in $\mathcal{F}$ are desirable in describing scattering processes in which the number of particles of various types changes. The general transformation rule for such states is obtained from (4.1.1) and (4.1.2):

$$(4.1.3)\quad U(\Lambda, a)|p_1, \sigma_1; \cdots\rangle =$$

$$= \mathrm{e}^{-\mathrm{i}a_\mu(p_1^\mu+\cdots)}\sqrt{(\Lambda p_1)^0 \cdots / p_1^0 \cdots}$$

$$\times \sum_{\sigma_1' \cdots} D^{(j_1)}_{\sigma_1'\sigma_1}(W(\Lambda, p_1)) \cdots |\Lambda p_1, \sigma_1'; \cdots\rangle.$$

(This is actually the transformation rule for massive particles where the $D^{(j)}$-matrices are unitary spin-j representations of SO(3). A slightly different rule obtains for the massless case.)

The raising and lowering operators $a^\dagger(q)$, $a(q)$ are defined as usual by their actions on N-particle states:

$$(4.1.4)\quad a^\dagger(q)|q_1 \cdots q_N\rangle \equiv$$

$$\equiv |q q_1 \cdots q_N\rangle, a(q)|q_1 \cdots q_N\rangle$$

$$\equiv \sum_{r=1}^{N} (\pm 1)^{r+1} \delta(q - q_r)|q_1 \cdots q_{r-1} q_{r+1} \cdots q_n\rangle,$$

where the ± 1 sign depends on whether the states are fermionic or bosonic and q labels momentum p and spin σ. In particular, acting on the vacuum state $|0\rangle$,

$$(4.1.5)\quad a^\dagger(q_1)a^\dagger(q_2)\cdots a^\dagger(q_N)|0\rangle = |q_1 q_2 \cdots q_N\rangle. \quad a(q)|0\rangle = 0.$$

The standard (anti-)commutation relations then result:

$$(4.1.6)\quad [a^\dagger(q'), a^\dagger(q)]_\pm = [a(q'), a(q)]_\pm = 0,$$

$$[a(q'), a^\dagger(q)]_\pm = \delta(q' - q),$$

where the $\pm$ indicates an anti-commutator/commutator, depending on whether the states are fermionic/bosonic. The transformation rules for $a^\dagger(q)$, $a(q)$ are obtained from (4.1.3), (4.1.5) and the condition that the vacuum state be Lorentz invariant:

$$(4.1.7)\quad U(\Lambda, a)a^\dagger(\mathbf{p}, \sigma)U^\dagger(\Lambda, a) =$$

$$= \mathrm{e}^{-\mathrm{i}(\Lambda p)\cdot a}\sqrt{\Lambda p_0/p_0}\sum_{\sigma'} D^{(j)*}_{\sigma'\sigma}(W(\Lambda, p))a^\dagger(\mathbf{p}_\Lambda, \sigma'),$$

$$U(\Lambda, a)a(\mathbf{p}, \sigma)U^\dagger(\Lambda, a) =$$

$$= \mathrm{e}^{\mathrm{i}(\Lambda p)\cdot a}\sqrt{\Lambda p_0/p_0}\sum_{\sigma'} D^{(j)}_{\sigma'\sigma}(W(\Lambda, p))a(\mathbf{p}_\Lambda, \sigma'),$$

where $\mathbf{p}_\Lambda$ is the 3-vector part of Λp.

At this point, standard treatments introduce field operators as Fourier transforms of $a^\dagger(q)$, $a(q)$. Weinberg does not do this, moving on instead to a discussion of scattering theory before further developing the formalism. Again, his strong claim amounts to the contention that the LI and CD constraints on the S-matrix force the introduction of local field operators. For completeness sake, however, I shall continue with the standard treatment and indicate how local quantum field operators are introduced within it. This can then be compared to Weinberg's treatment in Section 4.5.

The position basis for $\mathcal{F}$ is constructed by taking Fourier transforms of $a^\dagger(q)$, $a(q)$:

$$(4.1.8)\quad \psi^+_m(x) = \sum_\sigma (2\pi)^{-3/2}\int \mathrm{d}^3\mathbf{p}\, u_m(\mathbf{p}, \sigma)\mathrm{e}^{\mathrm{i}p\cdot x}a(\mathbf{p}, \sigma),$$

$$\psi^-_m(x) = \sum_\sigma (2\pi)^{-3/2}\int \mathrm{d}^3\mathbf{p}\, v_m(\mathbf{p}, \sigma)\mathrm{e}^{-\mathrm{i}p\cdot x}a^\dagger(\mathbf{p}, \sigma),$$

where m labels the components of the fields $\psi^+_m(x)$ and $\psi^-_m(x)$. In general, they are required to satisfy the the transformation rules,

$$(4.1.9)\quad U(\Lambda, a)\psi^\pm_m(x)U^\dagger(\Lambda, a) = M_{mn}(\Lambda^{-1})\psi^\pm_n(\Lambda x + a),$$

where the matrices $M_{mn}(\Lambda)$ are representations of the homogeneous Lorentz group.

The fields (4.1.8) satisfy the (anti-) commutation relations,

$$(4.1.10)\quad [\psi_m^+(x), \psi_n^-(y)]_\pm = (2\pi)^{-3} \int d^3\mathbf{p} u_m(\mathbf{p}, \sigma) v_n(\mathbf{p}, \sigma)\, e^{ip\cdot(x-y)}.$$

To construct a *local* field, i.e., one whose (anti-) commutator vanishes at spacelike distances, the linear combination $\psi_m(x) = \kappa_m \psi_m^+(x) + \lambda_m \psi_m^-(x)$ is taken, which then can be made to satisfy,

$$(4.1.11)\quad [\psi_m^\dagger(x), \psi_n(y)]_\pm = [\psi_m(x), \psi_n^\dagger(y)]_\pm = 0, \quad \text{for } (x-y) \text{ spacelike.}$$

This condition for field operators is normally referred to as micro-causality. If the spacelike (anti-) commutator of two fields did not vanish, this would imply that measurements of spacelike separated fields could interfere, violating causality.

