

- 1861. British Association for the Advancement of Science establishes Committee on Electrical Standards.

- *Committee members include Maxwell and Thomson:*



"the important applications of electromagnetism to telegraphy have... reacted on pure science by giving a commercial value to accurate electrical measurements, and by affording to electricians the use of apparatus on a scale which greatly transcends that of any ordinary laboratory. The consequences of this demand for electrical knowledge, and of these experimental opportunities for acquiring it, have been already very great, both in stimulating the energies of advanced electricians, and in diffusing among practical men a degree of accurate knowledge which is likely to conduce to the general scientific progress of the whole engineering profession."

B. Field Theory and Telegraphy

- 1853. New Anglo-Dutch cable experiences distortion: pulses through cables delayed and elongated; signals sent in rapid succession unreadable at receiver.

Recall Faraday's views:

- *Electric charge* = manifestation on surface of a conductor of a state of strain in the surrounding medium. (Not an accumulation of imponderable fluid that acts at a distance.)
- *Electric current* = consequence of continual breakdown of strain within a conductor. (Not a real flow of fluid.)
- So: Conduction is always preceded by the induction of a state of strain, and only after a charge has been induced can a current rise to full strength.

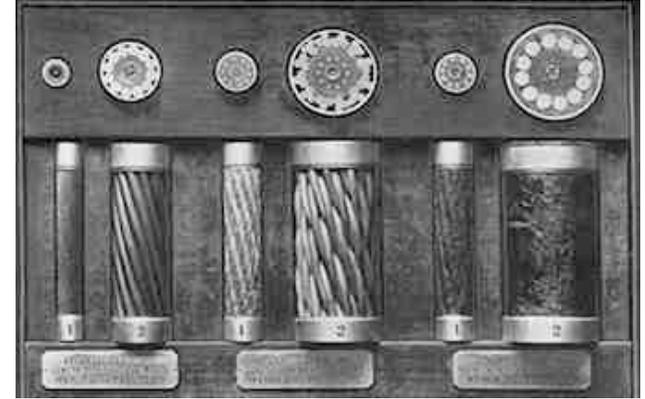


Faraday's explanation of distortion in telegraph cables:

- Process of induction followed by conduction happens too quickly in normal conductors to be detected (*low capacitance*, so can be charged and begin to carry current almost instantaneously).

- But: Submarine cables have *large* capacitance.

- *Cable consists of central wire separated from outer conductor of water/damp soil by thin layer of dielectric insulation (effectively a large condenser).*



- So: When voltage is applied, it takes time for electrostatic strain to spread through the insulation along entire length of cable; so current is retarded.
- And: When voltage is disconnected, more time is needed for strain to relax; subsequent discharge along cable prolongs current, leading to blurring of signals.
- Note: Action-at-a-distance theories suggest induction should happen instantly along entire cable.

- 1855. Thomson's Law of Squares: Retardation increases with both resistance and capacitance of a cable; hence is proportional to square of cable's length.

- So: If cable 200 miles long shows retardation of 1/10 sec, then cable 2000 miles long should show retardation of 100 sec.
- Now: Increasing diameter of conductor wire reduces resistance, and increasing diameter of insulating coating reduces capacitance.
- So: Increasing both diameter of wire and insulating coating in proportion to length should keep retardation of a long cable the same as that of a short one.

- 1856. E. O. Whitehouse challenges Thomson's views.
 - *Suggests cable of small diameter and high voltage is optimal.*
- 1858. First Atlantic cable laid to Whitehouse's specifications. Fails!
- 1866. New Atlantic cable laid to Thomson's specifications succeeds!

"Submarine cable did not *produce* Faraday's field ideas, but it provided a market for them -- a market that, like the cable industry, was almost entirely British... only in Britian was there sustained experience with cables and retardation, and only in Britian were Faraday's ideas taken up and elaborated." (Hunt, pg. 65.)

