14. The Maxwellian Heyday.

Hunt (1991), Chaps 7 & 8

Heinrich Hertz
(1857-1897)

A. Hertz's Waves.

- *1887*. Hertz generates and detects EM waves.

EM waves produced by spark here...

... and spark in gap here.

... cause current in loop there...

Hertzian radiator

Hertzian resonator

- *1888*. Publishes results. Demonstrates interference between waves traveling along a wire and those transmitted through the air.
- *And*: Demonstrates that EM waves can be reflected, refracted, diffracted and polarized, exactly like shorter wave-length visible light.

- *1888*: Bath meeting of the BAAS. Hertz upstages Lodge.

"When going away from Liverpool on a holiday, I read in the train the July number of Wiedemann's *Annalen*, and there found that Dr. Hertz had obtained much better and more striking evidence of these electromagnetic waves."

"I consider that no more important experiment has been made this century. Your experiment will be called 'Hertz's classical experiment that decided between theories of electromagnetic action at a distance and by means of the ether'."

Key point for Maxwellians: Existence of EM waves entails finite speed of EM effects: Action-at-a-distance theories are no longer tenable.

- "The discovery of electromagnetic waves greatly enhanced and broadened the prestige of the entire Maxwellian program, and the emergence of Maxwell's tehory as *the* theory of electromagnetism, and of FitzGerald, Lodge, and Heaviside as its leading interpreters, dates from 1888." (Hunt, pg. 162.)

B. *Practice versus Theory.*

- *Late 1880's*. Conflict between established community of electrical engineers ("practical men") and Maxwellians.
- *Funamental Issue*: Authority.
  - "Who was to decide that self-induction was or was not important, the 'practical men' or what one of Preece's friends called 'the Maxwellian clique'?" (Hunt, pg. 168.)

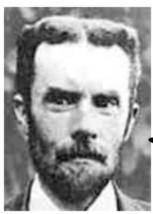
- *1888*. Lodge versus Preece on lightning conductors at Bath BAAS.
- Preece on self-induction:

A "bug-a-boo"!

- But conflict (supposedly) isn't with established theoreticians like Thomson:

> "[I]n the present day, what with Technical Institutes, what with the innumerable students that were sent into the world, especially into the electrical world, every year, they found some of those young fellows coming out every year with a smattering of mathematics; they wrote Papers for the technical journals, and they thrust upon the electrical world conditions and conclusions arrived at by their mathematics with a coolness and an effrontery that were simply appalling. It was to try and check that sort of thing, that was growing to a very serious extent, that he [Preece] had made the few remarks he had."

- Heaviside digs back:

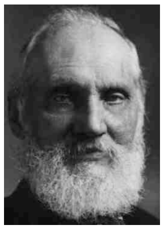


> "Self-induction's 'in the air', Everywhere, everywhere; Waves are running to and fro, Here they are, there they go. Try to stop 'em if you can You British Engineering man!"

- *Post 1888*. "There was growing acceptance not only of Heaviside's dictum that it was 'the duty of the theorist to try to keep the engineer... straight', but also of the principle that the engineer had a duty to heed the theorist on matters of basic electrical doctrine." (Hunt, pg. 174.)

C. The FitzGerald Contraction.

- *Nov. 1888*. Thomson letter to Heaviside.
  - *Rate of propagation of electric potential might be found by moving a charged object back and forth and testing whether electrometers at different distances from it are affected simultaneously or after a time lag.*



Lord Kelvin

"I don't agree that velocity of propagation of electric potential is a merely metaphysical question."