For any local quantum field, the coefficients κ and λ and the expansion coefficients $u_m(\mathbf{p}, \sigma)$ and $v_m(\mathbf{p}, \sigma)$ are completely determined by the representation of the homogeneous Lorentz group under which it transforms and by the micro-causality condition. I now turn to a brief discussion of scattering processes and the Dyson perturbative expansion of the S-matrix.

4.2. *Multiparticle Interactions and Perturbative S-matrix Theory*

Consider a scattering process involving M in-coming particles with ith momentum p_i and N out-going particles with ith momentum p_i'. The state of the system at $t = -\infty$, before the scattering event has occurred, is represented by a localized "in" multiparticle state $|\alpha\rangle_{\text{in}} \equiv |p_1, \sigma_1; \ldots p_M, \sigma_M\rangle_{\text{in}}$. At $t = +\infty$, this state has evolved into a localized "out" multiparticle state given by $|\alpha\rangle_{\text{out}} \equiv |p_1', \sigma_1'; \ldots p_N', \sigma_N'\rangle_{\text{out}}$. These in/out states are considered asymptotically free, transforming under the rule (4.1.3), and can thus be expanded in a superposition by means of a time-evolving Hamiltonian H. Schematically,

$$(4.2.1)\quad e^{-iHt} \int d\alpha\, g(\alpha)|\alpha\rangle_{\text{in/out}} = \int d\alpha\, e^{-iE_\alpha t} g(\alpha)|\alpha\rangle_{\text{in/out}}.$$

where, for instance, $d\alpha = d^3\mathbf{p}_1 \cdots d^3\mathbf{p}_M$ (for in states) and $g(\alpha)$ represents a product of Gaussian functions $g(\mathbf{p}_i)$, $i = 1 \cdots M$, each of which serves to localize the corresponding single-particle state as a wave packet peaked at $\mathbf{p}_i$. To describe the scattering interaction, H is split into a "free" part H_0 and an interaction part V in such a way that the eigenstates $|\alpha\rangle$ of H_0 have the same eigenvalues as the in/out states:

$$(4.2.2)\quad H|\alpha\rangle_{\text{in/out}} = E_\alpha|\alpha\rangle_{\text{in/out}}, \qquad H_0|\alpha\rangle = E_\alpha|\alpha\rangle.$$

The requirement that the localized in/out states are asymptotically free can then be written, using (4.2.1), as,

$$\text{(4.2.3)}\quad e^{-iHt}\int d\alpha\, g(\alpha)|\alpha\rangle_{\text{in/out}} \underset{t\to\mp\infty}{\to} e^{-iH_0t}\int d\alpha\, g(\alpha)|\alpha\rangle,$$

or schematically as $|\alpha\rangle_{\text{in/out}} = \Omega(\mp\infty)|\alpha\rangle$, where $\Omega(t) \equiv e^{iHt}\, e^{-iH_0t}$.

Elements of the S-matrix $S_{\beta\alpha}$ are probability amplitudes for transitions between in- and out-states:

$$\begin{aligned}\text{(4.2.4)}\quad S_{\beta\alpha} \equiv_{\text{out}} \langle\beta|\alpha\rangle_{\text{in}} &= \langle\beta|\Omega^{\dagger}(+\infty)\Omega(-\infty)|\alpha\rangle\\ &= \langle\beta|U(+\infty,-\infty)|\alpha\rangle,\end{aligned}$$

where the evolution operator $U(t, t_0)$ is given by $U(t, t_0) = e^{iH_0t}\, e^{-iH(t-t_0)}\, e^{-iH_0t_0}$. The task now is to obtain an explicit form for $U(t, t_0)$, and hence a form for the S-matrix. This is done by solving the differential equation,

$$\begin{aligned}\text{(4.2.5)}\quad (\partial/\partial t)U(t, t_0) &= e^{iH_0t}(iH_0 - iH)\, e^{-iH(t-t_0)}\, e^{-iH_0t_0}\\ &= -iV_{\text{int}}(t)U(t, t_0),\end{aligned}$$

where $V_{\text{int}}(t) \equiv e^{iH_0t}\, V\, e^{-iH_0t}$. For small interactions, solutions to (4.2.5) with initial condition $U(t_0, t_0) = 1$ may be expanded in powers of V_{int} yielding,

$$\begin{aligned}\text{(4.2.6)}\quad U(t, t_0) = &\sum_{n=0}^{\infty}(-i^n/n!)\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \cdots \int_{t_0}^{t}\\ &\times dt_n\, T\{V_{\text{int}}(t_1)V_{\text{int}}(t_2)\cdots V_{\text{int}}(t_n)\}.\end{aligned}$$

where the time-ordered product $T\{V_{\text{int}}(t_1)\ \cdots\ V_{\text{int}}(t_n)\}$ orders the $V_{\text{int}}(t)$ according to $t_1 > t_2 > \cdots > t_n$. The S-matrix operator S can now be identified with $U(+\infty, -\infty)$. Substituting into (4.2.4), the Dyson series expansion of the S-matrix is given by

$$\begin{aligned}\text{(4.2.7)}\quad S_{\beta\alpha} = &\sum_{n=0}^{\infty}(-i^n/n!)\int_{-\infty}^{\infty} dt_1\, dt_2 \cdots dt_n\\ &\times\langle\beta|T\{V_{\text{int}}(t_1)V_{\text{int}}(t_2)\cdots V_{\text{int}}(t_n)\}|\alpha\rangle.\end{aligned}$$

I now move on to Claims (A)–(C) of Section 2. In Sections 4.3 and 4.4, I explain Claims (A) and (B) which describe the conditions under which (4.2.7) satisfies the two constraints of Lorentz Invariance and Cluster Decomposition separately. In Section 4.5, I explain Claim (C) which describes the conditions under which (4.2.7) satisfies the two constraints jointly.

4.3. *Lorentz Invariance of the S-matrix*

Weinberg's first constraint is that the S-matrix be invariant under inhomogeneous Lorentz transformations. Formally, for the S-matrix operator S, $U(\Lambda, a)SU^{\dagger}(\Lambda, a) = S$.

