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CPT Invariance, the Spin-Statistics Connection, and the Ontology of Relativistic Quantum Field Theories

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Abstract CPT invariance and the spin-statistics connection are typically taken to be essential properties in relativistic quantum field theories (RQFTs), insofar as the CPT and Spin-Statistics theorems entail that any state of a physical system characterized by an RQFT must possess these properties. Moreover, in the physics literature, they are typically taken to be properties of particles. But there is a Received View among philosophers that RQFTs cannot fundamentally be about particles. This essay considers what proofs of the CPT and Spin-Statistics theorems suggest about the ontology of RQFTs, and the extent to which this is compatible with the Received View. I will argue that such proofs suggest the Received View's approach to ontology is flawed.

1 Introduction

This essay is concerned with the ontological status of two properties associated with relativistic quantum field theories: CPT invariance and the spin-statistics connection. I will argue that the typical way of viewing these properties in the physics literature is at odds with a Received View among philosophers, and in this clash, it is the latter view that should be checked.

CPT invariance is the property of being invariant under the combined transformations of charge conjugation (C), space inversion (P, for "parity"), and time reversal (T). The spin-statistics connection is the property that holds of a state just when, if the state obeys Fermi-Dirac statistics, then it possesses half-integer

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spin, and if the state obeys Bose-Einstein statistics, then it possesses integer spin.[1] In the physics literature, these properties are typically taken to be *essential* properties in relativistic quantum field theories (RQFTs), insofar as the CPT theorem and the Spin-Statistics theorem entail that any state of a physical system described by an RQFT must possess them. Moreover, read literally, these theorems entail that the *fundamental states* of an RQFT, i.e., the states that characterize what an RQFT is *about*, must possess CPT invariance and the spin-statistics connection. The physics literature also typically takes the bearers of these properties to be *particle* states. For instance, charge conjugation is usually described as a transformation between the states of particles and antiparticles, and the statistics associated with a state is usually described in terms of a method for counting the possible arrangements of particles associated with that state.[2] States that obey Bose-Einstein statistics are such that, if they differ only in the permutation of two or more particles, they are identical, and similarly for states that obey Fermi-Dirac statistics. In addition, states that obey Fermi-Dirac statistics are constrained by the Exclusion Principle: no two particles associated with such a state can have exactly the same (non-spatiotemporal) properties. These views are summarized by the following two theses:

(I) CPT invariance and the spin-statistics connection are essential properties of fundamental states in RQFTs.
(II) CPT invariance and the spin-statistics connection are properties of particle states.

While Thesis (I), arguably, may be supported by appeals to the CPT and Spin-Statistics theorems, Thesis (II) is a bit more controversial. In the philosophy of physics literature, there is a Received View that claims RQFTs cannot be fundamentally about particles (Clifton and Halvorson 2001; Halvorson and Clifton 2002; Arageorgis et al. 2003; Fraser 2008). This view requires that particles be localizable and countable, and that these characteristics be given mathematical expression in the forms of local and unique total number operators. Various results then indicate that formulations of RQFTs do not support such operators. The Received View concludes that since the mathematical representations of particles are not supported by RQFTs, these theories cannot be said to be fundamentally about particles.

The goal of this essay is to question this Received View. Whereas the Received View first adopts pre-theoretic intuitions about particles, and then concludes that RQFTs cannot represent these pre-theoretic intuitions, the present essay suggests that intuitions about particles should be informed, at least in part, by the theories in

[1] Readers who might object to viewing the spin-statistics connection as a property may view it instead as a principle, law, or disposition in the following discussion. The issue at stake is not so much how to characterize it, but rather whether it is essential, and what it is predicated of.

[2] Typical statements to this effect are found in Peskin and Schroeder (1995): "This conclusion is part of a more general result, first derived by Pauli… particles of integer spin obey Bose-Einstein statistics, while particles of half-odd-integer spin obey Fermi-Dirac statistics "(pp. 57–58)." "At the same time that we discuss P and T, it will be convenient to discuss a third (non-spacetime) discrete operation: *charge conjugation*, denoted by C. Under this operation, particles and antiparticles are interchanged" (p. 64). See, also, Weinberg (1995, pp. 191, 238), Sterman (1993), Jost (1965, pp. 100, 106).

which reference to them occurs. In particular, given that CPT invariance and the spin-statistics connection are essential properties of fundamental states in RQFTs, we should ask what RQFTs say these fundamental states are states of. This suggests that we should look to proofs of the CPT and Spin-Statistics theorems in order to inform the debate over whether the fundamental states of RQFTs describe particles.

Sections 2 and 3 attempt to justify Thesis (I) by reviewing four alternative approaches to proofs of the CPT Spin-Statistics theorems. These approaches differ on their assumptions, and in particular, on what they take to be the essential characteristics of an RQFT. I claim, however, that they should all be interpreted as supporting (I). Section 4 considers the extent to which these approaches support Thesis (II). Finally, Sect. 5 considers options for the Received View. I will argue that the Received View must deny either (I), or (II), or both, and that none of these options is particularly appealing.

2 Essential Properties: The CPT and Spin-Statistics Theorems

The motivation for Thesis (I) stems from the CPT theorem and the Spin-Statistics theorem. These theorems argue from basic assumptions about what constitutes an RQFT to the conclusions that the states of physical systems characterized by these assumptions must possess CPT invariance and the spin-statistics connection. Thus in addition to supporting Thesis (I), these theorems also address the question of what CPT invariance and the spin-statistics connection are properties of; i.e., what the fundamental states of an RQFT are states of. The answer to the latter is complicated by the fact that there is no consensus on what the essential characteristics of an RQFT are. This is manifest in at least four distinct approaches to proofs of the CPT and Spin-Statistics theorems. These include an axiomatic approach, an approach due to Steven Weinberg, a textbook Lagrangian approach, and an algebraic approach. The remainder of this section offers brief accounts of the salient features of these proofs. The immediate aim is to explicitly identify the assumptions that underlie each approach in order to facilitate comparison in the following sections.

It will first help to consider two ways that statistics can be encoded in an RQFT.

(i) Statistics can be encoded on creation and annihilation operators a, $a^\dagger$ that act on multiparticle states in a Fock space by requiring,

$$[a(\mathbf{p}), a^\dagger(\mathbf{p}')]_\mp = \delta(\mathbf{p} - \mathbf{p}') \tag{1}$$

for 3-momenta $\mathbf{p}$, where "$\mp$" indicates a commutator/anti-commutator, depending on whether the particle states are bosonic (i.e., obey Bose-Einstein statistics) or fermionic (i.e., obey Fermi-Dirac statistics). Creation/annihilation operators that commute will create/annihilate multi-particle states that are symmetric under permutation of single-particle substates, whereas creation/annihilation operators that anti-commute will create/annihilate multi-particle states that are anti-symmetric under such permutation. In both cases, such multi-particle states are permutation invariant.[3] In addition, the symmetric case allows, whereas the anti-symmetric case does not allow, the presence of single-particle substates representing identical particles (in the sense of agreeing on all their non-spatiotemporal properties).

(ii) Statistics can be encoded on local field operators by requiring,

$$[\phi^\dagger(x), \phi(y)]_\mp = 0, \quad \text{for spacelike } (x - y) \qquad \text{(Local Commutativity)}$$

where "$\mp$" indicates a commutator/anti-commutator, depending on whether the fields are bosonic or fermionic. This condition is referred to as *Local Commutativity*. It guarantees that, when a Fock space formulation is available, the creation/annihilation operators corresponding to the fields satisfy (1) above. Thus, to say that a field is bosonic (*resp*. fermionic), could mean either that, *by definition*, the field satisfies *Local Commutativity*, or that, when a Fock space formulation is available, the corresponding creation/annihilation operators are associated with particle states that are bosonic (*resp*. fermionic).[4]

Procedure (i) suggests the bearers of statistics are particles, insofar as it encodes statistics in a way that refers explicitly to the behavior of particle states under permutations. Procedure (ii) suggests the bearers of statistics are fields, insofar as it encodes statistics in a way that refers explicitly to the behavior of fields. As we shall see, with one exception, the approaches reviewed below can be distinguished by which of (i) or (ii) they adopt.