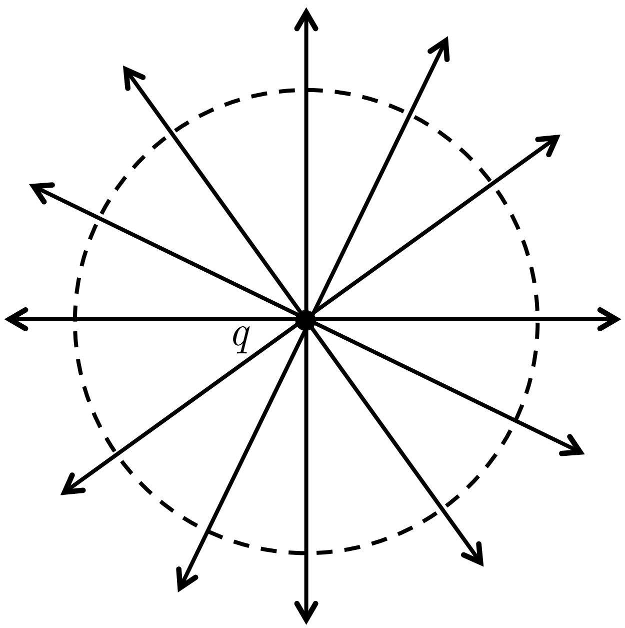
- *Significance of letter*: "It turned the debate over the propagation of the electric potential into one about the state of the electromagnetic field around a moving charge." (Hunt, pg. 185.)
- *Challenge for Maxwellians*: Determine the electromagnetic field of a moving charged object.

- *Dec. 1888*. Heaviside publishes formulas for electric and magnetic fields of a moving charge.

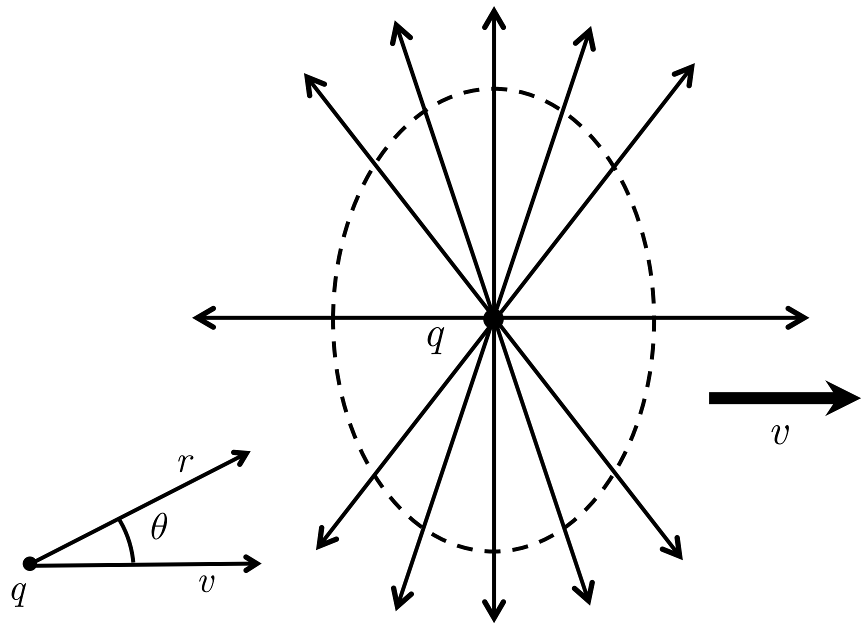
$$\varepsilon E = \frac{q}{r^2}\,\frac{1-(v^2/c^2)}{\left[1-(v^2/c^2)\sin^2\theta\right]^{3/2}}$$

$$H = \varepsilon E v \sin\theta$$

ε = *permitivity of medium*
q = *charge*
v = *charge velocity through ether*
c = *speed of light*
r = *distance from charge to a point*
θ = *angle to the line of motion*

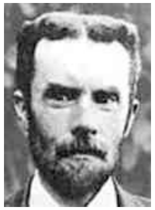


Electric field of charge q at rest.



Electric field of charge q with velocity v.

- Electric equipotential surfaces of moving charge contract in direction of motion as a function of v^2/c^2!



> "If we terminate the field described in [the formulas] on a spherical surface of radius a, instead of continuing it up to the charge q at the origin, we have the case of a perfectly conducting sphere of radius a possessing a total charge q, moving steadily at speed v through the dielectric ether. As the speed is increased to v, the charge all accumulates at the equator of the sphere."