Claim (A) (Weinberg 1995, 144): The S-matrix is Lorentz invariant if the interaction can be written as,

$$(4.3.1)\quad V_{\text{int}}(t) = \int \mathrm{d}^3x\,\mathcal{H}_{\text{int}}(\mathbf{x}, t),$$

where $\mathcal{H}_{\text{int}}(x)$ is a Lorentz scalar,

$$(4.3.2)\quad U(\Lambda, a)\mathcal{H}_{\text{int}}(x)U^{\dagger}((\Lambda, a) = \mathcal{H}_{\text{int}}(\Lambda x + a),$$

satisfying,

$$(4.3.3)\quad [\mathcal{H}_{\text{int}}(x), \mathcal{H}_{\text{int}}(x')] = 0, \qquad \text{for } (x - x') \text{ spacelike}.$$

Proof. Substituting (4.3.1) into the form (4.2.6) for the S operator yields,

$$S = \sum_{n=0}^{\infty}(-\mathrm{i}^n/n!)\int_{-\infty}^{\infty} \mathrm{d}^4x_1 \cdots \mathrm{d}^4x_n T$$
$$\times \{\mathcal{H}_{\text{int}}(x_1)\mathcal{H}_{\text{int}}(x_2) \cdots \mathcal{H}_{\text{int}}(x_n)\},$$

which is manifestly Lorentz invariant except for the time-ordering term. However, since the time-ordering of two points x_1, x_2 is Lorentz invariant unless $(x_1 - x_2)$ is spacelike, $T\{\mathcal{H}_{\text{int}}(x_1) \cdots \mathcal{H}_{\text{int}}(x_n)\}$ will be Lorentz invariant if (4.3.3) holds (i.e., (4.3.3) guarantees that time-ordering does not matter for spacelike fields). QED

Lorentz invariance of $S_{\beta\alpha}$ also requires that it be proportional to an overall 4-momentum conserving delta function $\delta^4(p_\beta - p_\alpha) = \delta(E_\beta - E_\alpha)\delta^3(\mathbf{p}_\beta - \mathbf{p}_\alpha)$ (where $p_\beta(p_\alpha)$ is the sum of all 4-momenta in the multiparticle state $\beta(\alpha)$). To see this, note that, from (4.1.1) and the fact that Lorentz transformations are unitary, we have, for translations, $S_{\beta\alpha} =_{\text{out}} \langle\beta|U^{-1}(1, a)U(1, a)|\alpha\rangle_{\text{in}} = \mathrm{e}^{\mathrm{i}a\cdot(p_\beta - p_\alpha)}S_{\beta\alpha}$. Since there is no a^μ-dependence on the left-hand side, the a^μ-dependence on the right must vanish; i.e., $S_{\beta\alpha}$ must conserve 4-momentum. In the next section, a much stronger constraint on $S_{\beta\alpha}$ is enforced that requires its *connected* components to be proportional to a *single* 3-momentum delta-function. I turn now to this constraint.

4.4. *Cluster Decomposition of the S-matrix*

The second constraint Weinberg imposes on the S-matrix is Cluster Decomposition (CD), which states the following:

For $\mathcal{N}$ multiparticle processes $\alpha_1 \to \beta_1$, $\alpha_2 \to \beta_2$, ..., $\alpha_{\mathcal{N}} \to \beta_{\mathcal{N}}$ in $\mathcal{N}$ very distant laboratories, the S-matrix element for the overall process factorizes:

$$(4.4.1)\quad S_{\beta_1+\beta_2+\cdots+\beta_{\mathcal{N}},\alpha_1+\alpha_2+\cdots+\alpha_{\mathcal{N}}} \to S_{\beta_1\alpha_1} S_{\beta_2\alpha_2} \cdots S_{\beta_{\mathcal{N}}\alpha_{\mathcal{N}}},$$

where, for $i \neq j$, all the particles in states α_i and β_i are at a great spatial distance from all of the particles in states α_j and β_j.[20]

Claim (B) (Weinberg 1995, 182): Let state β be comprised of N particles with ith momentum $\mathbf{p}'_i$ and state α be comprised of M particles with ith momentum $\mathbf{p}_i$. Then $S_{\beta\alpha}$ satisfies CD if

$$(4.4.2)\quad H = \sum_{N,M=0}^{\infty} \int \mathrm{d}q'_1 \cdots \mathrm{d}q'_N\, \mathrm{d}q_1 \cdots \mathrm{d}q_M$$

$$\times\, a^\dagger(q'_1) \cdots a^\dagger(q'_N) a(q_M) \cdots a(q_1) h_{NM}(q'_1 \cdots q_m),$$

where the coefficients h_{NM} contain a single delta function $\delta^3(\sum \mathbf{p}'_i - \sum \mathbf{p}_i)$:

$$h_{NM}(q'_1 \cdots q'_N, q_1 \cdots q_M) = \delta^3(\mathbf{p}'_1 + \cdots + \mathbf{p}'_N - \mathbf{p}_1 - \cdots - \mathbf{p}_M)$$

$$\times f_{NM}(q'_1 \cdots q'_N, q_1 \cdots q_M),$$

where f_{NM} are smooth functions of the momenta.[21]

To prove this claim, a condition on the connected components of the S-matrix equivalent to CD is first demonstrated. The connected components of the S-matrix are defined by the decomposition:

$$(4.4.3)\quad S_{\beta\alpha} = \sum_{\text{part}} (\pm) S^C_{\beta_1\alpha_1} S^C_{\beta_2\alpha_2} \cdots,$$

where the sum is over all possible partitions of the particles in α and β into clusters $\alpha_1\alpha_2 \ldots$ and $\beta_1\beta_2 \ldots$, counting clusters α_i and α_j the same if they differ only by a permutation of particles. (The $(\pm)$ depends on whether the rearrangements $\alpha \to \alpha_1\alpha_2\ldots$, $\beta \to \beta_1\beta_2\ldots$ involve even/odd interchanges of fermion states.) Equation (4.4.3) defines the connected components recursively. Given $S^C_{00} = 0$ and $S_{00} = 1$, we have (assuming stable particles that do not decay or interact with the vacuum),

$$\text{(4.4.4)}\quad S_{q'q} = \delta(q'-q) \equiv S^C_{q'q}$$

$$S_{q'_1q'_2,q_1q_2} = S^C_{q'_1q_1}S^C_{q'_2q_2} \pm S^C_{q'_1q_2}S^C_{q'_2q_1} + S^C_{q'_1q'_2,q_1q_2}$$

