

Three Myths About the Origin of Special Relativity:

- Myth #1 (Genius): The discovery of relativity was a single stroke of genius that defies rational analysis.
- Myth #2 (Empiricism): The failure of aether-drift experiments led to the Principle of Relativity, and the Michelson-Morley experiment led to the Light Postulate.
- Myth #3 (Idealism): The discovery of relativity was due to Einstein's philosophical critique of fundamental concepts.

- But: Consider the title of Einstein's (1905): "On the Electrodynamics of Moving Bodies".
- And: The electrodynamics of moving bodies was a widely discussed topic in the period 1895-1905.

1. Maxwell's Theory, Recap.

- A *phenomenological* theory that described the *macroscopic* states \mathbf{E} , \mathbf{D} , \mathbf{H} , \mathbf{B} of the electromagnetic aether in combo with matter moving at velocity \mathbf{v} :

$$\nabla \times \mathbf{E} = -D\mathbf{B}/Dt \quad \nabla \times \mathbf{H} = \mathbf{j} + DD/Dt$$

$$\nabla \cdot \mathbf{D} = \rho \quad \nabla \cdot \mathbf{B} = 0$$

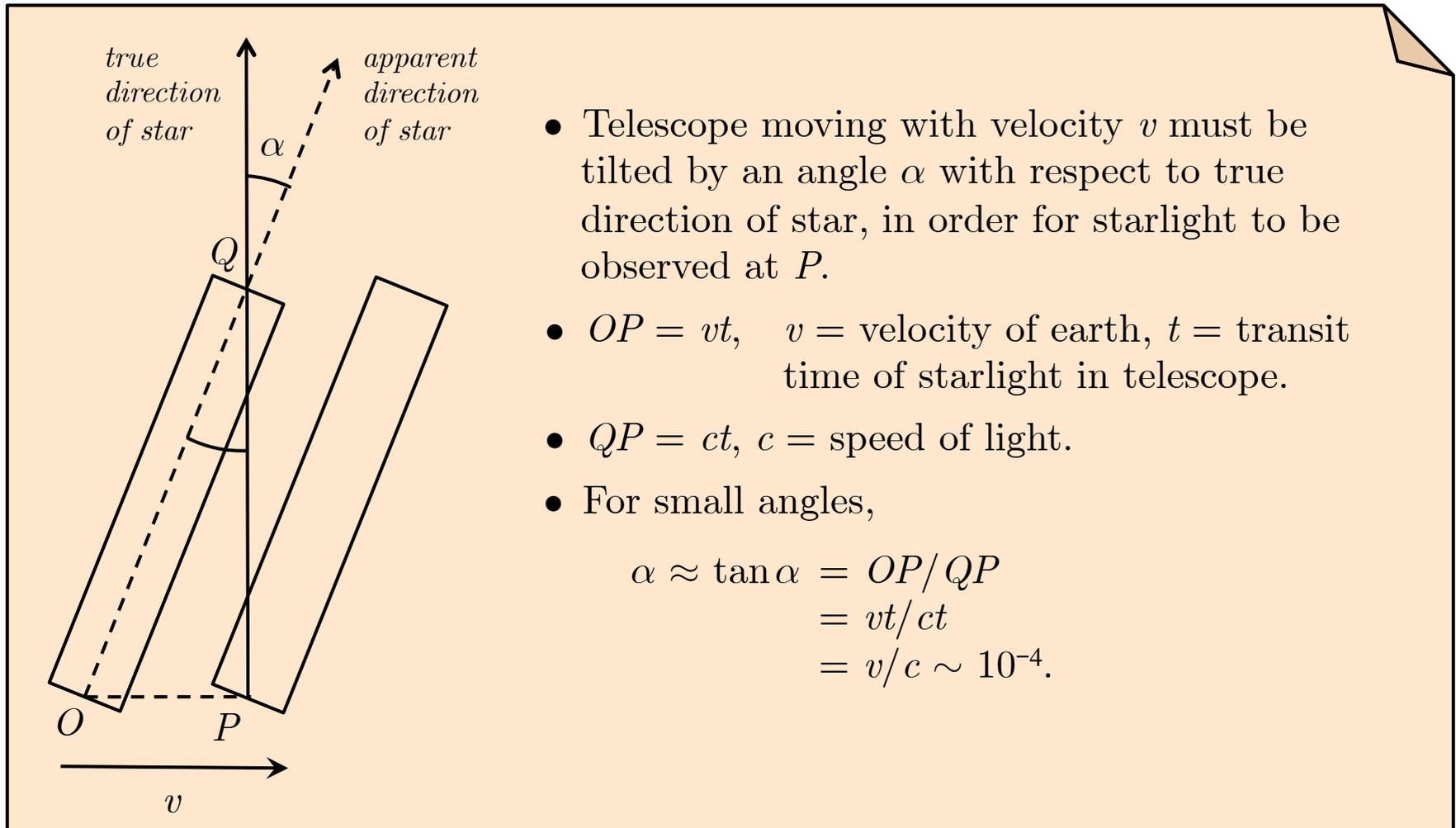
- Conduction current \mathbf{j} = dissipative relaxation of polarization in a conducting medium.
- Charge density ρ = rate of change of polarization \mathbf{D} in direction of \mathbf{D} .
- D/Dt = "convection derivative" = $\partial/\partial t - \nabla \times (\mathbf{v} \times) + \mathbf{v}(\nabla \cdot)$
- "Forces" \mathbf{E} , \mathbf{H} related to polarizations \mathbf{D} , \mathbf{B} via $\mathbf{D} = \epsilon\mathbf{E}$, $\mathbf{B} = \mu\mathbf{H}$.