> "But after that? This brings us to the... case of $v > c$, and here I have so-far failed to find any solution which will satisfy all the necessary conditions without unreality."

- *1889*. Heaviside and FitzGerald exchange letters.
  - *FitzGerald mentions possible relevance of formulas to a theory of intermolecular forces and suggests that the velocity of light might be a physical limit to speed.*

- "It is clear that by early 1889, FitzGerald knew that Heaviside had shown that Maxwell's theory implies that the electromagnetic field around a moving charge would be compressed by an amount depending on $1 - (v^2/c^2)$." (Hunt, pg. 190.)

1887. Michelson and Morley's ether drift experiment.

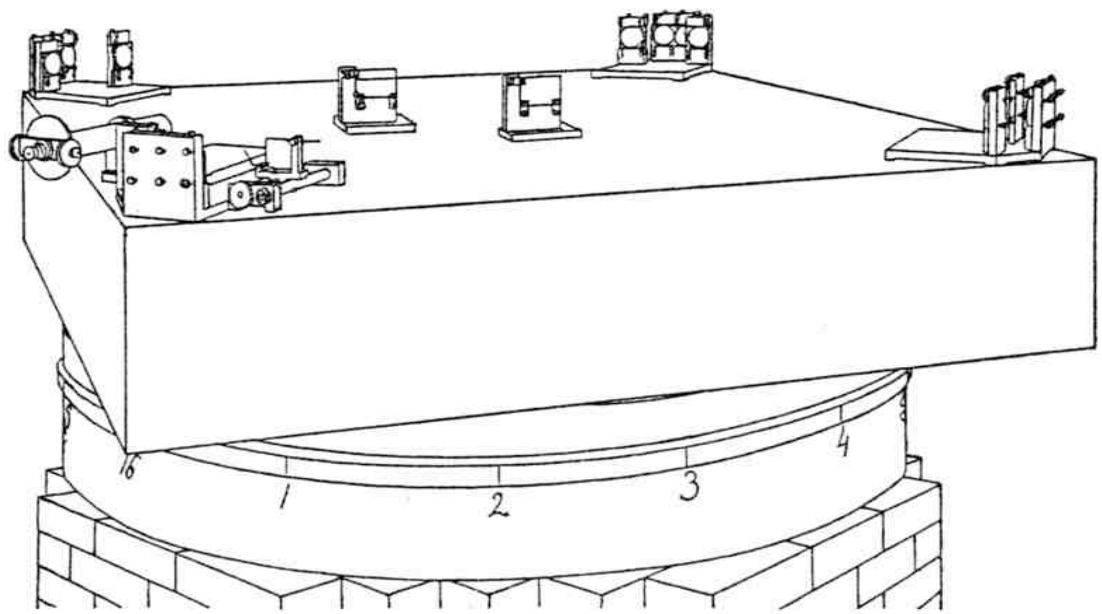
Albert Michelson (1852-1931)