2.1 The Wightman Axiomatic Approach

In the axiomatic approach, the CPT theorem was derived originally by Jost (1957) and the Spin-Statistics theorem by Lüders and Zumino (1958) and Burgoyne (1958). This approach is based on the fact that an RQFT characterized by a particular set of axioms can be encoded in the vacuum expectation values of products of its fields, or Wightman functions $F_n(x_1, \ldots, x_n) = \langle\Omega|\phi(x_1)\ldots\phi(x_n)|\Omega\rangle$.[5] The fields are characterized by invariance under the restricted Lorentz group $L_+^\uparrow$ (the subgroup of the

[3] Suppose $|\Phi\rangle$ is a multi-particle state, and let $|\Phi'\rangle$ be a multi-particle state obtained from $|\Phi\rangle$ by permuting its single-particle substates. $|\Phi\rangle$ is *symmetric* just when $|\Phi'\rangle = |\Phi\rangle$. $|\Phi\rangle$ is *anti-symmetric* just when $|\Phi'\rangle = -|\Phi\rangle$. $|\Phi\rangle$ is *permutation invariant* just when, for any linear operator A representing an observable quantity, the expectation value of A is the same for $|\Phi\rangle$ and $|\Phi'\rangle$: $\langle\Phi|A|\Phi\rangle = \langle\Phi'|A|\Phi'\rangle$.

[4] Haag (1996, p. 97) takes the first route. Streater and Wightman (2000, p. 147) suggest the second route: "A natural way to arrive at Bose-Einstein statistics is to describe the system in question by a field which commutes for space-like separations, while the analogous way for Fermi-Dirac statistics is to use a field which anti-commutes for space-like separations." The "natural way", evidently, would be to demonstrate that *Local Commutativity* entails that Fock space creation/annihilation operators corresponding to the fields (when they exist) satisfy (1).

[5] Here $\phi(x)$ is intended to represent a generic quantum field of arbitrary spin and $|\Omega\rangle$ is the corresponding unique vacuum state. In a more precise formulation, the field would be defined as an operator-valued distribution, and the corresponding Wightman function as a tempered distribution. Expositions of the CPT and Spin-Statistics theorems in the axiomatic approach are given in Araki (1999), Haag (1996), and Streater and Wightman (2000).

Lorentz group connected to the identity), which consists of Lorentz boosts, but does not contain parity or time reversal transformations. The fields also obey the *Spectrum Condition*:[6]

The energy of all states is positive (Spectrum Condition)

The *Spectrum Condition* and invariance under $L_+^\uparrow$ entail that Wightman functions can be analytically extended to complex-analytic functions that are invariant under the proper complex Lorentz group $L_+(\mathbb{C})$, which does contain parity and time reversal transformations. Moreover, the domain on which such extended Wightman functions are defined contains real points of analyticity ("Jost points"), and at these points the Wightman functions obey the following "PT" invariance property:

$$F_n(\xi_1, \ldots, \xi_{n-1}) = (-1)^M F_n(-\xi_{n-1}, \ldots, -\xi_1) \tag{PT}$$

where $\xi_i = x_{i+1} - x_i$, and M encodes the spin of the fields: if M is even, the fields have integer spin and if M is odd, they have half-integer spin. The invariance property PT is a special case of invariance under $L_+(\mathbb{C})$; namely, it is invariance under parity and time reversal.[7] Now suppose we impose the wrong spin-statistics connection on PT. To do this, we first encode statistics in the fields by assuming *Local Commutativity* (LC). We next assume the *wrong* spin-statistics connection; i.e., we assume half-integer-spin fields commute and integer-spin fields anti-commute. One can then demonstrate that the fields are identically zero (Streater and Wightman 2000, pp. 148–150). Thus nontrivial fields must possess the spin-statistics connection.

Now suppose that instead of LC, the fields obey *Weak Local Commutativity*:

$$\langle\Omega|\phi(x_1)\ldots\phi(x_n)|\Omega\rangle = i^K\langle\Omega|\phi(x_n)\ldots\phi(x_1)|\Omega\rangle \tag{WLC}$$

where K is the number of fermionic fields. Intuitively, WLC requires the fields to satisfy the appropriate statistics only when they appear in vacuum expectation values. One can then demonstrate that this guarantees CPT invariance in the sense of the existence of an antiunitary operator that combines the actions of C, P, and T transformations on Wightman functions, leaving them invariant (Streater and Wightman 2000, p. 146).

To recap, in the axiomatic approach, we have the following schematic entailments:[8]

A1. [(restricted Lorentz invariance of fields) and (Spectrum Condition)] ⇒ (PT)
A2. (PT & LC) ⇒ (spin-statistics connection)
A3. (PT & WLC) ⇒ (CPT invariance)

[6] More precisely, the spectrum of the momentum operator associated with $L_+^\uparrow$ is confined to the forward lightcone.

[7] As Wightman (1999, p. 744) points out, this is a remarkable fact: "…although only invariance under $L_+^\uparrow$ was assumed for the n-point vacuum expectation values, the associated analytic function is invariant under space–time inversion."

[8] One must also assume that the fields have finitely many components to avoid counterexamples of infinite fields that do not possess the spin-statistics connection or CPT invariance (Streater 1967; Oksak and Todorov 1968).

2.2 Weinberg's Approach

Weinberg's approach is given in Weinberg (1964, 1995).[9] Emphasis is placed on the S-matrix, which describes scattering interactions involving asymptotic particle states. One begins with the definition of single-particle states as finite irreducible representations of the restricted Lorentz group and builds quantum fields out of them in order to construct an S-matrix with two essential properties. Weinberg (1964, p. 1318) originally based this approach on three assumptions:

(i) *Perturbation Theory*. The S-matrix is calculated from Dyson's perturbative expansion formula, which depends on time-ordered products of an interaction Hamiltonian density $\mathfrak{H}_{\text{int}}(x)$.

(ii) *Restricted Lorentz Invariance of the S-Matrix*. The S-matrix is invariant under restricted Lorentz transformations. A sufficient condition for this is that $\mathfrak{H}_{\text{int}}(x)$ is a Lorentz scalar, and it commutes at spacelike distances: $[\mathfrak{H}_{\text{int}}(x), \mathfrak{H}_{\text{int}}(y)] = 0$, for spacelike $(x - y)$.[10]

(iii) *Particle Interpretation*. $\mathfrak{H}_{\text{int}}(x)$ is constructed out of the creation and annihilation operators a, $a^\dagger$ for free particles.

In Weinberg (1995), (iii) is replaced by an additional constraint on the S-matrix:

(iii′) *Cluster Decomposition of the S-matrix*. The S-matrix satisfies cluster decomposition (briefly, scattering experiments in spacelike separated regions of spacetime do not interfere). A sufficient condition for this is (iii).

The creation and annihilation operators a, $a^\dagger$ in (iii) are defined by their actions on multi-particle states in a Fock space constructed out of single-particle states. This definition entails that they satisfy the (anti-)commutation relations (1). Weinberg then demonstrates that a sufficient condition ("the only known way" Weinberg 1964, p. 1318) for (iii) to be compatible with (ii) is that $\mathfrak{H}_{\text{int}}(x)$ be constructed out of restricted Lorentz invariant fields $\psi_m(x)$ which are linear combinations of the creation and annihilation operators and which satisfy *Local Commutativity*. Schematically, these fields take the general form (Weinberg 1995, p. 198),

$$\psi_m(x) = \kappa_m \psi_m^+(x) + \lambda_m \psi_m^-(x) \tag{2}$$

where $\psi_m^+(x), \psi_m^-(x)$ are linear combinations of a, $a^\dagger$, respectively, with coefficients chosen such that the fields transform appropriately under the restricted Lorentz group (the index m denotes the particular type of field). The constants κ_m, λ_m are chosen so that the fields satisfy *Local Commutativity* (LC). Weinberg (1995, pp. 202, 223, 238) then demonstrates that the choice of κ_m and λ_m that guarantees LC *also* guarantees that the fields possess the spin-statistics connection. Thus, for Weinberg, the spin-statistics connection is a property of fields that (a) are built from creation/annihilation operators that encode particle statistics via (1); (b) transform as

[9] Massimi and Redhead (2003) compare Weinberg's approach to the Spin-Statistics theorem with the standard textbook approach reviewed in Sect. 2.3 below.

[10] The latter commutativity condition guarantees that time-ordered products of $\mathfrak{H}_{\text{int}}(x)$ are restricted Lorentz invariant.

irreducible representations of the restricted Lorentz group; and (c) satisfy LC. Moreover, these conditions are all consequences of assumptions (i)–(iii) above.