"Maxwell had himself noted that his phenomenological approach leads to wrong predictions when applied to optical dispersion, to magneto-optics, and to the optics of moving bodies. In these cases, he suspected that the molecular structure of matter had to be taken into account." (Darrigol 2005, pg. 4.)

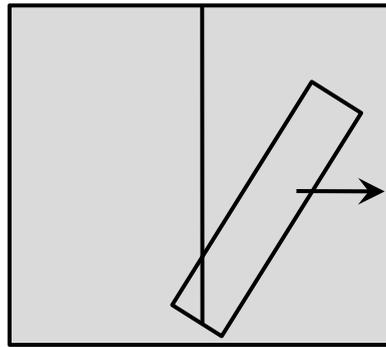
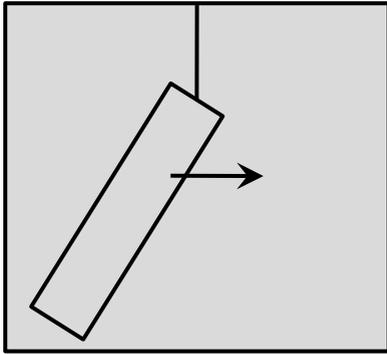
2. Stellar Aberration and the Electromagnetic Aether.

Janssen and Stachel (2004)

- *Aberration of starlight* = The direction of a fixed star appears to vary periodically in the course of a year, by an amount of the same order as the ratio (10^{-4}) of the orbital velocity of the earth to the velocity of light.

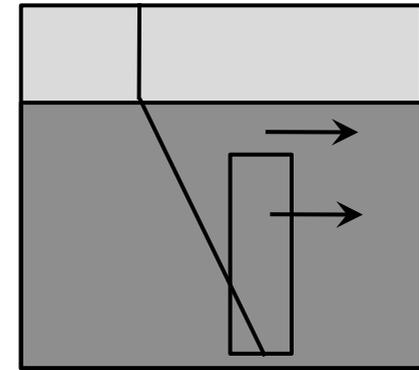
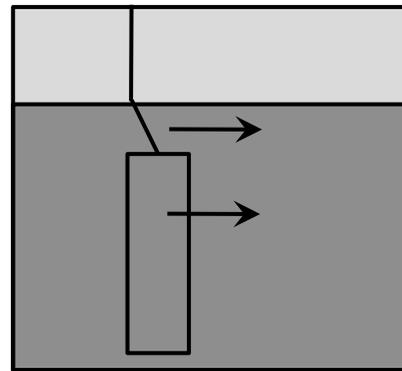
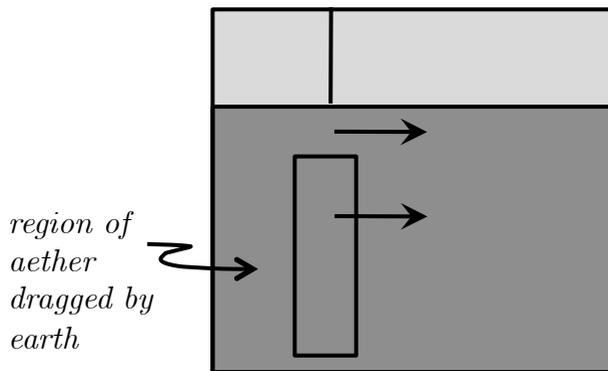


- Problem: Maxwell's aether must be *dragged* by moving objects.
- But: Stellar aberration seems to require aether to be *immobile*.



Immobile aether prediction:

- Suppose light travels in straight lines in aether.
- Then for starlight to go straight down, *telescope must be tilted*.



Dragged aether prediction:

- Suppose light travels in straight lines in aether.
- Then if aether is dragged by earth, path of starlight is dragged, too.
- Thus *telescope need not be tilted* in order for starlight to traverse its length.

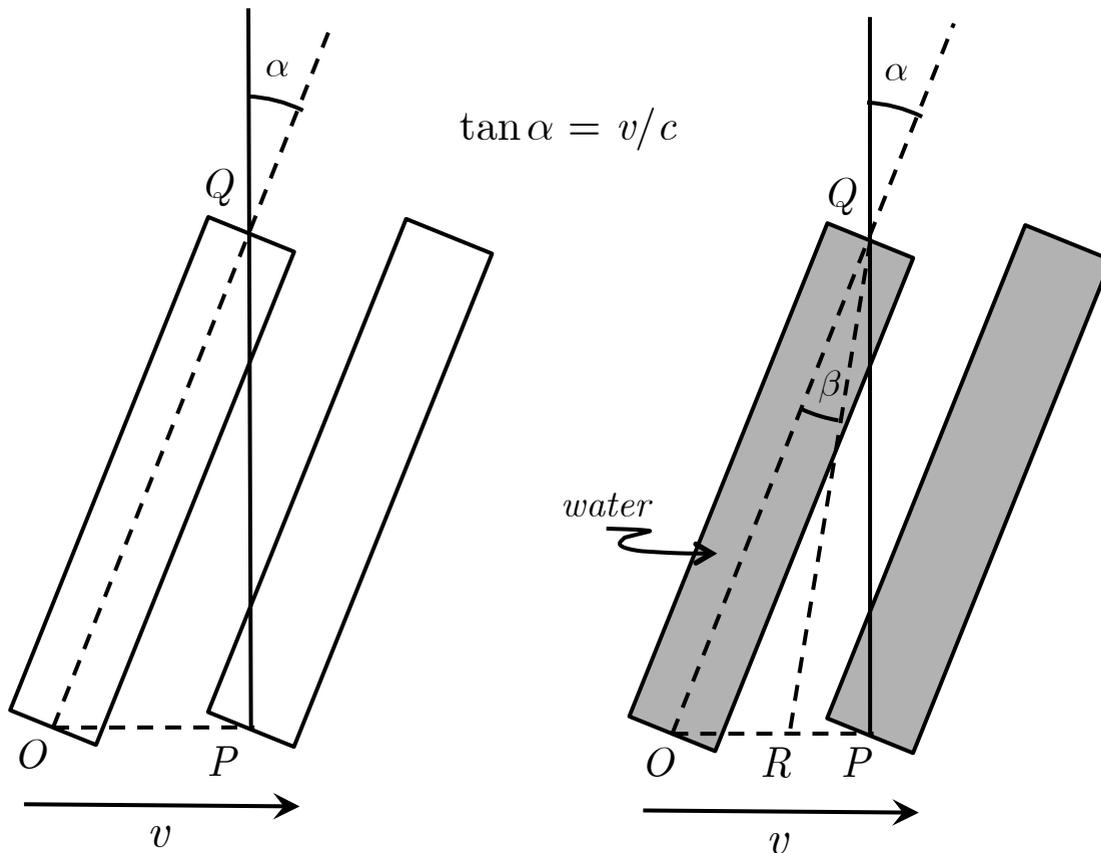
- We observe stellar aberration (*i.e.*, we must tilt telescopes to observe stars).
- Thus: The aether must be immobile!