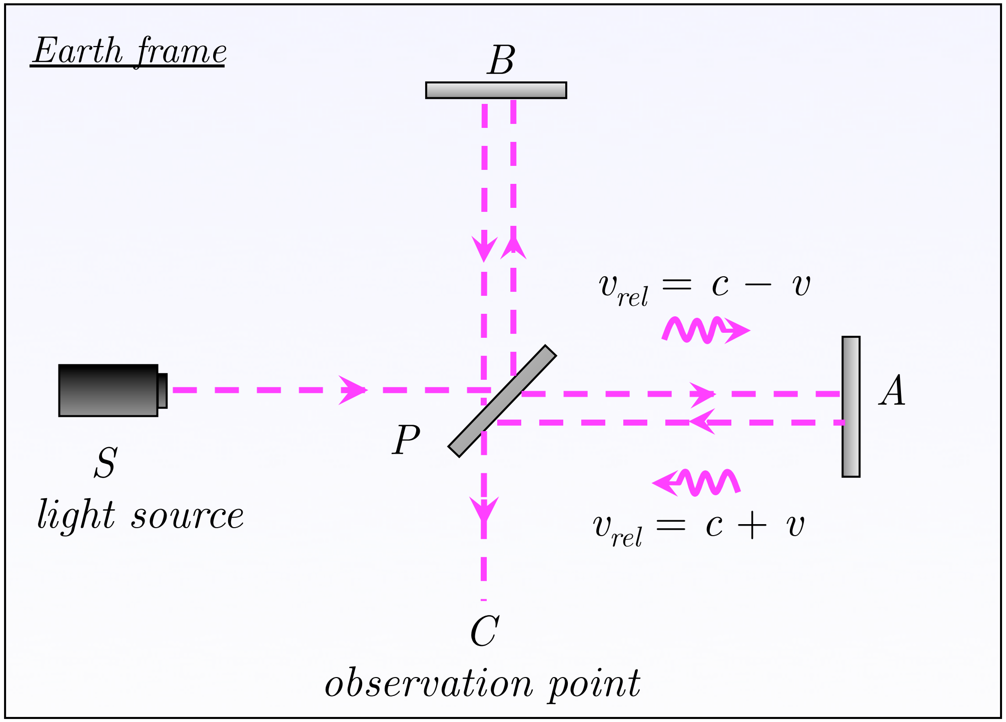
Edward Morley (1838-1923)

- *Purpose:* Measure speed of Earth through ether.
  - *Assumption 1*: EM waves travel at constant speed c with respect to ether, regardless of direction.

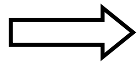
- *Procedure:* Measure the time it takes a light signal to travel a given distance L in the direction of the Earth's motion through the ether, and in a direction perpendicular to this direction.
  - *Assumption 2*: The speed of light as measured on the Earth will be different from c, depending on the Earth's speed and direction of travel.

- *Set-up:* Michelson interferometer.





Ether frame



$v = ?$

c = speed of light *w.r.t.* aether

v = speed of interferometer *w.r.t.* aether

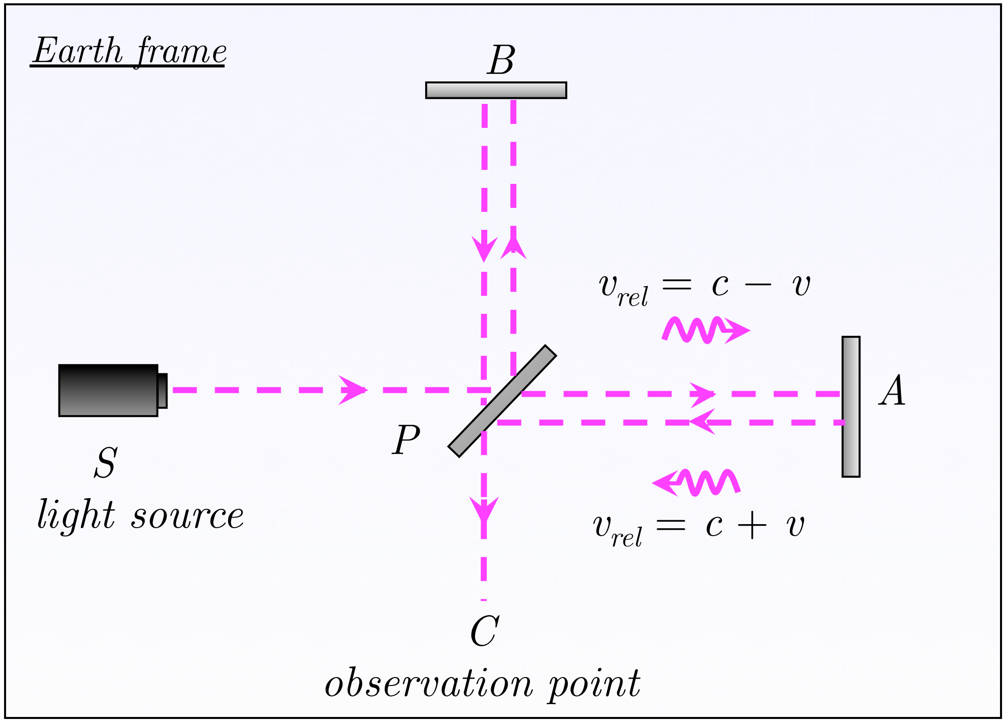
v_{rel} = speed of light *w.r.t.* interferometer

L = length of arms PA and PB *w.r.t.* interferometer.