C. Heaviside on Propagation

- 1868. Heaviside hired to work on newly laid cable between Newcastle and Denmark (company later merges into Great Northern Telegraph Company).
 - *Left school at 16; self-taught thereafter.*



Oliver Heaviside
(1850-1925)

- 1874. Quits telegraph job, returns home and devotes time to private study and research.
- 1876. On distortion in cables:

"The only way, so far as I am aware, is to follow the method given by Sir William Thomson in 1855."



- But: Law of Squares doesn't take into account *self-inductance*.

Faraday's concept of self-inductance

- Basic property of electric currents: A tendency for currents to oppose any changes in their strength.
- Contributed to idea that the magnetic field around a current is a storehouse of energy and motion:
 - "Victorian physicists often pictured this field as filled with some sort of whirling machinery in the ether; when a current was started or reversed, it had to react against the momentum of this whirling machinery or, in electrical terms, against the self-induction. Self-induction thus acted much like a flywheel to oppose any change in the current." (Hunt, pg. 67.)

- Thomson: Self-inductance can be ignored in cables since its effect is swamped by large retardation caused by electrostatic capacity.
- Heaviside: A fully general theory of electromagnetic propagation must take self-inductance into account.

Heaviside's Equation:

$$\frac{d^2v}{dx^2} = ck \frac{dv}{dt} + sc \frac{d^2v}{dt^2}$$

v = voltage at a point x on the line;

c = capacitance/length; k = resistance/length;

s = inductance/length

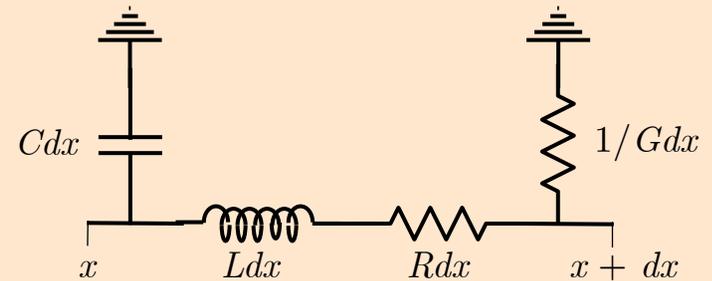
- Implication: For $s = 0$, current ($i = v/k$) diffuses along the line like heat (Thomson's result); but for $s \neq 0$, current oscillates back and forth before settling into a steady state.
- Significance: Beginnings of reconceptualization of telegraphic propagation in terms of oscillatory waves.



"We may be sure that, in virtue of the property of the electric current which Professor Maxwell terms its 'electromagnetic momentum,' whenever any sudden change of current or of charge takes place in a circuit possessing an appreciable amount of self-induction, the new state of equilibrium is arrived at through a series of oscillations in the strength of the current, which may be noticeable under certain circumstances."

Heaviside's equation (the Telegraph Equation): Contemporary View

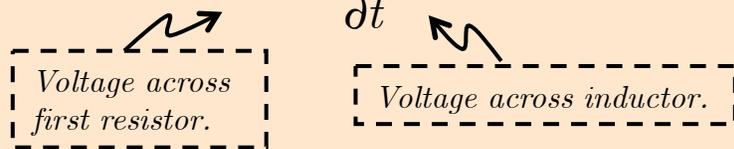
- Infinitesimal section of telegraph wire = an electrical circuit consisting of a resistor (with resistance Rdx), and an inductor (with self-inductance Ldx).



- Suppose: Current $i(x, t)$ can escape to ground either through a second resistor (with conductance Gdx), or through a capacitor (with capacitance Cdx).

- Then: Change in voltage between ends is: $dv = -iRdx - \frac{\partial i}{\partial t} Ldx$.

- Or: $\frac{\partial v}{\partial x} + Ri + \frac{\partial i}{\partial t} L = 0$. (1)



- And: Change in current due to loss to ground is: $di = -vGdx - \frac{\partial v}{\partial t} Cdx$.

- Or: $C \frac{\partial v}{\partial t} + Gv + \frac{\partial i}{\partial x} = 0$. (2)



- From (2): $\frac{\partial^2 i}{\partial x \partial t} = -C \frac{\partial^2 v}{\partial t^2} - G \frac{\partial v}{\partial t}$.