Weinberg (1995, pp. 198–199) further demonstrates that if the fields carry a non-zero value of a conserved charge, then to every particle state there must correspond an antiparticle state.[11] Note that this is not quite a demonstration of CPT invariance. To prove the latter, from Weinberg's point of view, requires an explicit demonstration that the full Hamiltonian density is invariant with respect to the composition of C, P, and T operators (Weinberg 1995, pp. 244–246). The demonstration ultimately rests on the transformation properties of the creation and annihilation operators a, $a^\dagger$ under C, P, and T separately, which determine how the fields (2) transform, and hence how the Hamiltonian density transforms. The P and T transformations follow from the behavior of a, $a^\dagger$ under restricted-Lorentz transformations, while the C transformation is posited to hold between a, $a^\dagger$ on the one hand, and antiparticle operators a^c, $a^{c\dagger}$ on the other, where the existence of the latter is entailed by the existence of antiparticles.

To recap, in Weinberg's approach, we have the following schematic entailments:

B1. [(restricted Lorentz invariance of *S*-matrix) & (cluster decomposition of *S*-matrix)] ⇒ (LC)
B2. (LC) ⇒ (spin-statistics connection)
B3. [(restricted Lorentz invariance of *S*-matrix) and (cluster decomposition of *S*-matrix) & (existence of conserved charges)] ⇒ (CPT invariance)

2.3 The Textbook Lagrangian Approach

In Weinberg's approach, one begins with the definition of single-particle states as irreducible representations of the restricted Lorentz group, and then constructs quantum fields out of them. In textbook accounts of RQFTs, one begins with a classical Lagrangian field theory, and then "second quantizes" the fields to obtain quantum fields. Second quantization involves the construction of a Fock space from the properties of the solution space of a classical field equation. Part of this process involves positing *Local Commutativity* (LC) for the fields, which then entails the appropriate commutation relations (1) for the Fock space creation and annihilation operators. So far, this is a reversal of Weinberg's procedure, in which LC is derivative, in part, of (1). To derive the spin-statistics connection, however, the textbook approach takes a slightly different tack. Following Fierz (1939) and Pauli (1940), it introduces a causality constraint (sometimes referred to as "microcausality"):

[11] The existence of a conserved charge entails that $\mathfrak{H}_{\text{int}}(x)$ must commute with the charge operator Q. This entails that $\mathfrak{H}_{\text{int}}(x)$ must be formed out of fields ψ_m that have simple commutation relations with Q. To accomplish this, it suffices to construct ψ_m as a sum $\psi_m = \psi_m^+(x) + \psi_m^{+c\dagger}(x)$, where $\psi_m^+(x)$ and $\psi_m^{+c}(x)$ are linear combinations of creation/annihilation operators a, a^c for particle states with the same mass and spin, but opposite charge; i.e., ψ_m is a sum of fields associated with particles and their antiparticles.

The observable quantities associated with an RQFT commute at spacelike distances. (Causality)

Pauli (1940, p. 721) justifies this constraint in the following manner:

> We shall, however, expressively postulate in the following *that all physical quantities at finite distances exterior to the light cone (for* $|x_0{}' - x_0{}''| < |\mathbf{x}' - \mathbf{x}''|$*) are commutable*… The justification for our postulate lies in the fact that measurements at two space points with a space-like distance can never disturb each other, since no signals can be transmitted with velocities greater than that of light.

Note that *Causality* is distinct from *Local Commutativity* (LC); in particular, *Causality* only requires commutativity, and it does not explicitly refer to statistics (i.e., fermions and bosons). Moreover, it suggests that fermionic fields that anti-commute according to LC are not observable quantities. Sterman (1993, p. 167) provides the standard explanation in the specific case of spin-1/2 Dirac fields:

> If the commutators of Dirac fields do not vanish at spacelike distances, what becomes of causality? We recall, however, that spinors are double-valued representations of the rotation group. As such, a spinor is not itself directly observable, since a rotation by 2π changes its sign. On the other hand, operators that are bilinear in the field—such as components of the energy–momentum tensor—do not change sign, and are observables. More generally, we may consider any operator of the form $B_i(x) = \bar{\psi}(x)O_i\psi(x)$, where O_i is some matrix, possibly combined with differential operators. We can easily show that equal-time commutators between the B_i vanish, if the fields obey [anti-commutation relations].

This suggests that the observable quantities that *Causality* refers to be identified either with bosonic fields or bilinears in fermionic fields. The textbook approach then demonstrates that imposing anti-commutators on the creation/annihilation operators of a restricted Lorentz invariant integer spin field (i.e., "second quantizing" the field with the "wrong statistics") violates *Causality*; and imposing commutators on the creation/annihilation operators of a restricted Lorentz invariant half-integer spin field violates either *Causality* or the *Spectrum Condition* (see, e.g., Kaku 1993, pp. 87, 90; Peskin and Schroeder 1995, pp. 52–58).

In the textbook approach, CPT invariance requires a demonstration that the Hamiltonian density associated with the Lagrangian density of an RQFT is invariant under the operation CPT. As Kaku (1993, pp. 120–123) outlines, this requires two assumptions:

(i) The Lagrangian density $\mathfrak{L}(x)$ is a local, Hermitian Lorentz scalar.
(ii) The spin-statistics connection holds for the fields that appear in $\mathfrak{L}(x)$.

Assumption (i) is necessary and sufficient for $\mathfrak{L}(x)$ to be CPT invariant, and assumption (ii) is necessary and sufficient to subsequently show that the Hamiltonian density derived from $\mathfrak{L}(x)$ is CPT invariant.[12]

[12] This assumes there is a corresponding Hamiltonian density.

To recap, in the standard textbook Lagrangian approach, we have the following schematic entailments:

C1. [(restricted Lorentz invariance of fields) and (Spectrum Condition) and (Causality)] $\Rightarrow$ (spin-statistics connection for fermions)
C2. [(restricted Lorentz invariance of fields) and (Causality)] $\Rightarrow$ (spin-statistics connection for bosons)
C3. [(spin-statistics connection) and (restricted Lorentz invariance of fields) and (local Hermitian Lagrangian)] $\Rightarrow$ (CPT invariance)

2.4 The Algebraic Approach

A final formulation of the CPT and Spin-Statistics Theorems was given by Guido and Longo (1995) in the context of algebraic quantum field theory. The basic object in this approach is a net of von Neumann algebras, $\mathcal{O}\mapsto\Re(\mathcal{O})$, that assigns a local algebra of observables $\Re(\mathcal{O})$ to every double-cone region $\mathcal{O}$ of Minkowski spacetime (a double-cone region is the intersection of the causal future of a point with the causal past of another point to the future of the first). The local algebras are required to satisfy isotony: if $\mathcal{O}_1 \subset \mathcal{O}_2$, then $\Re(\mathcal{O}_1) \subset \Re(\mathcal{O}_2)$; and this entails that they generate a quasi-local algebra $\Re$. The elements of $\Re$ can be represented as bounded linear operators that act on a separable Hilbert space $\mathcal{H}_0$ with a cyclic and separating vacuum vector Ω.[13] Three additional assumptions are then required. The first is the algebraic analog of *Causality*, call it *Microcausality*, which requires observables associated with spacelike separated regions to commute:

$$\text{For } A_1 \in \Re(\mathcal{O}_1), A_2 \in \Re(\mathcal{O}_2), \quad \text{and} \quad \mathcal{O}_1, \mathcal{O}_2 \text{ spacelike} \quad \text{separated}, [A_1, A_2] = 0. \qquad \text{(Microcausality)}$$

The second assumption is *Weak Additivity*, which requires that the quasilocal algebra $\Re$ be generated by the local algebras associated with regions obtained by arbitrary translations from any given double-cone $\mathcal{O}$:

$$\Re = \bigcup\nolimits_x \Re(\mathcal{O} + x). \qquad \text{(Weak Additivity)}$$

The third assumption is what Guido and Longo call *Modular Covariance* (MC). This requires that, for any wedge region W of Minkowski spacetime, the *modular operator* Δ_W^{it} of the local algebra $\Re(W)$ of the wedge implements Lorentz boosts on $\Re$.[14] Formally, for any wedge W and any double-cone $\mathcal{O}$,

[13] Ω is cyclic for $\Re$ just when $\{A\Omega : A \in \Re\}$ is dense in $\mathcal{H}_0$. Ω is separating for $\Re$ just when $A\Omega = 0$ and $A \in \Re$ entails $A = 0$.