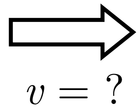
- *Suppose*: PA is in direction of Earth's motion through aether
- *Let*: T_1 = time for light signal to travel path PAP.
  T_2 = time for light signal to travel path PBP.
- *Then:* If $T_1 - T_2 = 0$, then no *interference* will be observed at C.
  If $T_1 - T_2 \neq 0$, then *interference* will be observed at C.
- *Can show*: $T_1 - T_2$ depends explicitly on v.
- *So*: v can be calculated if we can determine the width of the interference pattern $c(T_1 - T_2)$ at C.

How to calculate $(T_1 - T_2)$

- *In Earth frame*: $t_{PA} = \dfrac{L}{v_{rel}} = \dfrac{L}{c-v}$ and $t_{AP} = \dfrac{L}{c+v}$.
- *So:* $T_1 = \dfrac{L}{c-v} + \dfrac{L}{c+v} = \dfrac{2L}{c\left(1-\frac{v^2}{c^2}\right)} = \dfrac{2L}{c}\gamma^2$ where $\gamma = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.
- *In aether frame*: $c^2 = {v_{rel}}^2 + v^2$, or $v_{rel} = \dfrac{c}{\gamma}$.
- *So*: $t_{PB} = t_{BP} = \dfrac{L}{v_{rel}} = \dfrac{L}{c}\gamma$.
- *Thus*: $T_2 = \dfrac{2L}{c}\gamma$.
- *The binomial theorem entails*: For $v \ll c$, $\gamma \approx 1 + \frac{v^2}{2c^2}$ and $\gamma^2 \approx 1 + \frac{v^2}{c^2}$.
- *So*: $T_1 - T_2 = \dfrac{2L}{c}\gamma^2 - \dfrac{2L}{c}\gamma \approx \dfrac{2L}{c}\left(\dfrac{v^2}{2c^2}\right) = \dfrac{Lv^2}{c^3}$.



Ether frame



c = speed of light *w.r.t.* aether

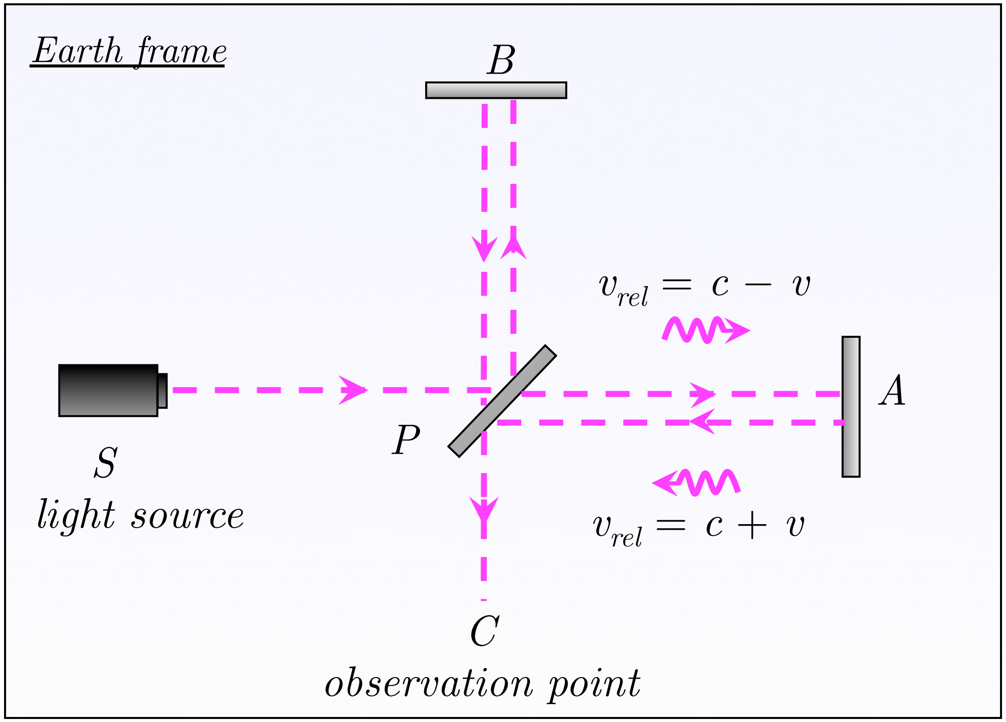
v = speed of interferometer *w.r.t.* aether