$$S_{q'_1q'_2q'_3,q_1q_2q_3} = S^C_{q'_1q_1}S^C_{q'_2q_2}S^C_{q'_3q_3} \pm \text{perm.} + S^C_{q'_1q'_2,q_1q_2}S^C_{q'_3q_3}$$

$$\pm\text{perm.} + S_{q'_1q'_2q'_3,q_1q_2q_3}$$

$$\vdots$$

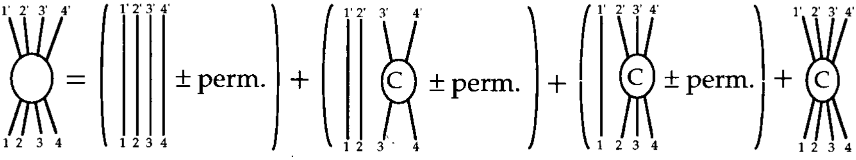

Figure 1. Graphical representation of the *S*-matrix for 4-4 scattering. The "C" denotes a connected component.

Graphically, for 4-4 scattering, (4.4.3) can be represented by Figure 1. The connected components are those matrix elements describing complete interactions between all particles involved. The role of the raising and lowering operators is to make this connectedness property explicit. The idea is to expand the basis states $|\alpha\rangle$, $|\beta\rangle$ appearing in the *S*-matrix (4.2.7) in terms of raising/lowering operators acting on the vacuum, and represent the operator $V(t)$ as a sum of products of raising/lowering operators. The resulting expression is evaluated by commuting all operators through each other, using the (anti-) commutation relations (4.1.6) to get all raising operators on the left and all lowering operators on the right (since such terms vanish). After integration, (4.2.7) is thus reduced to a sum of factors from the operators $V(t)$ and 3-momentum delta functions from the (anti-) commutation relations (4.1.6). The connected components of the *S*-matrix turn out to be those terms for which every initial and final particle and every operator $V(t)$ is connected to the others by a sequence of raising/lowering operators.

The factorization condition (4.4.1) now corresponds to the condition,

(4.4.5) $S^C_{\beta\alpha} \to 0$,

when the spatial distance between any pair of particles in states β and/or α goes to infinity. To see this for 2-2 scattering, for instance, suppose q_1 and q_1' are spacelike separated. Then $S^C_{q_1'q_2',q_1q_2} = S^C_{q_1'q_1} S^C_{q_2'q_2} = 0$, and $S_{q_1'q_2',q_1q_2} = S^C_{q_1'q_1} S^C_{q_2'q_2} = S_{q_1'q_1} S_{q_2'q_2} = \delta(q_1' - q_1)\delta(q_2' - q_2)$, which is the desired CD result.

Weinberg now demonstrates that (4.4.2) is sufficient for (4.4.5) and hence CD. The argument is given schematically by the following steps:

1. A necessary and sufficient condition for (4.4.5) is that $S^C_{\beta\alpha}$ contains a *single* 3-momentum delta function factor $\delta^3(\mathbf{p}_\beta - \mathbf{p}_\alpha)$, where, $\mathbf{p}_\beta(\mathbf{p}_\alpha)$ is the sum of all particle momenta in the state $\beta(\alpha)$.
2. If the connected matrix component $H^c_{\beta\alpha}$ of the Hamiltonian (described below) contains a single factor of $\delta^3(\mathbf{p}_\beta - \mathbf{p}_\alpha)$, then so does $S^C_{\beta\alpha}$.
3. To obtain $H^C_{\beta\alpha}$, it is first demonstrated that any operator H on $\mathcal{F}$ can be expanded as a sum of products of raising and lowering operators. The connected N-M scattering component of H is then given by $\langle q_1' \cdots q_N' | H^C | q_1 \cdots q_M \rangle = h_{NM}$, where h_{NM} is the NMth coefficient function occurring in the expansion.

Claim (B) is thus established, since condition (4.4.2) implies that $H^C_{\beta\alpha}$ contains a single $\delta^3(\mathbf{p}_\beta - \mathbf{p}_\alpha)$. Proofs of Steps (1) and (3) are given in the appendix. A rigorous proof of Step (2) is given in Weinberg (1964). The connected components H^C of the Hamiltonian H are those matrix elements of H that are given by the interaction part V of H and describe particle interactions in which all particles participate. (Those matrix elements of H that depend solely on H_0 will involve free particles that do not interact.) The relation between H^C and S^C is given by the differential equation (4.2.5), taking the limits $t \to +\infty$, $t_0 \to -\infty$. Naively, this relation preserves connectedness and, moreover, does not affect delta function factors. A proof demonstrates that if $V^C(t)$ contains a single delta function, then so does $(i\partial/\partial t)U^C(t, t_0)$. Consequently, if $U^C(t, t_0)$ contains a single delta function at any given time, then it will contain a single delta function for all times; hence, specifically, for $t \to +\infty$ and $t_0 \to -\infty$. The proof is completed by observing that at $t = t_0$, $U(t_0, t_0) = 1$, hence $\langle \mathbf{p}' | U^C | \mathbf{p} \rangle = \delta^3(\mathbf{p}' - \mathbf{p})$.