- From (1): $\frac{\partial^2 v}{\partial x^2} + R \frac{\partial i}{\partial x} + \frac{\partial^2 i}{\partial x \partial t} L = 0$.

Combine to get Telegraph Equation:

$$\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2} + (RC + GL) \frac{\partial v}{\partial t} + RGv.$$

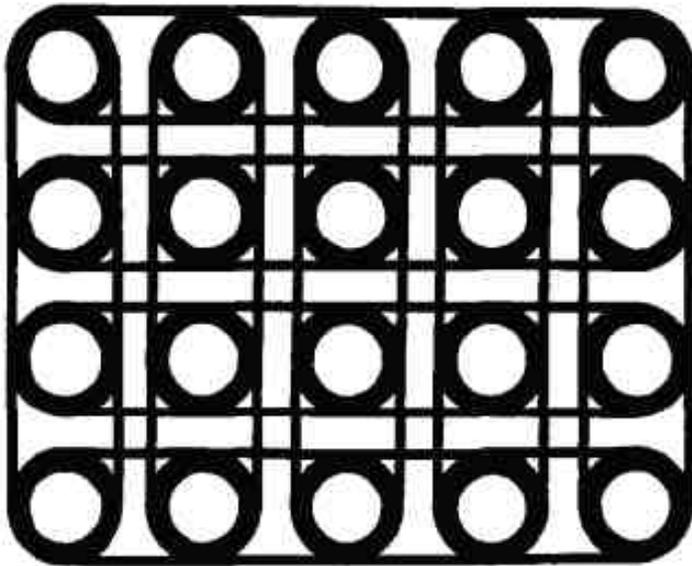
D. Ether Models

I. FitzGerald's wheel and band model (1885).

- Intended to account for Poynting's (1884) description of the flow of energy in an electromagnetic field.

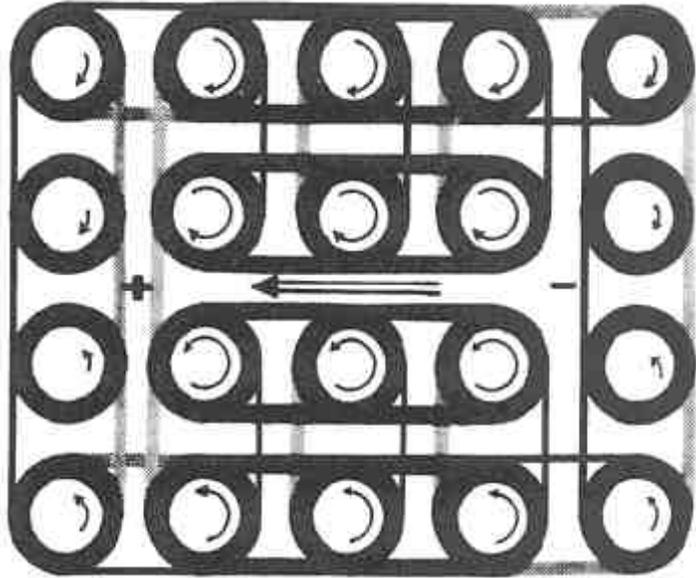


"... I want to illustrate Poynting's great discovery that the energy of an electric current must come in at its sides and is not carried along with the current... I propose a series of wheels of course on fixed axes all connected in pairs by india-rubber bands."