v_{rel} = speed of light *w.r.t.* interferometer

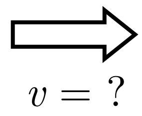
L = length of arms PA and PB *w.r.t.* interferometer.

- *Result of experiment*: No detectable interference!
- Is the Earth at rest in the ether? Does the Earth drag the ether with it?

One of the "outstanding problems" confronting Maxwell's theory.



Ether frame



$c =$ speed of light *w.r.t.* aether

$v =$ speed of interferometer *w.r.t.* aether

$v_{rel} =$ speed of light *w.r.t.* interferometer

$L =$ length of arms PA and PB *w.r.t.* interferometer.

- *1889*. FitzGerald visits Lodge in Liverpool.

> "So also occurred that brilliant suggestion of the change of shape or distortion due to motion through ether, now known as the FitzGerald-Lorentz hypothesis, which flashed on [FitzGerald] in the writer's study at Liverpool as he was discussing the meaning of the Michelson-Morley experiment." (Lodge, 1901.)

- *FiztGerald's suggestion*: The motion of bodies through the ether cause them to change in size by just the amount needed to account for the Michelson-Morely null result.

<u>*FitzGerald Contraction*</u>. Objects physically contract in the direction of their motion through the ether by an amount $\gamma = (1 - v^2/c^2)^{-1/2}$.

- <u>*How this explains MM experiment:*</u>
  - <u>*Recall*</u>: $cT_1 = 2L\gamma^2$ and $cT_2 = 2L\gamma$.
  - <u>*So*</u>: MM result $T_1 = T_2$ entails $2L\gamma^2 = 2L\gamma$.
  - <u>*Now*</u>: Suppose L is different for both paths:

$$
\begin{aligned}
cT_1 &= 2L_{||}\gamma^2 && (L_{||} = \text{length of path parallel to motion})\\
cT_2 &= 2L_{\perp}\gamma && (L_{\perp} = \text{length of path perpendicular to motion})
\end{aligned}
$$

  - <u>*Then*</u>: If $T_1 = T_2$, then $L_{\perp} = L_{||}\gamma$.
  - <u>*So*</u>: Arm of interferometer $L_{||}$ parallel to direction of motion is shorter than arm $L_{\perp}$ perpendicular to direction of motion (recall $\gamma > 1$).

- The FitzGerald contraction hypothesis: An *ad hoc* response to the Michelson-Morely experiment?

  - *Hunt (pg. 193)*: "...it should be stressed that [FitzGerald] had in mind the prerequisites for a plausible contraction hypothesis well before his discussion with Lodge precipitated its statement... FitzGerald thought it quite likely that intermolecular forces were electromagnetic; in any case, he was convinced they depended on the ether just as electromagnetic forces did and so would presumably be affected in the same way by motion. Most important, he had fresh in his mind Heaviside's formula giving just the $(1 - v^2/c^2)^{1/2}$ effect needed to account for Michalson and Morely's result."

- *In other words*: If the electric field of a moving charge contracts, and if intermolecular forces are electromagnetic in nature, then it's not unreasonable to posit a physical contraction of objects moving through the ether.