[14] A wedge region in Minkowski spacetime M is any Poincaré transformation of the region $\{x \in M : x_1 > |x_0|\}$, where (x_1, x_2, x_3, x_0) is an inertial coordinate system. That a modular operator exists is entailed by the Tomita-Takesaki theorem (see, e.g., Halvorson and Müger 2006, p. 738; Haag 1996, p. 217). The latter demonstrates that a von Neumann algebra $\Re$ of bounded linear operators on a Hilbert space $\mathcal{H}$ with a cyclic and separating vacuum vector Ω possesses a modular operator Δ and a modular conjugate operator J such that $J\Omega = \Omega = \Delta\Omega$, $\Delta^{it}\Re\Delta^{-it} = \Re$, and $J\Re J = \Re'$, where $\Re'$ is the commutant of $\Re$ (i.e., the set of bounded linear operators that commute with all elements of $\Re$).

$$\Delta_W^{it}\Re(\mathcal{O})\Delta_W^{-it} = \Re(\Lambda_W(t)\mathcal{O}) \qquad \text{(Modular Covariance)}$$

where $\Lambda_W(t)$ is the one-parameter group of Lorentz boosts that leave W invariant. MC is motivated by a theorem due to Bisognano and Wichmann (1976) which demonstrates that for a von Neumann algebra generated by local polynomial algebras of Wightman fields, the modular operator of the local algebra of any wedge implements Lorentz boosts, and the modular conjugate operator is given by the CPT operator that leaves the Wightman fields invariant.

Under the above three assumptions, Guido and Longo were able to show that CPT invariance holds for a particular type of representation of $\Re$, what are called *DHR representations*; and that the spin-statistics connection holds for a subset of such representations. DHR representations were defined by Doplicher et al. (1971, p. 200) in the following way,

DHR Representation: Let $(\mathcal{H}_0, \pi_0)$ be the vacuum representation of $\Re$ generated by the vacuum state ω_0.[15] Then a DHR representation with respect to ω_0 is a representation $(\mathcal{H}, \pi)$ such that $\pi|_{\Re(\mathcal{O}')}$ is unitarily equivalent to $\pi_0|_{\Re(\mathcal{O}')}$. for any double cone $\mathcal{O}$.

A DHR representation is unitary equivalent to the vacuum representation except for some bounded region of spacetime $\mathcal{O}$ (where $\mathcal{O}'$ is the causal complement of $\mathcal{O}$). Thus the states associated with DHR representations are supposed to represent localized states in so far as they differ from the vacuum only in some bounded region of spacetime. Doplicher et al. (1971) showed that DHR representations possess conjugates (which can be interpreted as representing antimatter), and admit representations of the permutation group (thus they can be characterized in terms of statistics). DHR representations that admit finite representations of the permutation group are referred to as possessing finite statistics.

Guido and Longo's (1995, pp. 530–531) derivation of the spin-statistics connection and CPT invariance then takes the following schematic form: For a von Neumann algebra $\Re$ of local observables with a cyclic vacuum representation,

D1. [(Microcausality) & (Weak Additivity) & MC] $\Rightarrow$ (CPT invariance for DHR representations)

D2. [(Microcausality) & (Weak Additivity) & MC] $\Rightarrow$ (spin-statistics connection for irreducible, restricted Poincaré-invariant DHR representations with finite statistics and masses)

In D1, CPT invariance refers to the existence of an anti-unitary operator Θ that implements parity and time reversal transformations on $\Re$ (i.e., $\Theta\Re(\mathcal{O})\Theta = \Re(-\mathcal{O})$), maps DHR representations to their conjugates, and is given, essentially, by the modular conjugate operator of the algebra of any wedge.[16] In D2,

[15] In general, a representation of $\Re$ consists of a pair $(\mathcal{H}, \pi)$ where $\mathcal{H}$ is a Hilbert space and π is a map that takes elements of $\Re$ to bounded linear operators on $\mathcal{H}$. A state ω on $\Re$ is a linear map that takes elements of $\Re$ to complex numbers. The GNS theorem entails that any state can be associated with a unique representation (Araki 1999, p. 34; Halvorson and Müger 2006, p. 734).

[16] More precisely, $\Theta = J_W R_W$, where J_W is the modular conjugate operator of $\Re$ restricted to the wedge W, and R_W implements rotations that leave W invariant.

restricted Poincaré invariance refers to invariance under the universal covering of the restricted Poincaré group, which includes Lorentz boosts and translations.[17]

3 Comparison

At this point it is informative to summarize some of the differences between the above alternative approaches to the CPT and Spin-Statistics theorems.

1. First, the approaches can be categorized in terms of how they treat interactions. The primary empirical evidence for RQFTs comes in the form of scattering experiments involving interactions, thus if one is concerned with the question of what the world would be like if RQFTs were true, one should look to interacting RQFTs for the answer. The method for treating interactions, typified by Weinberg's approach and the textbook Lagrangian approach, is to employ perturbation theory. Infamously, perturbative series expansions of physical quantities like scattering cross-sections typically are ill-defined mathematically: they involve infinities and require the apparatus of renormalization theory. This is part of the motivation for the axiomatic and algebraic formalisms, which attempt to provide mathematically rigorous foundations for RQFTs. Currently, however, they are incomplete: no non-trivial interacting RQFTs can be formulated in these approaches. We thus are faced with the following dilemma:[18]

(A) The Weinberg and textbook Lagrangian formalisms are complete but typically mathematically ill-formed.
(B) The axiomatic and algebraic formalisms are incomplete but mathematically well-formed.

Pragmatists who adopt (A) may claim that CPT invariance and the spin-statistics connection are properties of both interacting and non-interacting states in RQFTs. On the other hand, such pragmatism comes with a price. In practice, in the Weinberg and textbook Lagrangian approaches, interactions are treated by employing the LSZ reduction formula to calculate the S-matrix in terms of time-ordered vacuum expectation values of interpolating fields (see, e.g., Bain 2000). These fields interpolate between asymptotic particle states that are taken to be free

[17] Doplicher et al. (1974) derived the spin-statistics connection for irreducible, restricted Poincaré-invariant DHR representations with finite statistics, positive masses, and finitely many components, under the assumptions of *Microcausality*, *Haag Duality*, and *Property B* (for definitions of the latter, see Araki 1999, pp. 163–64, or Halvorson and Müger 2006, p. 784). Guido and Longo recover this result in the following way: They first demonstrate that *Microcausality*, *Weak Additivity*, and MC entail *Essential Duality*, which is a weaker form of *Haag Duality* that still allows Doplicher, Haag and Robert's analysis to go through. They further demonstrate that *Microcausality*, *Weak Additivity*, and MC entail the existence of a unique unitary representation of the restricted Poincaré group that acts on $\mathfrak{R}$ and satisfies the *Spectrum Condition*. This has two consequences. First, the uniqueness of this representation rules out counterexamples to the spin-statistics theorem of fields with infinitely many components (Guido and Longo 1995, p. 519). Second, *Microcausality*, the *Spectrum Condition*, and *Weak Additivity* entail *Property B* (Halvorson and Müger 2006, p. 748).

[18] For defenses of positions associated with (A) and (B), see Wallace (2011) and Fraser (2011), respectively.

at asymptotic times ($t \to \pm\infty$). The rigor of this formalism, however, is made problematic due to the consequences of Haag's theorem, which indicates that the Hilbert spaces for interacting and free states cannot be the same, thus a unitary S-matrix operator that transforms free states into interacting states does not exist (see., e.g., Earman and Fraser 2006).[19]

Purists who adopt (B) may be hard put in explaining how CPT invariance and the spin-statistics connection can be said to be properties of interacting states. They may find partial solace in an extension of the axiomatic approach known as Haag-Ruelle scattering theory. Briefly, under the assumption that the Hilbert space $\mathcal{H}$ of an interacting RQFT contains a single-particle subspace, this approach demonstrates the existence of "in" and "out" states in $\mathcal{H}$ which can be interpreted as free particle states at asymptotic times (Haag 1996, pp. 88–89). Assuming the subspaces $\mathcal{H}_{in}, \mathcal{H}_{out}$ spanned by these states satisfy *Asymptotic Completeness*: $\mathcal{H}_{in} = \mathcal{H}_{out} = \mathcal{H}$, one can show that the in and out states are related by a unitary S-matrix operator, and one can then employ the apparatus of the LSZ formalism to calculate S-matrix elements. Moreover, the in and out states are also restricted Lorentz invariant and possess both CPT invariance and the spin-statistics connection. Thus, on the basis of Haag-Ruelle scattering theory, purists may argue that asymptotic states in an interacting RQFT possess CPT invariance and the spin-statistics connection. On the other hand, again, none of the current empirically confirmed interacting RQFTs in particle physics admit formulations in terms of Haag-Ruelle scattering theory.