Thomas Young

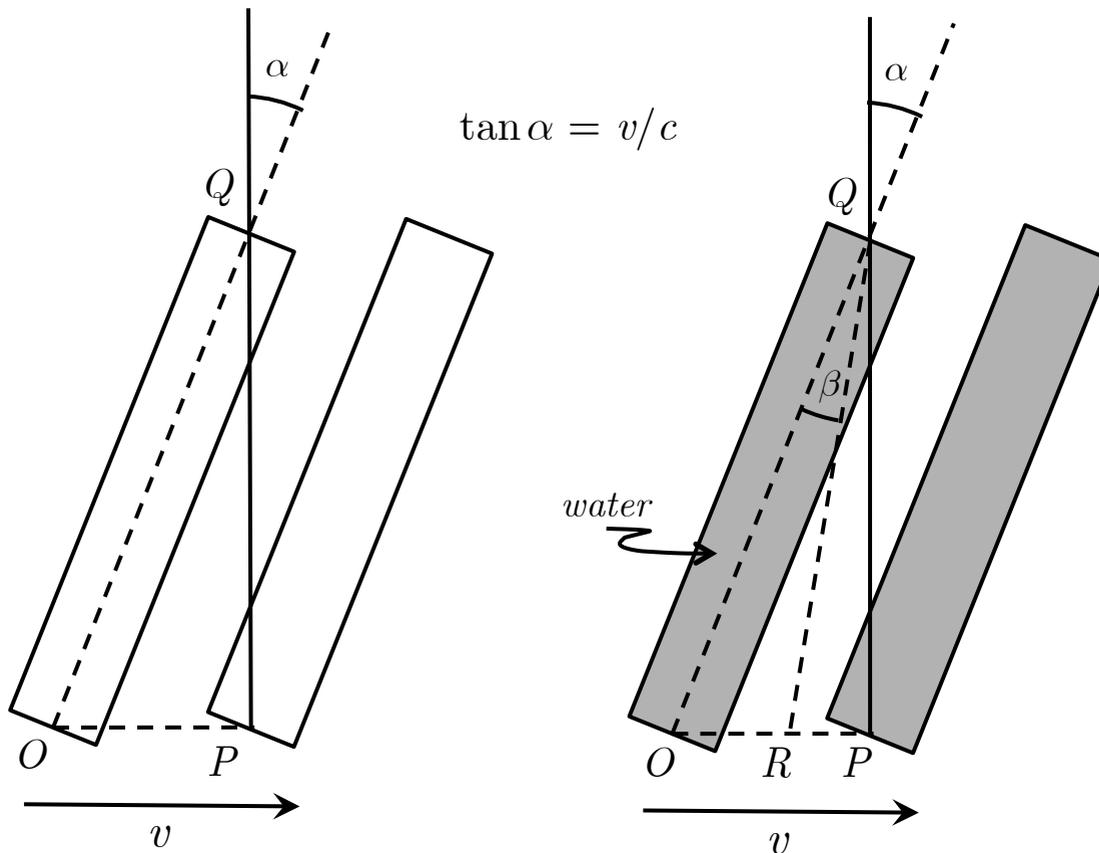
"Upon considering the phenomena of the aberration of the stars I am disposed to believe, that the luminiferous aether pervades the substance of all material bodies with little or no resistance, as freely perhaps as the wind passes through a grove of trees." (1804)

- But: 1810. Arago performs experiment to determine if light particles entering a prism are refracted differently due to their velocity.
 - Observes light from the same star over the course of a year.
 - Assumes changes in velocity of earth with respect to star will produce changes in refraction of starlight.
 - Result: No such effect is observed!
- Arago to Fresnel: Can wave theory explain this?
 - *What happens to light being refracted through an object moving through the aether?*



- Law of Refraction:
 $\sin \alpha = n \sin \beta$.
- $\alpha =$ angle of incidence
(aberration angle)
- $\beta =$ angle of refraction
- $\tan \beta \approx OR/RQ$
- $\tan \alpha = v/c$
- $OP = vt$, $t =$ transit time
- $RP = fvt$
- $RQ = (c/n)t$, ($c_{water} = c_{air}/n$)