4.5. *Lorentz Invariance, Cluster Decomposition and Local Quantum Fields*

We now determine the conditions under which Lorentz invariance of the S-matrix is compatible with Cluster Decomposition; i.e., the conditions under which (4.3.2) and (4.3.3) are compatible with (4.4.2). First, there is a problem in reconciling (4.3.2) with (4.4.2). The raising and lowering operators, $a^\dagger(q)$, $a(q)$, transform under inhomogeneous Lorentz transformations according to (4.1.7), in which the $D^{(j)}$-matrices depend on the 4-momentum p. Hence to construct a Lorentz scalar out of linear combinations of raising and lowering operators is problematic. The solution Weinberg proposes is to construct $\mathcal{H}_{\text{int}}(x)$ out of fields,

$$\text{(4.5.1)}\quad \psi_m^+(x) = \sum_\sigma \int \mathrm{d}^3\mathbf{p}\, u_m(x; \mathbf{p}, \sigma) a(\mathbf{p}, \sigma),$$

$$\psi_m^-(x) = \sum_\sigma \int \mathrm{d}^3\mathbf{p}\, v_m(x; \mathbf{p}, \sigma) a^\dagger(\mathbf{p}, \sigma),$$

with coefficients $u_m(x; \mathbf{p}, \sigma)$ and $v_m(x; \mathbf{p}, \sigma)$ chosen such that the fields transform *independently* of p:

$$\text{(4.5.2)}\quad U(\Lambda, a)\psi_m^\pm(x)U(\Lambda, a) = M_{mn}(\Lambda^{-1})\psi_m^\pm(\Lambda x + a).$$

As in (4.1.9), the matrices $M_{mn}(\Lambda)$ are representations of the homogeneous Lorentz group. In general, (4.5.2) is satisfied by absorbing the p-dependency of the $D^{(j)}$-matrices into the expansion coefficients $u_m(x; \mathbf{p}, \sigma)$ and $v_m(x: \mathbf{p}, \sigma)$. $\mathcal{H}_{\text{int}}(x)$ is now guaranteed to be a Lorentz scalar by expressing it as a sum of fields contracted with appropriate Lorentz invariant tensor coefficients:

$$\text{(4.5.3)}\quad \mathcal{H}_{\text{int}}(x) = \sum_{NM} t_{n_1 \cdots n_N, m_1 \cdots m_M} \times \psi_{n_1}^-(x) \cdots \psi_{n_N}^-(x)\psi_{m_1}^+(x) \cdots \psi_{m_M}^+(x),$$

where the constants $t_{m_1 m_2 \ldots}$ transform according to

$$t_{m_1 m_2 \ldots} M_{m_1 n_1}(\Lambda^{-1}) M_{m_2 n_2}(\Lambda^{-1}) \cdots = t_{n_1 n_2 \ldots}.$$

(This guarantees that if $\mathcal{H}_{\text{int}}(x)$ is a local scalar field since the latter form a ring under addition and multiplication.) It turns out that a necessary and sufficient condition for the fields to satisfy (4.5.2), given (4.5.1), is for them to take the form (4.1.8). Note however that the Fourier transform form

(4.1.8) is now a consequence of the LI constraint (4.3.2) on the S-matrix; namely, that $\mathcal{H}_{\text{int}}(x)$ transform as a scalar.

To see that (4.5.3) also satisfies CD, substitute (4.1.8) into it and integrate over x:

$$(4.5.4)\quad \int \mathrm{d}^3x\,\mathcal{H}_{\text{int}}(x) = V$$
$$= \sum_{NM} \mathrm{d}^3\mathbf{p}'_1 \cdots \mathrm{d}^3\mathbf{p}'_N \mathrm{d}^3\mathbf{p}_1 \cdots \mathrm{d}^3\mathbf{p}_M \sum_{\sigma'_1\cdots\sigma'_N} \sum_{\sigma_1\cdots\sigma_M}$$
$$\times a^\dagger(\mathbf{p}'_1\sigma'_1)\cdots a^\dagger(\mathbf{p}'_N\sigma'_N)a(\mathbf{p}_M\sigma_M)\cdots a(\mathbf{p}_1\sigma_1)$$
$$\times \mathcal{V}_{NM}(\mathbf{p}'_1\sigma'_1\cdots\mathbf{p}_M\sigma_M),$$

where,

$$\mathcal{V}_{NM}(\mathbf{p}'_1\sigma'_1\cdots\mathbf{p}'_N\sigma'_N,\mathbf{p}_1\sigma_1\cdots\mathbf{p}_M\sigma_M)$$
$$= \delta^3\left(\sum \mathbf{p}'_i - \sum \mathbf{p}_i\right) f_{NM}(\mathbf{p}'_1\sigma'_1\cdots\mathbf{p}_M\sigma_M),$$

and,

$$f_{NM}(\mathbf{p}'_1\sigma'_1\cdots\mathbf{p}_M\sigma_M) =$$
$$= (2\pi)^{3-3N/2-3M/2} t_{n_1\cdots n_N,m_1\cdots m_M} v_{n_1}(\mathbf{p}'_1\sigma'_1)\cdots u_{m_M}(\mathbf{p}_M\sigma_M).$$

It is clear that the f_{NM} are smooth functions of the momenta, as (4.4.2) requires.

To be compatible with condition (4.3.3), however, not any combination of fields in (4.5.3) will do. This is apparent from the (anti-) commutation relations (4.1.10). Again, the solution is to use the linear combination,

$$(4.5.5)\quad \psi_m(x) = \kappa_m\psi_m^+(x) + \lambda_m\psi_m^-(x),$$

which then satisfies,

$$(4.5.6)\quad [\psi_m^\dagger(x),\psi_n(y)]_\pm = [\psi_m(x),\psi_n^\dagger(y)]_\pm = 0,$$
$$\text{for spacelike } (x-y),$$

for judicious choice of constants κ and λ. This is the same micro-causality condition as (4.1.11), but where the latter was motivated by concerns over causality for the fields, (4.5.6) stems solely from the LI constraint (4.3.3) on the S-matrix.

Hence we have $\mathcal{H}_{\text{int}}(x) = T_{m_1 m_2 \cdots} \psi_{m_1} \psi_{m_2} \ldots$, where the constants $T_{m_1 m_2 \cdots}$ transform as Lorentz tensors. To reiterate, the S-matrix is Lorentz invariant and satisfies Cluster Decomposition given that the Hamiltonian density is a sum of products of local quantum fields (4.5.5); i.e., fields that satisfy the micro-causality condition (4.5.6) and are linear in $a^\dagger(q)$, $a(q)$ with coefficients that are smooth functions of momenta. Weinberg concludes:

> Thus the whole formalism of fields, particles, and antiparticles seems to be an inevitable consequence of Lorentz invariance, quantum mechanics, and cluster decomposition, without any ancillary assumptions about locality or causality. (1997, 6)

APPENDIX

THEOREM 1. The connected component $S^C_{\beta\alpha}$ of the S-matrix vanishes when the spatial distance between any pair of particles in states β and/or α goes to infinity *if and only if* $S^C_{\beta\alpha} = \delta^3(\mathbf{p}_\beta - \mathbf{p}_\alpha) C_p$, where C_p is a smooth function of momenta and $\mathbf{p}_\beta(\mathbf{p}_\alpha)$ is the sum of all 3-momenta in the state $\beta(\alpha)$.