2. A second way in which the approaches in Sect. 2 differ is over the assumption of restricted Lorentz invariance. While much could (and should) be said about the role of relativity in the CPT and Spin-Statistics theorems in general, for present purposes I will restrict discussion to the following brief comments. Restricted Lorentz invariance is an explicit assumption in all approaches except the algebraic approach.[20] For the latter, first note that the operative property is invariance under the restricted Poincaré group $P_+^\uparrow$ which consists of Lorentz boosts and translations. In the algebraic context, this is referred to as *Poincaré covariance*. Note also that under Guido and Longo's (1995) analysis, it is the states associated with DHR representations that possess the properties of CPT invariance and the spin-statistics connection. Thus one needs to distinguish between Poincaré covariance of the von Neumann algebra $\mathfrak{R}$, and Poincaré covariance of its DHR representations $(\mathcal{H}, \pi)$.[21] Three things can now be said:'

[19] Pragmatists may respond to the consequences of Haag's theorem by adopting what Wallace (2011) refers to as "conventional", or "cutoff" QFT. See Sect. 5.1 below for a discussion.

[20] In the axiomatic approach, the relation between restricted Lorentz invariance and CPT invariance is tighter in an interacting theory, appropriately construed, than a free theory. In the LSZ formalism, time-ordered Wightman functions (or "τ-functions") are used to calculate the elements of the S-matrix of an interacting theory. Greenberg (2002) demonstrates that violation of CPT invariance of any Wightman function entails that the corresponding τ-function is not restricted Lorentz invariant. Thus, "[i]f CPT invariance is violated in an interacting quantum field theory, then that theory also violates Lorentz invariance" (pp. 231602-1, 231602-2), where Greenberg takes Lorentz invariance as the condition that both Wightman and τ-functions be restricted Lorentz invariant.

[21] Taken as a subset $\mathfrak{R} \subset \mathfrak{B}(\mathcal{H}_0)$ of the concrete algebra of bounded linear operators on the vacuum Hilbert space $\mathcal{H}_0$, $\mathfrak{R}$ is Poincaré covariant just when there is a unitary representation $U_0 : P_+^\uparrow \to \mathfrak{B}(\mathcal{H}_0)$

(a) Poincaré covariance of either $\mathfrak{R}$ or its DHR representations is *not* necessary to show that DHR representations can be classified in terms of their statistics, and that they admit conjugates (Halvorson and Müger 2006, p. 784).
(b) Poincaré covariance of either $\mathfrak{R}$ or its DHR representations is *not* necessary to demonstrate CPT invariance of the latter.
(c) Poincaré covariance of the DHR representations of $\mathfrak{R}$ *is* necessary to demonstrate that states of the latter possess the spin-statistics connection.

With respect to (b) and (c), the assumption of *Modular Covariance* (MC) in Guido and Longo's approach provides the heavy lifting accomplished by Lorentz invariance in the other approaches. Now certainly Lorentz invariance is a well-motivated assumption for RQFTs, but the extent to which MC is likewise well-motivated is a matter of some debate. Recall from Sect. 2.4 that MC is the assumption that the actions of the modular operator of the wedge algebra $\mathfrak{R}(W)$ can be geometrically interpreted as Lorentz boosts. If $\mathfrak{R}$ is generated by Wightman fields, then MC follows from the Bisognano–Wichmann theorem, but it's not entirely clear what its status is in the more general algebraic case. It can be shown that the conjunction of MC, *Microcausality*, and *Weak Additivity* entails Poincaré covariance of $\mathfrak{R}$ (Guido and Longo 1995, p. 530).[22] Moreover, the conjunction of *Microcausality*, *Additivity*, and *conformal* invariance of $\mathfrak{R}$ entails MC (Brunetti et al. 1993, p. 212).[23] This indicates that MC is a weaker constraint than Lorentz invariance. Furthermore, if one assumes $\mathfrak{R}$ is Poincaré covariant, then MC follows under the additional assumptions of *Wedge Duality* and a *Reality Condition* (for definitions of the latter see Borchers 2000, pp. 29, 31–32).[24]

Physically, Guido and Longo (1995, p. 520) suggest MC may be motivated by appeal to the Unruh effect. Briefly, the Unruh effect occurs when an observer, in constant acceleration with respect to the Minkowski vacuum, experiences the latter as a thermal state.[25] A standard explanation runs as follows: The modular operator of the wedge algebra $\mathfrak{R}(W)$ generates a one-parameter group of automorphisms (called the modular group). One can show that the restriction $\omega_0|_{\mathfrak{R}(W)}$ of the Minkowski vacuum state ω_0 to the wedge is a Kubo–Martin–Schwinger (KMS)

Footnote 21 continued
such that $U_0(g)\mathfrak{R}(\mathcal{O})U_0(g)^* = \mathfrak{R}(g\mathcal{O}), g \in P^{\uparrow}_{+}$. A DHR representation $(\mathcal{H}, \pi)$ of $\mathfrak{R}$ is unitarily equivalent to a *localized morphism* $\rho : \mathfrak{R} \to \mathfrak{B}(\mathcal{H}_0)$ defined by $\rho(A) = V\pi(A)V^*$, for unitary map $V : \mathcal{H} \to \mathcal{H}_0$. A DHR representation is then said to be Poincaré covariant just when there is a unitary representation $U_\rho : \tilde{P}^{\uparrow}_{+} \to \mathfrak{B}(\mathcal{H}_0)$ of the universal covering $\tilde{P}^{\uparrow}_{+}$of the restricted Poincaré group such that $U_\rho(h)\rho(A)U_\rho(h)^* = \rho(U_0(\sigma(h))AU_0(\sigma(h))^*)$, where $h \in \tilde{P}^{\uparrow}_{+}$, and $\sigma : \tilde{P}^{\uparrow}_{+} \to P^{\uparrow}_{+}$, is the covering map.

[22] If $\mathfrak{R}$ is Poincaré covariant, so are its DHR representations, but the converse is not true: Guido and Longo (1992, p. 534) show that every DHR representation with finite statistics is Poincaré covariant with positive energy, provided $\mathfrak{R}$ has a certain regularity property.

[23] *Additivity* (as opposed to *Weak Additivity*) is the requirement $\mathfrak{R} = \bigcup_i \mathfrak{R}(\mathcal{O}_i)$.

[24] This result, combined with Guido and Longo's analysis, suggests another version of an algebraic CPT theorem; namely, for a von Neumann algebra of local observables with a cyclic vacuum representation, the conjunction of *Poincaré Covariance*, *Weak Additivity*, *Wedge Duality*, and the *Reality Condition* entails CPT invariance (Borchers 2000, p. 32).

[25] This is typically interpreted as a thermalized multi-particle state, although Earman (2011) and Arageorgis et al. (2003) argue that this unjustified.

state with respect to the modular group (for a definition of the KMS condition, see Haag 1996, p. 218). Now suppose KMS states are identified as thermalized states; i.e., equilibrium states at some finite temperature (for motivation, see Earman 2011, pp. 82–83). MC then entails that $\omega_0|_{\Re(W)}$ is a thermalized state with respect to Lorentz boosts on W. Since the orbits of the latter are worldlines of constant acceleration, one concludes that an observer in constant acceleration experiences $\omega_0|_{\Re(W)}$ as a thermalized state. Guido and Longo (1995, p. 520) go on to suggest,

> …[T]he equivalence principle in Relativity Theory then allows an interpretation of the thermal outcome as a gravitational effect. On this basis Haag has proposed long ago to derive the Bisognano–Wichmann theorem [and thus MC].