- If aether is immobile, then starlight should refract in water-filled telescope and travel from Q to R .
- But: Arago's experiment indicates starlight travels from Q to P !
- Suppose: Water drags aether with a fraction f of its velocity, just enough to result in the observed aberration angle α .
- Fresnel's task (1818): Determine this fraction f .



$$\tan \alpha = v/c$$

- Law of Refraction:
 $\sin \alpha = n \sin \beta$.
- $\alpha =$ angle of incidence (aberration angle)
- $\beta =$ angle of refraction
- $\tan \beta \approx OR/RQ$
- $\tan \alpha = v/c$
- $OP = vt$, $t =$ transit time
- $RP = fvt$
- $RQ = (c/n)t$, ($c_{\text{water}} = c_{\text{air}}/n$)

- For small angles, $\sin \theta \approx \tan \theta$.
- Law of refraction becomes $\tan \alpha = n \tan \beta$, or

$$\begin{aligned} v/c &= n(OR/RQ) = n(OP - RP)/RQ \\ &= nv(1 - f)/(c/n) \end{aligned}$$

- Thus: $f = 1 - (1/n^2)$. *Fresnel's Drag Coefficient.*
- Conclusion: Transparent matter drags aether inside it with a fraction f that depends on its index of refraction.

- 1845. Stokes's aether theory: Accounts for aberration and refraction in terms of *total* aether drag.

- Behaves as an incompressible fluid with respect to slow moving bodies immersed in it.
- Behaves as a rigid solid with respect to fast vibrations associated with light waves.
- Assumedly would be dragged by objects moving slowly through it.

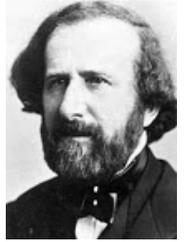


George Stokes
1819-1903

- Pre-1850s: Which theory is correct?
 - Stokes's aether theory: Total aether drag of moving objects.
 - Fresnel's aether theory: Immobile aether except in moving refractive media in which partial aether drag occurs.

3. Aether-Drag Experiments.

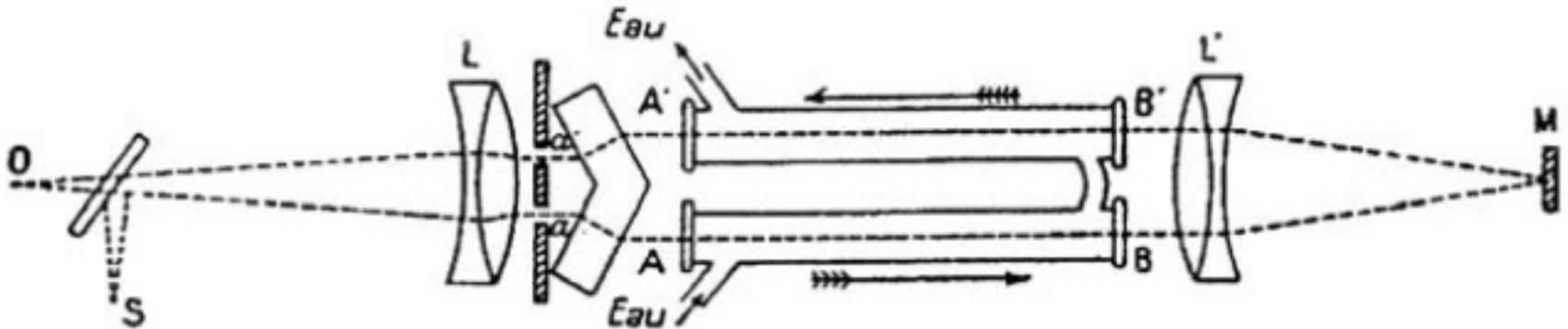
Janssen and Stachel (2004)



Hippolyte Fizeau
1819-1896

1850. Fizeau Experiment.

- Light beams moving with and against a flow of water exhibit a shift in their interference pattern predicted by Fresnel's drag coefficient for water.



- Light from source S divided into two beams at L .
- Upper beam travels against water stream in $A'B'$, is reflected back at M to travel against water stream in AB , and returns to O .
- Lower beam travels with water streams in AB and $A'B'$ and returns to O .
- Phase difference between two beams is judged by interference pattern at O .

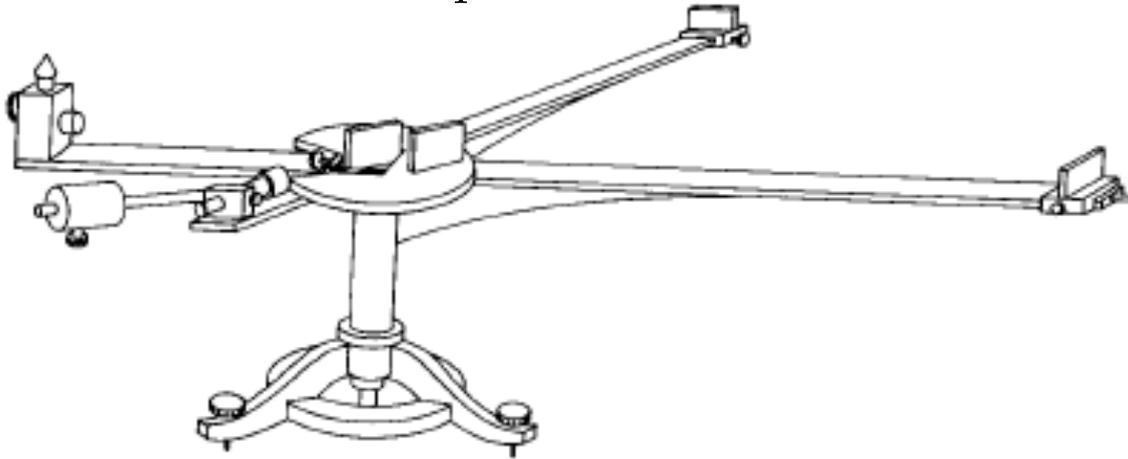
- Implication: Fresnel is correct!