Proof. (A) "$\Leftarrow$". Let state β be comprised of n particles with ith momentum $\mathbf{p}'_i$, and state α be comprised of m particles with ith momentum $\mathbf{p}_i$. By assumption,

$$S^C_{\mathbf{p}'_1 \cdots \mathbf{p}'_n, \mathbf{p}_1 \cdots \mathbf{p}_m} = \delta^3\left(\sum_{i=1}^{n} \mathbf{p}'_i - \sum_{i=1}^{m} \mathbf{p}_i\right) C_{\mathbf{p}'_1 \cdots \mathbf{p}'_n, \mathbf{p}_1 \cdots \mathbf{p}_m} \tag{A.1}$$

where the function $C_{\mathbf{p}'_1 \cdots \mathbf{p}'_n, \mathbf{p}_1 \cdots \mathbf{p}_m}$ does not contain additional 3-momentum delta functions. The Fourier transform of (A.1) is,

$$S^C_{\mathbf{y}_1 \cdots \mathbf{y}_n, \mathbf{y}_{n+1} \cdots \mathbf{y}_{n+m}} = \tag{A.2}$$

$$= \int \mathrm{d}^3\mathbf{p}'_1 \cdots \mathrm{d}^3\mathbf{p}'_n \, \mathrm{d}^3\mathbf{p}_1 \cdots \mathrm{d}^3\mathbf{p}_{m-1} C_{\mathbf{p}'_1 \cdots \mathbf{p}'_n, \mathbf{p}_1 \cdots \mathbf{p}_{m-1}} \, \mathrm{e}^{\mathrm{i} \sum_{j=1}^{n+m} \mathbf{K}_j \cdot \xi_j}$$

where $\xi_j = (\mathbf{y}_j - \mathbf{y}_{j+1})$ and $\mathbf{K}_j = \sum_{l=1}^{j} \mathbf{p}'_l$, for $j \leq n$, or $-\sum_{l=1}^{j-n} \mathbf{p}_l$, for $j > n$. If $C_{\mathbf{p}'_1 \cdots \mathbf{p}'_n, \mathbf{p}_1 \cdots \mathbf{p}_m}$ vanishes at $\pm\infty$, then by the Riemann–Lebesque Lemma, the integral in the last line of (A.2) vanishes as any of the coordinate intervals ξ_j go to $+\infty$

(B) "⇒". In momentum space,

$$\text{(A.3)}\quad \begin{aligned} S^C_{\mathbf{p}'_1\cdots\mathbf{p}'_n,\mathbf{p}_1\cdots\mathbf{p}_m} &= \int \mathrm{d}^3\mathbf{y}_1\cdots\mathrm{d}^3\mathbf{y}_{n+m}\,S^C_{\mathbf{y}_1\cdots\mathbf{y}_{n+m}} \\ &\quad\times \mathrm{e}^{-\mathrm{i}\mathbf{p}'_1\cdot\mathbf{y}_1}\cdots\mathrm{e}^{-\mathrm{i}\mathbf{p}'_n\cdot\mathbf{y}_n}\,\mathrm{e}^{\mathrm{i}\mathbf{p}_1\cdot\mathbf{y}_{n+1}}\cdots\mathrm{e}^{\mathrm{i}\mathbf{p}_m\cdot\mathbf{y}_{n+m}} \\ &= \delta^3\left(\sum\mathbf{p}'_i-\sum\mathbf{p}_i\right)C_{\mathbf{p}'_1\cdots\mathbf{p}'_n,\mathbf{p}_1\cdots\mathbf{p}_m} \end{aligned}$$

where $\int \mathrm{d}^3\mathbf{y}_1\cdots\mathrm{d}^3\mathbf{y}_{n+m-1} = \int \mathrm{d}^3\xi_1\cdots\mathrm{d}^3\xi_{n+m-1}$, $S^C_{\mathbf{y}_1\cdots\mathbf{y}_{n+m}} = S^C_{\xi_1\cdots\xi_{n+m-1}}$, and the function $C_{\mathbf{p}'_1\cdots\mathbf{p}'_n,\mathbf{p}_1\cdots\mathbf{p}_m}$ is given by

$$\text{(A.4)}\quad C_{\mathbf{p}'_1\cdots\mathbf{p}'_n,\mathbf{p}_1\cdots\mathbf{p}_m} = \int \mathrm{d}^3\xi_1\cdots\mathrm{d}^3\xi_{n+m-1}\,S^C_{\xi_1\cdots\xi_{n+m-1}}\,\mathrm{e}^{\mathrm{i}\sum_{j=1}^{n+m}\mathbf{K}_j\cdot\xi_j}.$$

This is a smooth function of the momenta since, by assumption, $S^C_{\xi_1\cdots\xi_{n+m-1}}$ vanishes as any of the coordinate intervals ξ_j go to $+\infty$. QED

THEOREM 2.[22] Any operator H acting on $\mathcal{F}$ may be expressed as a sum of products of raising and lowering operators $a^\dagger(q)$, $a(q)$:

$$\text{(A.5)}\quad H = \sum_{N,M}\int \mathrm{d}q'_1\cdots\mathrm{d}q'_N\,\mathrm{d}q_1\cdots\mathrm{d}q_M\,a^\dagger(q'_1)\cdots a^\dagger(q'_N)$$

$$\times a(q_M)\cdots a(q_1)h_{NM}(q'_1\cdots q'_N, q_1\cdots q_M)$$

Proof. The proof proceeds by induction. First determine h_{11} from the matrix elements of H between two single particle states:

$$\text{(A.6)}\quad \begin{aligned} \langle q'|H|q\rangle &= \int \mathrm{d}q'_1\,\mathrm{d}q_1 h_{11}(q'_1,q_1)\langle q'|a^\dagger(q'_1)a(q_1)|q\rangle \\ &= \int \mathrm{d}q'_1\mathrm{d}q_1 h_{11}(q'_1,q_1)\delta(q'_1-q')\delta(q_1-q) \\ &= h_{11}(q',q) \end{aligned}$$

where $|q\rangle = a^\dagger(q)|0\rangle$, $\langle q'| = \langle 0|a(q')$ and use has been made of the commutation relations (4.1.6). Next determine h_{22} from the matrix elements of H between four single particle states: $\langle q'_1q'_2|H|q_1q_2\rangle = h_{22}(q'_1,q'_2,q_1,q_2)$ + (terms involving h_{11}). Now continue in a similar manner for higher-order terms. QED