The suggestion, evidently, is that MC coupled with the equivalence principle entails that a gravitational field thermalizes the vacuum state of a von Neumann algebra of observables. The extent to which this constitutes a motivation for MC may initially depend on one's attitudes towards explanation and/or confirmation. For instance, to the extent that explanation requires derivation from first principles, MC might be claimed (in part) to explain the Unruh effect; and to the extent that confirmation requires derivation of evidence from hypothesis, evidence for the Unruh effect might be claimed to be evidence for MC. However, these considerations don't stand up to further scrutiny. Apart from questions concerning the feasibility of applying the equivalence principle in the context of flat Minkowksi spacetime, one may also question the cogency of the modular theory derivation of the Unruh effect. Earman (2011, pp. 87–88) raises the following concerns. First, the relevant KMS state is the restriction $\omega_0|_{\Re(W)}$ of the Minkowski vacuum state to the wedge algebra $\Re(W)$. This is the vacuum state experienced by an observer in perpetual constant acceleration who has access only to $\Re(W)$. On the other hand, an observer who maintains constant acceleration for any finite stretch of proper time τ, no matter how long, but is unaccelerated either at $\tau = +\infty$ or $\tau = -\infty$, will have access to the full quasi-local algebra $\Re$ and the corresponding Minkowski vacuum state ω_0, for which the KMS result does not hold. This suggests that whether or not an observer experiences the Unruh effect cannot be determined by facts about any finite portion of her history, and "[t]his makes it mysterious how to mesh the deliverances of modular theory with the registrations of laboratory instruments" (Earman 2011, p. 88). Second, to argue that the KMS states associated with the modular group of $\Re(W)$ are thermal states depends on an analogy between KMS states and Gibbs states in quantum statistical mechanics. In the first instance, this requires an assumption that the modular group parameter can be interpreted as inverse temperature. In some contexts, the justification for this assumption is based on systems characterized by KMS states and obtained by taking appropriate thermodynamic limits of ordinary thermodynamic systems. But no such limiting procedures are associated with the restriction of the Minkowski vacuum state to $\Re(W)$. Moreover, even if this assumption is accepted, one is still faced with the task of explaining, in the context of RQFT, how thermodynamic effects physically arise for accelerating observers. Assumedly such an explanation would require an account of how the vacuum state couples to accelerating observers, an account that modular theory by itself does not furnish.

The upshot of the above discussion is that it is still an open question as to whether *Modular Covariance* is a physically reasonable assumption. This consequently places a degree of doubt on any attempt to use the algebraic approach to the CPT and Spin-Statistics theorems to inform an answer to the question of what CPT invariance and the spin-statistics connection are properties of.

3. A third and final way in which the approaches in Sect. 2 differ is over whether they identify *Local Commutativity* (LC) or *Causality/Microcausality* as a fundamental assumption for the spin-statistics connection. The axiomatic approach alone adopts LC.[26] The textbook Lagrangian and algebraic approaches adopt the *Causality/Microcausality* criterion. In Weinberg's approach, neither LC nor *Causality* is an explicit assumption. For Weinberg, LC is a derived condition from more fundamental assumptions related to properties of the *S*-matrix. However, one of these properties (Weinberg's assumption (ii)) might be construed as requiring *Causality* to hold of the interacting Hamiltonian.

4 Particle Properties?

To what extent do the approaches to the CPT and Spin-Statistics theorems reviewed in Sect. 2 support Thesis (II), the claim that CPT invariance and the spin-statistics connection are properties of particle states? Consider first the pragmatist of Sect. 3 who adopts either Weinberg's approach or the textbook Lagrangian approach. In both of these cases, statistics are encoded in the (anti-)commutation relations (1) of creation and annihilation operators that act on (free) particle states in a Fock space. This suggests that the spin-statistics connection in these approaches is a property of particles. Weinberg makes this explicit in his assumption (iii) (see Sect. 2.2) and his instrumentalist interpretation of fields: "This article treats a quantum field as a mere artifact to be used in the construction of an invariant *S*-matrix" (p. B1319). Moreover, Weinberg's proof of the CPT theorem requires demonstrating that creation and annihilation operators are invariant under C, P, and T transformations; and the textbook Lagrangian's proof of the CPT theorem requires the spin-statistics connection to hold. This suggests that in both approaches, CPT invariance is also a property of particles.

On the surface, Thesis (II) also seems to be supported by a purist who adopts the algebraic approach, insofar as CPT invariance and the spin-statistics connection are derived for (a subset of) DHR states interpreted as localized particle states. This reference to particle states is explicit in Doplicher et al. (1974) version of the Spin-Statistics Theorem: "We did not treat in Doplicher et al. (1971) any of the particle aspects of the theory. This will be the essential objective of the present paper" (p. 50). Whether this amounts to naiveté in light of the analysis of the Received View will be considered in the next section.

[26] Greenberg (2006) demonstrates that restricted Lorentz invariance of τ-functions (see footnote 20) evaluated at Jost points entails LC. Thus, "…if we take Lorentz covariance of time-ordered products as the condition of Lorentz covariance of the field theory, then… local commutativity is not an independent assumption of the theory" (p. 087701-1).

It may not be as evident that Thesis (II) can be supported by purists who adopt the axiomatic approach. In particular, in the axiomatic approach, statistics are encoded in the *Local Commutativity* constraint on fields, which suggests that fields are the bearers of the spin-statistics connection. Recall, however, that purists, of either the axiomatic or algebraic variety, face the problem of incompleteness: no empirically successful RQFT admits a purist formulation. And recall, too, that a conciliatory axiomatic purist willing to adopt Haag-Ruelle scattering theory as a method of extending her approach to encompass interactions may take (well-defined) Haag-Ruelle asymptotic states to be the objects that bear CPT invariance and the spin-statistics connection. The question for this type of purist then becomes, What are the asymptotic states that bear the properties of CPT invariance and the spin-statistics connection states of?

Greenberg (1998) indicates one form a purist response might take. Recall that the approaches in Sect. 2 differ on whether they adopt *Local Commutativity* or *Causality* as a fundamental assumption in the Spin-Statistics theorem. Greenberg suggests that these are ontologically distinct assumptions, the former associated with fields and the latter associated with particles. In particular, he distinguishes between *two* theorems: The "Spin-Statistics" theorem, which states that "…*particles* that obey Bose statistics must have integer spin and *particles* that obey Fermi statistics must have odd half-integer spin", and the "Spin-Locality" theorem, which states that "…*fields* that commute at spacelike separation must have integer spin and *fields* that anticommute at spacelike separation must have odd half-integer spin" (Greenberg 1998, p. 144). Greenberg identifies the Spin-Statistics theorem with the proofs due to Fierz (1939) and Pauli (1940) that inform the textbook Lagrangian approach,[27] and takes the *Causality* assumption to be the requirement that "…the bilinears constructed from the (free) asymptotic fields [i.e., the in- and out-fields of Haag-Ruelle scattering theory] commute at spacelike separation" (p. 145). He identifies the Spin-Locality theorem with the proofs due to Lüders and Zumino (1958) and Burgoyne (1958) that inform the axiomatic approach, and distinguishes it from the Spin-Statistics theorem solely on its replacement of *Causality* with *Local Commutativity*.

This is a distinction that makes a difference, insofar as a field can violate the Spin-Statistics theorem (and thus have non-local observables) while satisfying the Spin-Locality theorem (and thus not vanish identically). Jost (1965, pp. 103–104) provides an example of a non-vanishing spin-0 scalar field solution ϕ to the Klein-Gordon equation that obeys Fermi-Dirac statistics in the sense that its corresponding

[27] Greenberg (1998, p. 145) also associates the "Spin-Statistics" theorem with Weinberg's approach: "[Fierz and Pauli] used locality of observables as the crucial condition for integer-spin particles and positivity of the energy as the crucial condition for the odd half-integer case. Weinberg showed that one can use the locality of observables for both cases if one requires positive-frequency modes to be associated with annihilation operators and negative-frequency modes to be associated with creation operators." However, for Weinberg, "locality of observables" (i.e., commutativity at spacelike separated distances) is only imposed on the interacting Hamiltonian density $\mathfrak{H}_{\mathrm{int}}(x)$ and only to formally secure Lorentz invariance of the S-matrix (see Sect. 2.2). For Weinberg, "causality" as applied to observable quantities other than the S-matrix is explicitly renounced: "The point of view taken here is that [LC] is needed for the Lorentz invariance of the S-matrix, without any ancillary assumptions about measurability or causality" (Weinberg 1995, p. 198).

creation/annihilation operators anti-commute. This entails that ϕ does not anti-commute at spacelike separated distances; hence it does not violate the Spin-Locality theorem (such a violation would occur if ϕ anti-commuted). The field violates *Causality*, however; in particular, it possesses a non-local energy density (Greenberg 1998, p. 148). Moreover, it can be shown that, while ϕ does not satisfy *Local Commutativity*, it does satisfy *Weak Local Commutativity*; hence it possesses CPT invariance.