1881. Michelson Experiment.

- Light beams moving along perpendicular arms of interferometer exhibit no shift in interference pattern.



Albert Michelson
(1852-1931)



- Time of travel in arm \parallel to earth's motion through aether: $2L/c(1 + v^2/c^2)$.
- Time of travel in arm \perp to earth's motion through aether: $2L/c(1 + v^2/2c^2)$.
- Difference in times of travel: $L/c(v^2/c^2)$. *Second-order* in v/c .
- Should cause shift in interference pattern when beams recombine.



"The interpretation of these results is that there is no displacement of the interference bands. The result of the hypothesis of a stationary ether is thus shown to be incorrect, and the necessary conclusion follows that the hypothesis is erroneous."

- Implication: Fresnel is incorrect, Stokes is correct!

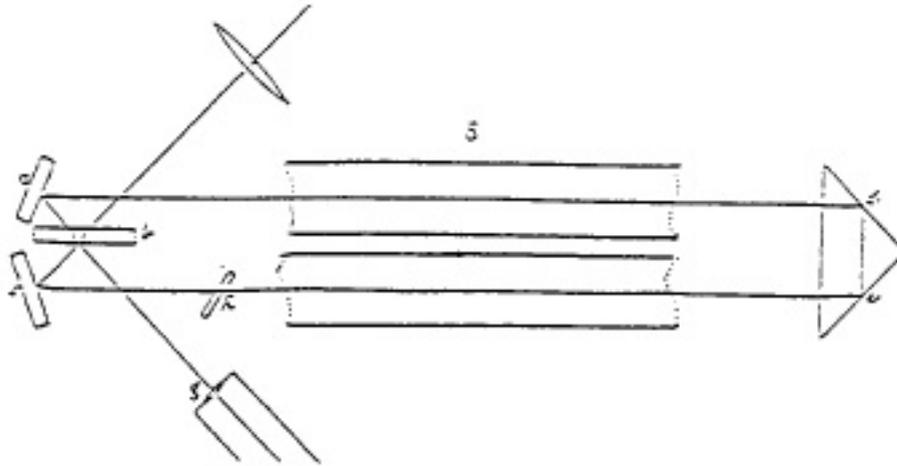
1886. Michelson-Morley Repetition of Fizeau Experiment.



Albert Michelson



Edward Morley
(1838-1923)

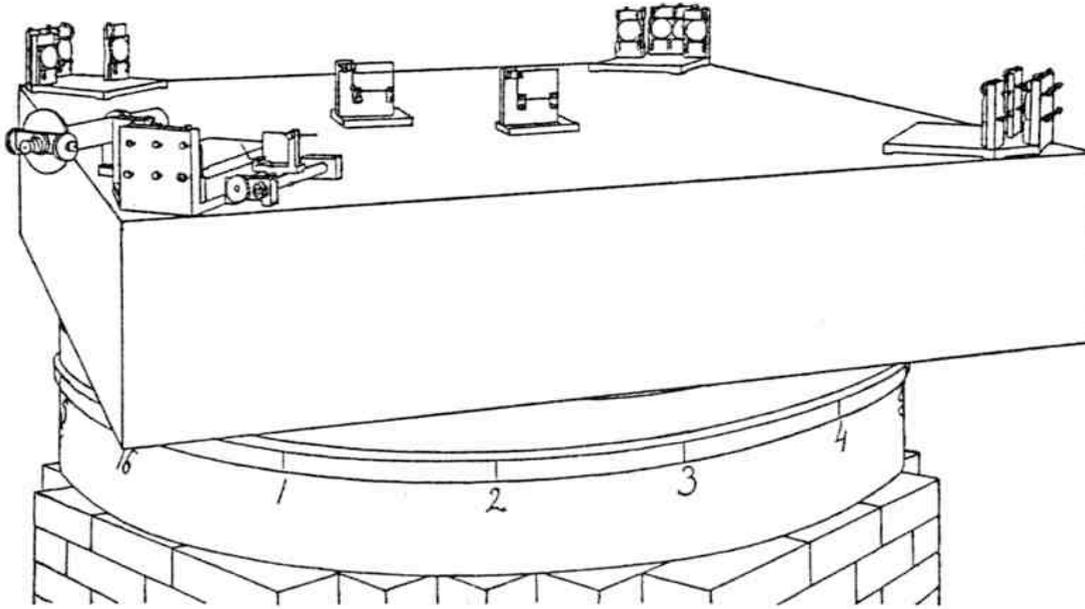


"The result of this work is therefore that the result announced by Fizeau is essentially correct: and that *the luminiferous ether is entirely unaffected by the motion of the matter which it permeates.*"

- Implication: Fresnel is correct, Stokes is incorrect!

1887. Michelson-Morley Experiment.

- Repetition of 1881 Michelson experiment.



"It appears, from all that precedes, reasonably certain that if there be any relative motion between the earth and the luminiferous ether, it must be small."

- Implication: Both Fresnel and Stokes are incorrect!