- *1889*. FitzGerald publishes letter to *Science* describing hypothesis.

"I have read with much interest Messrs. Michelson and Morley's wonderfully delicate experiment attempting to decide the important question as to how far the ether is carried along by the earth. Their result seems opposed to other experiments showing that the ether in the air can be carried along only to an inappreciable extent. I would suggest that almost the only hypothesis that can reconcile this opposition is that the length of material bodies changes, according as they are moving through the ether or across it, by an amount depending on the square of the ratio of their velocities to that of light. We know that electric forces are affected by the motion of the electrified bodies relative to the ether, and it seems a not improbable supposition that the molecular forces are affected by the motion, and that the size of a body alters consequently. It would be very important if secular experiments on electrical attractions between permanently electrified bodies, such as in a very delicate quadrant electrometer, were instituted in some of the equatorial parts of the earth to observe whether there is any diurnal and annual variation of attraction - diurnal due to the rotation of the earth being added and subtracted from its orbital velocity; and annual similarly for its orbital velocity and the motion of the solar system."

- *1892*. Lorentz independently develops same idea.
  - *1895*. Mentions FitzGerald in *Versuch*.
  - Claim now referred to as the Lorentz-FitzGerald Contraction hypothesis.

Hendrik Lorentz
(1853-1928)

D. *What is Maxwell's Theory?*

"I know of no shorter or more definite answer than the following: Maxwell's theory is Maxwell's system of equations."

Ulterior motive: "Hertz wanted to justify regarding both Helmholtz's equations and his own as valid expressions of 'Maxwell's theory'. To do this, he had to separate the mathematical content of Maxwell's theory, to which he believed his own and Helmholtz's equations to be formally equivalent, from its physical basis, with which Hertz's and Helmholtz's theories were in conflict." (Hunt, pg. 198.)

- *But*:

"Helmholtz's theory seemed to me as if he had read all of Maxwell at once, then gone to bed and had a bad dream about it, and then put it down on paper independently; his theory being Maxwell's run mad."

- *1893*. Unsigned review of Hertz's *Electric Waves*:

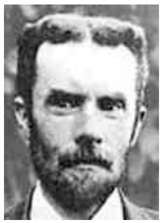
"Any exposition of Maxwell's theory which does not clearly put before the reader that energy is stored in the ether by stresses working on strains, is a very incomplete representation of Maxwell's theory."

Key point for Maxwellians: Heart of Maxwell's theory are the stresses, strains, and energy in the ether.

- "Both force and displacement, or stress and strain, were needed to account dynamically for the storage of energy in the ether." (Hunt, pg. 200.)

"I am afraid I consider it of the essence of that theory to distinguish between Force and Displacement. It is generally almost ignored and the ether in consequence is also ignored, everything resting on symbols to the destruction of a chance of explaining the struture of the ether and also to the confusion of learners who, I think, require some geometrical and dynamical analogues to hang their arguments on. I would call any reasonable modification of this fundamental idea, Maxwell's Theory, though perhaps this germ is really Faraday's."

- *Moreover*: Even Maxwell himself should not be followed slavishly when it comes to determining what Maxwell's Theory is.



"The first approximation to the answer is to say, There is Maxwell's book as he wrote it; there is his text, and there are his equations: together they make his theory. But when we come to examine it closely, we find that this anwer is unsatisfactory... There are many obscurities and some inconsistencies. Speaking for myself, it was only by changing its form of presetnation that I was able to see it clearly, and so as to avoid the inconsistencies."

"By a Maxwellian, I mean one who follows Maxwell as interpreted by O. H."

- *Maxwellian textbooks on electromagnetism*:
  - T. Preston *The Theory of Light.* 1890 (1st edition), 1895 (2nd ed.), 1901 (3rd ed.), 1912 (4th ed.).
  - A. Föpple (1894) *Einfuhrung in die Maxwell'sche Theorie der Elektricität.* (Textbook from which Einstein learned Maxwell's theory.)