THEOREM 3. The connected $N'-M'$ scattering component of the Hamiltonian H is given by $\langle q'_1\cdots q'_{N'}|H^C|q_1\cdots q_{M'}\rangle = h_{N'M'}(q'_1\cdots q'_{N'}, q_1$

$\cdots q_{M'})$ where $h_{N'M'}$ is the $N'M'$th coefficient function occurring in the expansion (A.5).

Proof. Consider a general NM matrix element of H. It will decompose into a sum of disconnected elements and a fully connected element (see Figure 1). For elements with $N < N'$ and/or $M < M'$, there are not enough a^+, a operators to affect all N particles in the initial state and/or all M particles in the final state. Such elements thus contribute only to the disconnected components. For elements with $N > N'$ and/or $M > M'$, there are too many $a^\dagger$, a operators; some will eventually end up annihilating the vacuum. Hence the only part of H to contribute to the fully connected matrix element is $h_{N'M'}$. QED

NOTES

* Thanks to Tony Duncan, John Norton and John Earman for comments and suggestions.

1 In this essay, the spacetime metric has signature (1, 1, 1, −1) and 3-vectors are indicated in bold type.

2 Hudson (1997) claims that underdetermination can be resurrected for the case of Planck's hypothesis. However, this appears to be based on a spurious distinction between "particle" and "degree of freedom" interpretations of the equipartition theorem.

3 Option (a) is adopted by Horwich's (1982a, 1986) global conventionalism. Elsewhere, I argue that option (bi) in the form of a weakened version of semantic realism is viable under the name structural realism.

4 Horwich (1982a) and Laudan and Leplin (1991) maintain that such a distinction cannot be made. Kukla (1994a) claims that it can (although adopting it, he maintains, is just as question-begging for the anti-realist as adopting a non-empirical virtue is for the realist). For the purposes of this essay, I shall assume the distinction is unproblematic. It can, but does not have to be, based on an observable/unobservable split in the vocabulary of the language in which the theory is expressed. In general, it suffices to allow that theoretical claims are expressed in a language that out-strips the language in which evidence statements are made. If such a syntactic distinction is objectionable, allow a distinction in kind between unobserved observables and in-principle-unobservable unobservables, the latter being the subject of theoretical claims. The basis of the anti-realist's underdetermination argument is simply that inferences to the former are warranted whereas inferences to the latter are not.

5 Bayesian updating, arguably, fits this category (while the selection of priors is notoriously subjective, Gaifman/Snir-type convergence theorems provide some measure of objectivity). Inferences based on eliminative induction also fit.

6 For comparison, hypothetico-deductive inferences take the following general form:

(1) If hypothesis h is true, then predictions $a, b, c, \ldots,$ etc. are true.
(2) Predictions $a, b, c, \ldots,$ etc. are true.

---

(3) Hypothesis h is true.

The implicit rule of inference here is that evidence supports a theory just when the evidence is entailed by the theory.

7 More precisely, for the record, a measurable quantity Q corresponds to a self-adjoint operator $\hat{Q}$ on $\mathcal{H}$. The possible values q_i of Q one obtains upon measuring for Q are given by the spectrum of $\hat{Q}$. To obtain the probability that a given state $|\alpha\rangle \in \mathcal{H}$ exhibits a value q for the quantity Q, one expands $|\alpha\rangle$ in the eigenbasis $|q_i\rangle$ of $\hat{Q}$ and takes the squared norm of the overlap $\langle q|\alpha\rangle$.

8 Both approaches fall under what is generally referred to as canonical quantization. By no means does this exhaust methods of quantization that result in local quantum field theories. Kaku (1993, 62) lists the following: Canonical quantization, Gupta–Bleuler (covariant) quantization, the functional integral method, Becchi–Rouet–Stora–Tyupin (BRST), Batalin–Vilkovisky (BV), and stochastic quantization.

9 Some expositions construe the equivalence of the two approaches as indicating the dual natures of the “particle” picture (I) and the field picture (II). This should be discouraged for a number of reasons. First, the elementary states Wigner identifies with the irreducible representations of ISO(3, 1) are not necessarily elementary partides; the ground state of the hydrogen atom, for instance, counts as an elementary state. Second, Wigner’s approach works only in spacetimes that are static; in nonstatic spacetimes, it is notoriously difficult to maintain a consistent particle interpretation. Furthermore, even in interacting field theory in well-behaved spacetimes the duality between particles and fields breaks down if the duality thesis rests on the claim that, to every particle there is a corresponding field, and vice versa. On the one hand, for any asymptotically well-defined particle state, there exists an infinite number of interpolating fields, any one of which can be used to construct the appropriate S-matrix. On the other hand, there are fields that admit no asymptotic particle states; namely, quark fields. With these caveats in mind, I shall continue to use the appellation “particle” in referring to Wignerian elementary states, for the sake of convenience.

10 By this I mean invariant under *both* Lorentz rotations *and* translations, viz. invariant under IO(3, 1), as indicated above. Sometimes Lorentz invariance refers to invariance under only the homogeneous Lorentz group O(3, 1).

11 For example, take

$$V = \int \prod_{i=1}^{4} \frac{d^3 p_i}{\sqrt{2E_i}} \delta^4(p_1 + p_2 - p_3 - p_4) h(p_1, p_2, p_3, p_4) |p_1 p_2\rangle\langle p_3 p_4|$$

and require that $h(p_i) = h * (p_i)$ be a function of Lorentz invariant scalar products of the momenta p_i. Then it is not too difficult to show that V is unitary and satisfies LI. However, V does not satisfy CD, since it projects only onto the 2-particle subspace. As a consequence, interactions exist only when there are 2 particles in the universe and vanish when a third is introduced, *anywhere.* (Technically, the 3-3 connected part of V contains more than one delta function, which is a violation of CD (see below Section 4.4).) This example is discussed by T. Duncan, unpublished lecture notes 1996. See, also, Weinberg 1995, 187.