Hendrik Lorentz
(1853-1928)

4. Lorentz's Theory (1890s).

- Charge, current, and polarization viewed as the accumulation, flow, and displacement of electrons interacting with the aether.
- "Lorentz force" \mathbf{f} = force experienced by electron as it moves with velocity \mathbf{v} through aether.

$$\nabla \times \mathbf{e} = -c^{-1} \partial \mathbf{b} / \partial t \quad \nabla \times \mathbf{b} = c^{-1} [\rho_m \mathbf{v} + \partial \mathbf{e} / \partial t]$$

$$\nabla \cdot \mathbf{e} = \rho_m \quad \nabla \cdot \mathbf{b} = 0$$

$$\mathbf{f} = \rho_m [\mathbf{e} + c^{-1} \mathbf{v} \times \mathbf{b}]$$

- ρ_m = microscopic charge density of electrons ("ions").
- \mathbf{f} = density of force acting on electrons.
- Assumption: Aether surrounding electrons has same properties as aether in a vacuum. Thus only two independent fields \mathbf{e} , \mathbf{b} describing states of aether.

- Under assumption of immobile aether, can derive Fresnel's drag coefficient.
- Interpretation: Partial drag of EM waves by a refractive medium moving in a stationary aether.

Lorentz's Theorem of Corresponding States

- A mathematical aid to simplify complicated calculations.

1895. "First-order" theorem of corresponding states.

- Aether frame variables $(t_0, \mathbf{e}_0, \mathbf{b}_0)$ and moving frame variables $(t, \mathbf{e}, \mathbf{b})$.
- Replace $(t, \mathbf{e}, \mathbf{b})$ in moving frame with auxiliary variables $(t', \mathbf{e}', \mathbf{b}')$:

$$t' = t - (\mathbf{v}/c^2)x \quad (\text{"local time"})$$

$$\mathbf{e}' = \mathbf{e} + c^{-1}(\mathbf{v} \times \mathbf{b})$$

$$\mathbf{b}' = \mathbf{b} - c^{-1}(\mathbf{v} \times \mathbf{e})$$

- Equations for $(t', \mathbf{e}', \mathbf{b}')$ are same as equations for $(t_0, \mathbf{e}_0, \mathbf{b}_0)$ in aether frame to first-order in v/c .
- Problem: No explanation of *second-order* MM experiment.

Lorentz's Theorem of Corresponding States

- A mathematical aid to simplify complicated calculations.

1899/1904. Exact theorem of corresponding states.

- Aether frame variables $(t_0, x_0, \mathbf{e}_0, \mathbf{b}_0)$ and moving frame variables $(t, x, \mathbf{e}, \mathbf{b})$.
- Replace $(t, x, \mathbf{e}, \mathbf{b})$ in moving frame with auxiliary variables $(t', x', \mathbf{e}', \mathbf{b}')$:

$$t' = \gamma^{-1}t - \gamma vx/c^2 \quad (\text{"local time"})$$

$$x' = \gamma x$$

$$\gamma = 1/(1 - v^2/c^2)^{-1/2}$$

- Equations for $(t', x', \mathbf{e}', \mathbf{b}')$ are *exactly* same as equations for $(t_0, x_0, \mathbf{e}_0, \mathbf{b}_0)$.
- Moreover: Explanation of second-order MM experiment: Lorentz-FitzGerald contraction hypothesis. Matter in motion through the aether contracts by a factor of γ^{-1} .

"It is important to understand that for Lorentz the transformed coordinates and fields were mathematical aids with no direct physical significance. They were only introduced to facilitate the solution of complicated differential equations." (Darrigol, pg. 11.)

5. Poincaré's Criticism.

Sorbonne lectures of 1888, 1890, 1899.

- Three general principles of mechanics: *Principle of Relativity*, *Principle of Reaction*, *Principle of Least Action*.
- On the aether:



Henri Poincaré
(1854-1912)



"It matters little whether the ether really exists: that is the affair of the metaphysicians. The essential thing for us is that everything happens as if it existed, and that this hypothesis is convenient for us for the explanation of the phenomena."

"...Poincaré never abandoned the aether. But he refused to regard it as an ordinary kind of matter whose motion could affect observed phenomena. In his view, the principle of reaction and the principle of relativity had to apply to matter alone." (Darrigol, pg. 13.)

- On the relativity principle:

"I consider it very probable that optical phenomena depend only on the relative motion of the material bodies present--light sources and apparatus--and this not only to first or second order but exactly."



1900. "La théorie de Lorentz et le principe de la réaction."

- Interpretation of Lorentz's local time: The time measured by moving observers if they synchronize their clocks in a particular manner:

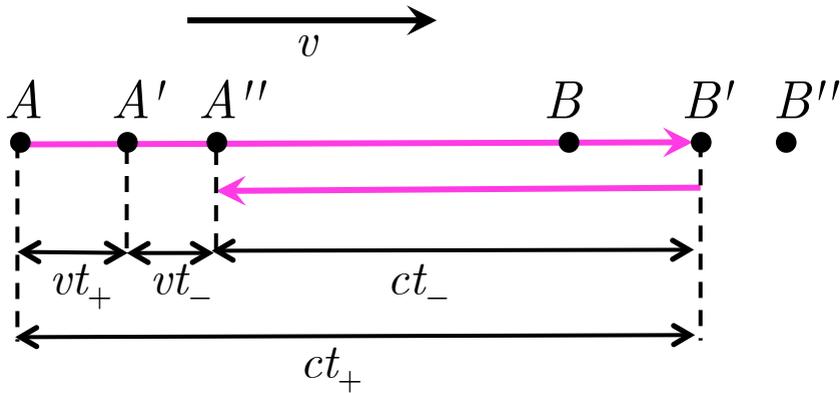


"I suppose that observers placed in different points set their watches by means of optical signals; that they try to correct these signals by the transmission time, but that, ignoring their translational motion and thus believing that the signals travel at the same speed in both directions, they content themselves with crossing the observations, by sending one signal from A to B , then another from B to A ."