Thus, for Greenberg, the following three properties should be made distinct: the *spin-statistics connection*, the *spin-locality connection*, and *CPT invariance*. The *spin-statistics connection* is a property of particles: it holds just when bosons (particles that obey Bose-Einstein statistics) possess integer spin, and fermions (particles that obey Fermi-Dirac statistics) possess half-integer spin. The spin-locality connection is a property of fields: it holds just when fields that commute at spacelike distances possess integer spin, and fields that anti-commute possess half-integer spin. Finally, CPT invariance may be taken to be a property of both fields and particles. In this way Thesis (II) can be upheld for purists who adopt the axiomatic approach. The question remains, however, which property is more fundamental: the spin-statistics connection, or the spin-locality connection. To see how Thesis (I) might be upheld in this context, recall that we are considering Haag-Ruelle scattering theory as an extension of the axiomatic approach. One could argue that the fields, in their role as interpolating fields for the *S*-matrix, are unobservable, purely mathematical constructs that serve to interpolate between more fundamental in- and out- asymptotic particle states. In fact, a theorem due to Borchers demonstrates that to every asymptotic particle state there corresponds an equivalence class of interpolating fields (Haag 1996, p. 103).

At this point, all I have suggested is that Thesis (II) is compatible with the approaches to the CPT and Spin-Statistics Theorems reviewed in Sect. 2, at least if we take them at their face values. This is not to say that these approaches *entail* Thesis (II). In fact, many philosophers of physics claim that RQFTs cannot be given a particle interpretation. It's now time to address this view.

5 The Received View

According to a Received View among philosophers (Clifton and Halvorson 2001; Halvorson and Clifton 2002; Arageorgis et al. 2003; Fraser 2008), in order to admit a particle interpretation, a quantum field theory (QFT) must satisfy the following two conditions.

(a) The QFT must admit a Fock space formulation in which local number operators appear that can be interpreted as acting on a state of the system associated with a bounded region of spacetime and returning the number of particles in that region.
(b) The QFT must admit a unique Fock space formulation in which a total number operator appears that can be interpreted as acting on a state of the system and returning the total number of particles in that state.

These conditions are supposed to encode two pre-theoretic intuitions about particles; namely, that they possess the characteristics of *localizability* and *countability*, respectively. These intuitions require that, for a system of particles distributed over various regions of space, an adequate theory must be able to (a) identify the number of particles located in each region;[28] and (b) identify a *unique* value for the total number of particles, counted over all regions.

One can now demonstrate that Conditions (a) and (b) fail in RQFTs (see Bain 2011 for details). Briefly, failure of Condition (a) is a consequence of the fact that there can be no non-trivial operators associated with a bounded region of Minkowski spacetime that annihilate the vacuum.[29] Failure of Condition (b) is a consequence of the existence of unitarily inequivalent Fock space representations of the canonical (anti-) commutation relations of an RQFT. For a non-interacting RQFT, this entails there is no unique total number operator. Moreover, as noted in Sect. 3 above, Haag's theorem entails that a Fock space representation of interacting particles cannot be adopted from the corresponding non-interacting theory.

I'd now like to consider how the Received View fairs with respect to Theses (I) and (II). On the surface (I) and (II) entail that particles are fundamental states in RQFTs: If, by (II), particles are the bearers of CPT invariance and the spin-statistics connection, and if, by (I), these are essential properties born by fundamental states in RQFTs, then particle states must be fundamental in RQFTs. Thus if one accepts the Received View on particles, one must either reject (I), or reject (II), or reject both (I) and (II). In the following, I will call these Options A, B, and C, respectively.

5.1 Option A: Accept Thesis (II)

Consider first Option A, which claims that CPT invariance and the spin-statistics connection are properties of particles; but particles are not fundamental. Arguably, this is the view expressed by Wallace (2009) in an article on the concept of antimatter. On the one hand, he claims that RQFTs are not fundamentally about particles; rather, "particles are emergent phenomena, which emerge in domains where the underlying quantum field can be treated as approximately linear" (Wallace 2009, p. 219). On the other hand, he takes the charge conjugation transformation, C, to be a transformation between particle and antiparticle states (Wallace 2009, p. 218), and this suggests CPT is a property possessed by particles.

On the surface, this view appears to reject Thesis (I): If CPT invariance (and, assumedly, the spin-statistics connection) are properties of particles, and the latter are emergent or approximate states in an RQFT, then it seems to follow that CPT invariance and the spin-statistics connection are not essential properties of fundamental states. In fact Wallace (2011, 2006) defends a version of the Lagrangian formalism in which a momentum cutoff is imposed to address the

[28] This follows the intuitions of Halvorson and Clifton (2002, pp. 17–18). This aspect of the Received View should thus be made distinct from concepts of localized particles that require the existence of position operators and/or localized states.

[29] Streater and Wightman (2000, p. 139). This, in turn, is a consequence of the Reeh-Schlieder theorem and *Local Commutativity*.

problems associated with the perturbative description of interactions (this version is referred to as "conventional" or "cutoff" quantum field theory, CQFT). In this approach, the cutoff is used to regularize the ultraviolet divergences that appear in perturbative calculations, and is interpreted realistically, thus it is not taken to infinity at the end of such calculations (as it is in typical renormalization schemes). This method of regularization is analogous to placing a high-energy continuum theory on a discrete lattice. It thus results in an effective field theory that violates restricted Lorentz invariance. As a consequence, the CPT and Spin-Statistics theorems fail (recall the Lagrangian approach to these theorems requires restricted Lorentz invariance), and we thus lose the primary motivation for Thesis (I).

In this version of Option A, the CPT and Spin-Statistics theorems might be viewed as limited to idealized, linear, non-interacting RQFTs. This suggests a modified version of Thesis (I); namely,

(I′) CPT invariance and the spin-statistics connection are essential properties of fundamental states in idealized, linear, non-interacting RQFTs.

Thesis (I′) suggests that CPT invariance and the spin-statistics connection are idealized, approximate properties. On the one hand, this is consistent with Wallace's (2006) attitude towards quantum field theories: "QFTs as a whole are to be regarded only as approximate descriptions of some as-yet-unknown deeper theory, which gives a mathematically self-contained description of the short-distance physics" (2006, p. 45). On the other hand, Thesis (I′) also suggests that CPT invariance and the spin-statistics connection do not hold in *interacting* RQFTs.

This view seems problematic, in so far as the evidence for CPT invariance and the spin-statistics connection is typically taken to come from interacting RQFTs. Note, further, that Wallace's view is supposed to warrant taking interacting RQFTs seriously in the sense that, within the domain specified by its cutoff, an interacting RQFT is well-behaved and offers an approximation of the ontology of an as yet unknown "deeper" theory. But, under Thesis (I′), CPT invariance and the spin-statistics connection are not properties of states associated with such an approximation; rather, it appears that they must be viewed as properties of states associated with an approximation of such an approximation: under Thesis (I′), they hold of states in non-interacting RQFTs, which are approximations of interacting RQFTs, which themselves are approximations of some underlying deeper theory. This seems a bit odd, given the central role that CPT invariance and the spin-statistics connection play in typical depictions of RQFTs.

Note, finally, that one can impose a cutoff on interacting Lagrangian RQFTs and interpret it realistically, as Wallace suggests, but avoid violating restricted Lorentz invariance. This can be done by replacing momentum cutoff regularization with dimensional regularization as the method of taming divergent integrals in perturbative calculations. Realistically interpreting the cutoff in renormalization schemes that employ dimensional regularization results in what Georgi (1993) refers to as a "continuum effective field theory". Such a theory is manifestly restricted Lorentz invariant, and thus (assumedly) admits formulations of the CPT and Spin-Statistics theorems. Adopting such an approach would allow a pragmatist to consistently uphold both Theses (I) and (II) (*al beit* at the expense of the Received View).

To recap, the version of Option A for the Received View considered above interpreted CPT invariance and the spin-statistics connection as idealized properties that do not hold in interacting RQFTs. In principle there may be other ways to adopt Option A; but all such versions will be faced with the difficult task of interpreting the CPT and Spin-Statistics Theorems in RQFTs in such a way that the properties of CPT invariance and the spin-statistics connection come out as non-essential in RQFTs.