12 Note that the context in which CD, as a locality constraint, is enforced differs from EPR/Bell- type experiments in which locality constraints are called into question. In the CD context, one considers clusters of scattering events and CD enforces causal independence of spacelike separated clusters. In the EPR context, one considers a single source emitting two products and investigates the relation between these products when they become spacelike separated. It turns out that, under certain assumptions, locality conditions are violated. However, insofar as the entire system of source and products is considered a single cluster,

CD is not violated. If this gloss is objectionable, it is still the case that there are two types of (logically independent) locality principle at play in the EPR/Bell setting, viz. what are referred to in the philosophical literature as outcome independence (alternatively, “Jarrett completeness”) and parameter independence (alternatively, “hidden locality”). Both are required in order to derive a Bell inequality. Arguably, only a violation of the second can be associated with a violation of NSC. Hence, NSC is compatible with violations of Bell inequalities if one denies outcome independence. CD as a locality constraint is thus reconcilable with Bell inequalities, given one associates it with a version of parameter independence.

[13] For instance, for scalar fields, the LSZ formula reads

$$
\begin{aligned}
{}_{\text{out}}\langle \mathbf{p}'_1 \cdots \mathbf{p}'_N \mid \mathbf{p}_1 \cdots \mathbf{p}_M \rangle_{\text{in}} &= (i/\sqrt{Z})^{N+M} \\
&\times \int d^4x_1 \cdots d^4y_M\, f^*_{\mathbf{p}'_1}(x_1) \cdots f^*_{\mathbf{p}'_N}(x_n)(\overrightarrow{\partial}^2_\mu + m^2)_{x_1} \cdots (\overrightarrow{\partial}^2_\mu + m^2)_{x_N} \\
&\times \langle \Omega \mid T\{\phi(y_1) \cdots \phi(y_M)\phi(x_1) \cdots \phi(x_N)\} | \Omega \rangle \\
&\times (\overleftarrow{\partial}^2_\mu + m^2)_{y_1} \cdots (\overleftarrow{\partial}^2_\mu + m^2)_{y_M} f_{\mathbf{p}_1}(y_1) \cdots f_{\mathbf{p}_M}(y_M)
\end{aligned}
$$

where the S-matrix element occurs on the left and the corresponding $(N+M)$-point τ-function occurs on the right with $|\Omega\rangle$ the vacuum state of the full Hamiltonian H, and

$$
f_{\mathbf{p}_j}(y_j) = \frac{1}{(2\pi)^{3/2}(2E(\mathbf{p}_j))^{1/2}}\, \mathrm{e}^{-\mathrm{i}p_j \cdot y_j}.
$$

[14] Briefly, if $f(\omega)$ is a smooth function which vanishes as $\omega \to \pm\infty$, then its Fourier transform vanishes in the limit $t \to \infty$: $\int_{-\infty}^{+\infty} \mathrm{d}\omega f(\omega)\, \mathrm{e}^{-\mathrm{i}\omega t} \to_{t\to\infty} 0$.

[15] If $\mathcal{L}[\psi_i]$ has no constriants, then $\mathcal{H}(x)$ can be obtained via a Legendre transformation. If $\mathcal{L}[\psi_i]$ possesses gauge symmetries, then the system is a constrained Hamiltonian system and one must use Dirac’s procedure to obtain the corresponding Hamiltonian density. (See Weinberg 1995, Chap. 7.6, for a discussion.)

[16] For the 2-point τ-function $\langle T\{AB\}\rangle$, CD stipulates that $\langle T\{AB\}\rangle = \langle A\rangle\langle B\rangle$ when the fields A and B are spacelike separated. In this case, we also have $\langle T\{BA\}\rangle = \langle A\rangle\langle B\rangle$; hence $\langle T\{AB\}\rangle = \langle T\{BA\}\rangle$; hence $[A, B] = 0$, when A and B are spacelike separated. Now extend this for n-point τ-functions.

[17] Granted, this assumes that string theory is the only counter-example to the argument. Here one might argue that it is the only currently viable counter-example. To avoid all *possible* counter-examples, one must demonstrate that any alternative high-energy theory reduces to a local QFT in the low- energy limit. Such a proof has yet to be constructed. Renormalisation group techniques would play a central role here (see Huggett and Weingard 1995 for a discussion of such techniques).

[18] Indeed, Cao and Sweber (1993) offer an interpretation of the effective field theory programme that is anti-foundationalist in its epistemic claims and instrumentalist in its ontological claims.

[19] Cf. Weinberg (1995, 50–51) for a proof. Strictly speaking, the operators $U(\Lambda, a)$ are projective representations of the proper orthochronous Poincaré group (denoted ISO(3, 1)↑); i.e., they are unique up to a phase. To obtain an ordinary representation, one takes representations of the covering group ISL(2, $\mathbb{C}$)↑ of ISO(3, 1)↑. This technical detail

will be omitted in the following. Furthermore, the components of the Poincaré group not connected with the identity (corresponding to the parity and time-inversion operators) must be represented by anti- unitary operators. These, too, will be ignored for brevity's sake.

[20] Alternatively, in the special relativistic context, all the particles in states α_i, β_i are spacelike separated from all the particles in states α_j, β_j, for $i \neq j$. Any spacelike interval $(\mathbf{x} - \mathbf{x}')^2 - (t - t')^2 > 0$ can be sent by a Lorentz transformation into a purely spatial interval $(\bar{\mathbf{x}} - \mathbf{x}')^2 \neq 0$, $(\bar{t} - \bar{t}') = 0$.

[21] Smoothness here requires only that f_{NM} does not depend on additional delta functions of momenta. Note that $\int \mathrm{d}q_i$ represents a sum over spin σ_i and an integration over $d^3\mathbf{p}_i$.

[22] T. Duncan, Univ. Pittsburgh lecture notes 1996, unpublished.

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