- In other words: Lorentz's (first-order) local time is given by

$$t' = t - (v/c^2)x.$$

- And: This is a correction to the real time t by the factor $(v/c^2)x$.
- And: This factor is due to an incorrect clock synchronization procedure!



A, B = locations when A sends signal.
 A', B' = locations when B receives and resends signal.
 A'', B'' = locations when A receives signal.

- A, B moving at velocity v with respect to aether.
- A sends signal to B , who then sets clock to 0 and resends signal back to A .
- Upon reception, A sets clock to $\tau/2$, where τ = time of signal round-trip.
- Claim: Clocks will be out of synch by $\tau/2 - t_-$, where t_- = time for signal to travel from B to A .
 - Why? Because as the signal travels from B back to A , A is moving towards it; so it will take less time than $\tau/2$ to reach A .
- Now:
 - $t_+ = AB/(c - v)$ = time for signal to travel from A to B .
 - $t_- = AB/(c + v)$ = time for signal to travel from B to A .
 - $\tau = t_+ + t_-$ = time for signal to travel from A to B back to A .
- So: $\tau/2 - t_- = (t_+ - t_-)/2 = (v/c^2)AB$, which is the local time correction.

1905. "Sur la dynamique de l'électron".

- "Relativity Postulate":



"It seems that the impossibility of experimentally detecting the absolute motion of the earth is a general law of nature; we naturally incline to assume this law, which we shall call the Postulate of Relativity, and to do so without any restriction."

- Identifies "Lorentz transformations" that leave Maxwell-Lorentz equations invariant. Shows that they form a group.
- Shows that quadratic form $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz transformations.
 - *Can thus interpret Lorentz transformations as rotations in a 4-dim space.*
- Obtains appropriate Lorentz-invariant law for the addition of velocities.
- Proposes modifications of Newton's law of gravity to make it Lorentz invariant.

- Darrigol, pg. 17. "Thus there is no doubt that Poincaré regarded Lorentz invariance as a general requirement for the laws of physics, and that he identified this formal condition with the principle of relativity."

"The reason why we can, without modifying any apparent phenomenon, confer to the whole system a common translation, is that the equations of an electromagnetic medium are not changed under certain transformations, which I shall call the Lorentz transformations; two systems, one at rest, the other in translation, thus become exact images of one another."



- But: "The same lectures and later talks on the 'mécanique nouvelle' show that Poincaré nonetheless maintained the ether and the ordinary concepts of space and time." (Darrigol, pg. 17.)

6. Einstein's Theory.



Albert Einstein
(1879-1955)

"I am more and more convinced that the electrodynamics of moving bodies, as it is presented today, does not agree with the truth, and that it should be possible to present it in a simpler way. The introduction of the name 'ether' into the electric theories has led to the notion of a medium of whose motion one could speak of without being able, I believe, to associate a physical meaning to this statement." (1899.)

"I believe that electric forces can be directly defined only for empty space... Further, electric currents will have to be regarded not as 'the vanishing of electric polarization in time' but as motion of true electric masses, whose physical reality seems to result from the electrochemical equivalents... Electrodynamics would then be the science of the motions in empty space of moving electricities and magnetisms." (1899.)



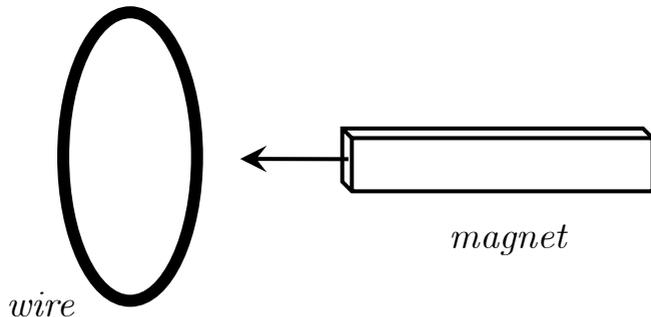
1905. "On the Electrodynamics of Moving Bodies."

- Central Motivation:

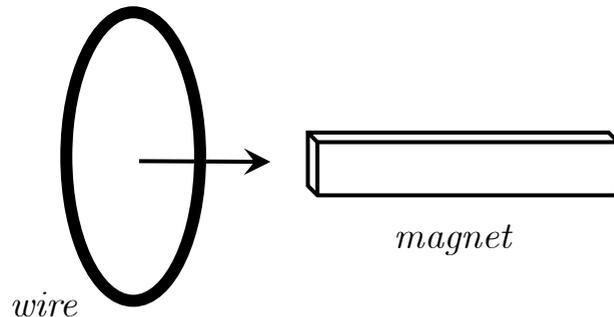


"It is known that Maxwell's electrodynamics--as usually understood at the present time [*i.e.*, Lorentz's theory]--when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena."

- When a magnet moves through a stationary conducting coil, or when the coil moves about the stationary magnet, the induced current is the same.
- But: The theoretical explanation is different!
 - First case: Moving magnet induces electric field in coil (Faraday's Law).
Electric force makes electrons move in coil
 - Second case: Lorentz force due to magnetic field of magnet causes electrons to move in coil.



$$\nabla \times \mathbf{e} = -c^{-1} \partial \mathbf{b} / \partial t$$



$$\mathbf{f} = \rho_m [c^{-1} \mathbf{v} \times \mathbf{b}]$$



"Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relatively to the 'light medium' suggest..."