5.2 Option B: Reject Thesis (II)

Now consider Option B for the Received View. This is the claim that CPT invariance and the spin-statistics connection are not properties of particles; but are properties of fundamental states in RQFTs. This view is suggested by Baker and Halvorson (2010) who defend the concept of antimatter against a particle interpretation.[30] In the context of algebraic quantum field theory, they suggest that DHR states and their conjugates be interpreted as representing matter and antimatter states. Moreover, they argue that DHR states are more general than the particle states of the Received View on particles: they cite the example of a 2-dimensional RQFT of charged fermions and neutral bosons interacting via a Yukawa potential as a theory that possesses states that satisfy the DHR criteria but do not satisfy the Received View's conditions of adequacy for particles (pp. 116–117).

An advocate of Option B might thus attempt to justify Thesis (I) by arguing that (i) DHR states are the fundamental physically possible states in RQFTs; (ii) DHR states possess the properties of CPT invariance and the spin-statistics connection; and (iii) DHR states are more general than particle states, under the Received View. The following concerns suggest this approach to Option B is problematic:

(a) First, recall that in Guido and Longo's algebraic derivation of the Spin-Statistics theorem, the bearers of the spin-statistics connection are massive, Poincaré-invariant DHR states with finite statistics. This excludes nonlocal electromagnetic states (which do not satisfy the DHR criterion in general), as well as the states of massless gauge bosons; and both of these types of states are considered as physically possible in the Standard Model.
(b) Second, Guido and Longo's derivation requires the assumption of *Modular Covariance*, and, as Sect. 3 indicates, one might question whether this is a physically reasonable assumption.
(c) Finally, Baker and Halvorson's example none withstanding, the algebraic approach is incomplete in the sense of Sect. 3. In particular, 2-dimensional

[30] Note that Baker and Halvorson are not expressly concerned with Thesis (I), even restricted to the concept of antimatter. Rather, they are *only* concerned with divorcing the concept of antimatter from the concept of particle: "[T]here may be a fundamental matter–antimatter distinction to be drawn in QFT. Whether there is does *not* depend on whether particles play any part in the theory's fundamental ontology" (p. 94).

> Yukawa theory is a far cry from the interacting RQFTs employed in the Standard Model.

In the absence of alternative algebraic formulations of the CPT and Spin-Statistics theorems, these considerations might motivate an Option B'er to abandon the algebraic approach for the axiomatic approach. One might claim that CPT invariance and the spin-statistics connection (or spin-locality connection) are properties of fundamental fields. One might be motivated to do this by recalling that the axiomatic approach encodes statistics in the form of the *Local Commutativity* constraint on fields, and the key ingredient to the proof of the CPT theorem is the *Weak Local Commutativity* constraint, imposed on vacuum expectation values of fields. But is a field interpretation consistent with the Received View; in particular, with its denial of a particle interpretation? Baker (2009) argues that it is not. In particular, Baker observes that the Hilbert space of wavefunctional states typically associated with field interpretations is unitarily equivalent to the Fock space of particle states that the Received View requires for particle interpretations (the former is constructed from the space of solutions to a classical field theory, the latter is constructed from single-particle states identified as finite irreducible representations of the restricted Lorentz group).[31] Thus, to the extent that unitary equivalence entails translational equivalence, the arguments that the Received View mounts against particle interpretations are equally effective against field interpretations that employ a Hilbert space of wavefunctional states.

In a bit more detail, Baker demonstrates that just as there are unitarily inequivalent representations of the canonical (anti-) commutation relations of an RQFT that undermine the ascription of a unique particle state to a physical system, similarly there are unitarily inequivalent representations that undermine the ascription of a unique field configuration state to a physical system. Moreover, Haag's theorem indicates there is no Fock space for an interacting RQFT, and thus no wavefunctional space. Hence, under the same assumptions that indicate there are no particle states for an interacting RQFT, there are also no field configuration states. Of course the option remains for an advocate of the Received View on particles to build a field interpretation on mathematical structures that do not require a wavefunctional space. As Baker (2009, p. 606) points out, in the axiomatic approach it is possible to define field operators in the absence of a wavefunctional space; but exactly how to interpret them in terms of fields is unclear.

This suggests that an axiomatic Option B'er cannot succeed on a field interpretation of RQFT without further ado, but this leaves the door open to alternative interpretations of the axiomatic approach of neither the particle nor the field type. Baker (2009, p. 607), for instance, suggests seeking the fundamental quantities of an RQFT in its algebra of observables, either as represented by a concrete von Neumann algebra of operators, or as represented by an abstract Weyl algebra. The task for the Option B'er in this context is to resolve the tension between Thesis (I), on the one hand,

[31] A wavefunctional state $\Psi[\chi]$ is a probability distribution over classical field configurations $\chi(x)$ (see, e.g., Wallace 2006, pp. 40–41). A field operator $\hat{\phi}(x)$ (distinguished here with a hat) acts on $\Psi[\chi]$ and produces the field configuration $\chi(x)$.

and a literal construal of the CPT and Spin-Statistics theorems on the other, where the latter, at least on the surface, suggests *either* a particle ontology *or* a field ontology.

5.3 Option C: Reject Theses (I) and (II)

An advocate of the Received View may also adopt Option C and reject both Theses (I) and (II). The CPT and Spin-Statistics theorems in RQFTs suggest that rejecting Thesis (I) (or Thesis (I′)) will be problematic. But there may be precedent for this: there is a large body of literature that attempts to construct derivations of CPT invariance and the spin-statistics connection outside the framework of RQFTs. Some authors have attempted to construct non-relativistic derivations of the spin-statistics connection,[32] while other authors have presented classical (i.e., non-quantum–mechanical) derivations of the spin-statistics connection (Morgan 2004) and CPT invariance (Bell 1955; Greaves 2010). An advocate of Option C might thus claim that CPT invariance and the spin-statistics connection are not unique to RQFTs, and thus proofs in RQFTs do not necessarily inform us about the fundamental ontology of RQFTs. On the other hand, one might take this literature to imply that the reason why CPT invariance and the spin-statistics connection are fundamental properties in RQFTs has yet to be made clear. The task for the Option C'er is to sort through this literature to determine the extent to which it supports the former as opposed to the latter claim.

6 Conclusion

I have argued that the Received View against particle interpretations of RQFTs must either

(A) reject Thesis (I); i.e., the claim that CPT invariance and the spin-statistics connection are essential properties of fundamental states in RQFTs; or
(B) reject Thesis (II); i.e., the claim that the bearers of these properties are particle states; or
(C) reject both Theses (I) and (II).

Options (A) and (C) are made problematic by the CPT and Spin-Statistics theorems in RQFTs. These theorems suggest that deniers of Thesis (I) may have to view CPT invariance and the spin-statistics connection as idealized properties that do not hold in interacting RQFTs. But this is problematic insofar as the evidence for CPT invariance and the spin-statistics connection invariably comes from interacting RQFTs. Alternatively, deniers of Thesis (I) may pin their hopes on proofs of CPT and the Spin-Statistics theorems outside the purview of the RQFT framework, but work needs to be done in evaluating these proofs.

Option (B) is made problematic by the fact that the Received View's argument against particle interpretations is equally effective against the standard approach to field interpretations. Given the nature of the extant approaches to proofs of the CPT

[32] The literature on this is vast. A partial survey is given in Duck and Sudarshan (1997).

and Spin-Statistics theorems, it's not immediately clear what the alternatives to Thesis (II) could be. Read literally, these approaches suggest that CPT invaraince and the spin-statistics connection are properties either of particles or of fields. The Received View must therefore either provide a non-standard field interpretation, or explain how the CPT and Spin-Statistics theorems can be interpreted in a non-standard way.

This is not to say this constitutes a sound argument against the Received View, nor does it suggest that rejecting the Received View by adopting Theses (I) and (II) is unproblematic. To do so would require offering an alternative account of particles, or perhaps an account of why it may be appropriate to talk about particle fundamentality in the context of the CPT and Spin-Statistics theorems, but not in other contexts.[33] What this essay does suggest is that the Received View's approach to ontology is flawed. What we take RQFTs to be about should depend, in part, on what we take the essential properties of RQFTs to be. And what we take the essential properties of RQFTs to be should depend, in part, on results internal to RQFTs, like the CPT and Spin-Statistics Theorems. In analyzing such theorems, pre-theoretic intuitions that a priori militate against particle interpretations may be misleading.

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[33] One attempt at the former is Bain (2000). A view that might be identified with the latter is Ruetsche's (2011, p. 147) "coalescence" approach to the interpretation of RQFTs, which declares an RQFT appropriately interpreted only after an application is specified. One might thus attempt to argue that in applications of RQFTs in which CPT invariance and the spin-statistics connection are important, particle interpretations (of either the sort sanctioned by the Received View or otherwise) are available.

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