- 1st Postulate ("Principle of Relativity"):

"The same laws of electrodynamics and optics will be valid in all frames of reference for which the equations of mechanics hold good." "



- 2nd Postulate (Light Postulate):

"Only apparently irreconcilable with [the relativity postulate]."

"Light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body."



The Principle of Relativity.

"The same laws of electrodynamics and optics will be valid in all frames of reference for which the equations of mechanics hold good."

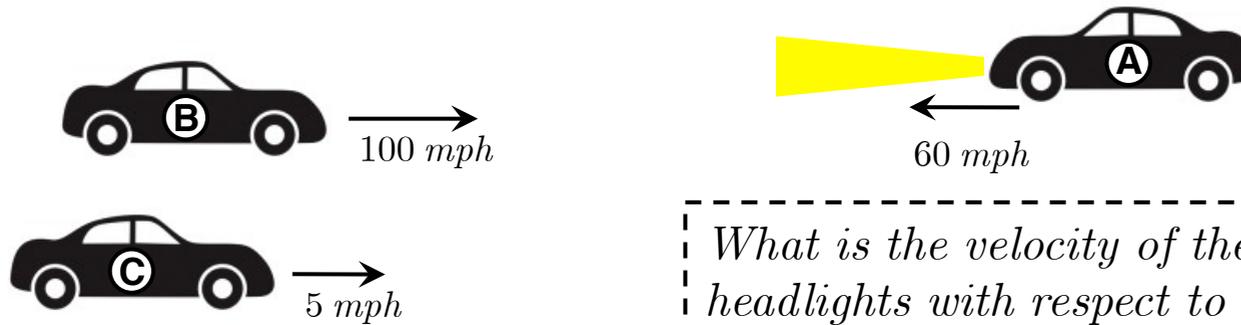
- Conceptual desire to avoid theoretical "asymmetries" in electrodynamics leads Einstein to reject the aether (the source of such asymmetries).
- "Frames for which the equations of mechanics hold good" = inertial frames = frames moving at constant velocity with respect to each other.
- Thus: The P. of R. says: There is no privileged aether frame in which the laws of electrodynamics and optics (*i.e.*, the Maxwell-Lorentz equations) hold; rather, these laws hold in all inertial frames.
- What this means: The laws of electrodynamics and optics and mechanics (*i.e.*, the laws of physics at the time) cannot distinguish one inertial frame from another: all inertial frames are physically indistinguishable with respect to the laws of physics.

The Light Postulate.

"Light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body."

- According to Maxwell-Lorentz electrodynamics, light is always propagated *in the aether* with a definite velocity c which is independent of the state of motion of the emitting body.
 - Light Postulate rephrases this in terms of "empty space".
- Why state this as a postulate?
 - "The constancy of the velocity of light no longer resulted from the existence of the ether, and had to be postulated separately." (Darrigol, pg. 23.)
 - It's the essential feature of electrodynamics that Einstein thought would survive in a theory describing *light quanta*...
- Why is it "apparently irreconcilable" with the Principle of Relativity?
 - Principle of Relativity and Light Postulate entail that the speed of light is the same in all inertial frames.
 - *And:* This violates the way of adding velocities on both emissionist and aether theories of light!

- Consider three cars on a highway:



What is the velocity of the light of car A's headlights with respect to cars B and C?

Emission theory: Velocity of light depends on velocity of its source.

- Car B says velocity of light from car A's headlights is $c + 60 \text{ mpg} + 100 \text{ mpg}$.
- Car C says velocity of light from car A's headlights is $c + 60 \text{ mpg} + 5 \text{ mpg}$.

Aether theory: Velocity of light with respect to aether is c , independent of its source.

- Car B says velocity of light from car A's headlights is $c + 100 \text{ mpg}$.
- Car C says velocity of light from car A's headlights is $c + 5 \text{ mpg}$.

Einstein's theory: Velocity of light is c in all inertial frames.

- Car B says velocity of light from car A's headlights is c .
- Car C says velocity of light from car A's headlights is c .

- Einstein's two postulates entail that B and C measure the same velocity for light, even though they are moving with respect to each other!

How to reconcile the two postulates?

- Spring 1905. In conversation with Michele Besso, Einstein considers redefinition of the concept of time...
- Read Poincaré's *La Science et l'Hypothèses* (1904 German edition).



"There is no absolute time. To say two durations are equal is an assertion which has by itself no meaning and which can acquire one only by convention. Not only have we no direct intuition of the equality of two durations, but we have not even direct intuition of the simultaneity of two events occurring in different places..."

"The simultaneity of two events or the order of their occurrence, and the equality of two time intervals must be defined so that the expression of the laws of physics should be the simplest possible; in other words, all those rules and definitions [conventions for time measurement] only are the fruits of an unconscious opportunism."



- To reconcile the Light Postulate with the Principle of Relativity, require that inertial observers measure time in different ways.
- Relativity of simultaneity: Events are simultaneous only with respect to an inertial frame.

- "Most of the components of Einstein's paper appeared in others' anterior works on the electrodynamics of moving bodies... None of these authors, however, dared to reform the concepts of space and time. None of them imagined a new kinematics based on two postulates. None of them derived the Lorentz transformations on this basis. None of them fully understood the physical implications of these transformations. It all was Einstein's unique feat." (Darrigol, pg. 25.)