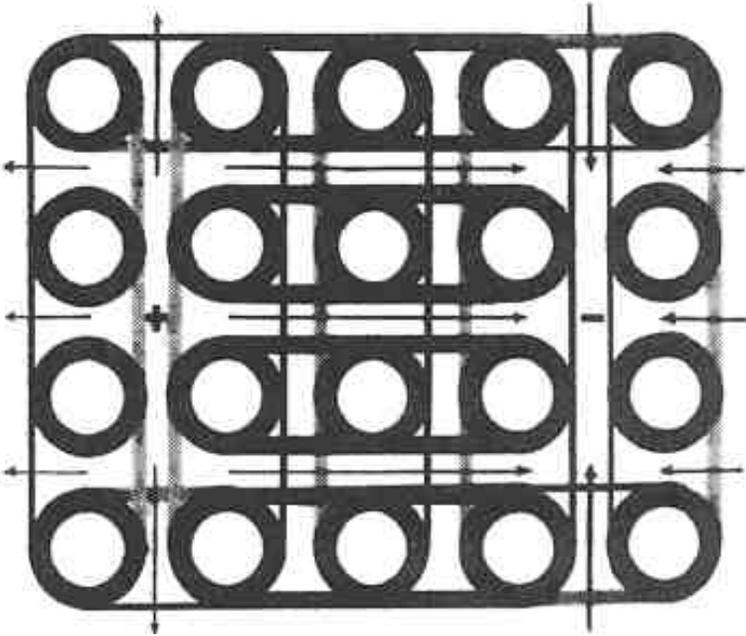


- *Spinning wheels = magnetic field.*
- *Rotational inertia of wheels = self-inductance.*
 - If wheels in a given row spin at same rate, then no strain on band.
 - If wheels in a given row spin at different rates, then one side of band tighter than the other.
- *Strain in bands = electric field.*
- *Elasticity of bands = inductive capacity of medium.*

Example 1: Charging a condenser.



- Two regions with bands removed = perfect conducting plates.
 - Line connecting regions with bands removed = wire.
 - Now: Turn wheels above wire one way, and wheels below wire the other way (corresponds to connecting a battery between plates).
-
- Result: Bands connecting wheels become strained (narrow lines represent tight bands, shaded lines represent loose bands).
 - In particular: Left plate bands become loose, right plate bands become tight.
 - Corresponds to: The conductors become oppositely "charged" ("charge" as reflection of conditions in the medium).
 - Charge is maintained only by continuing to pull along the wire with an impressed force (i.e., keeping battery connected).
 - Note: Represents charge as a reflection of conditions in the medium.

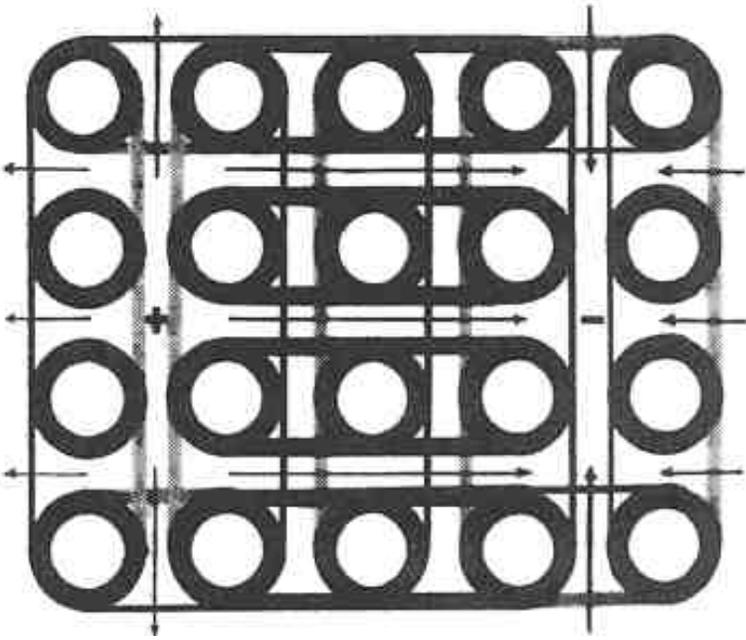


- Now: Reattach bands along wire (corresponds to insulating the plates by removing wire, thus forming a charged condenser).
 - Result: A "self-locked" system of strain is established (corresponds to a charged condenser).

- Tension in surrounding bands is partly spent in straining the reattached bands, which now become loose on one side and tight on the other.
- A vector drawn from the loose side to the tight side represents the electric displacement within each element.

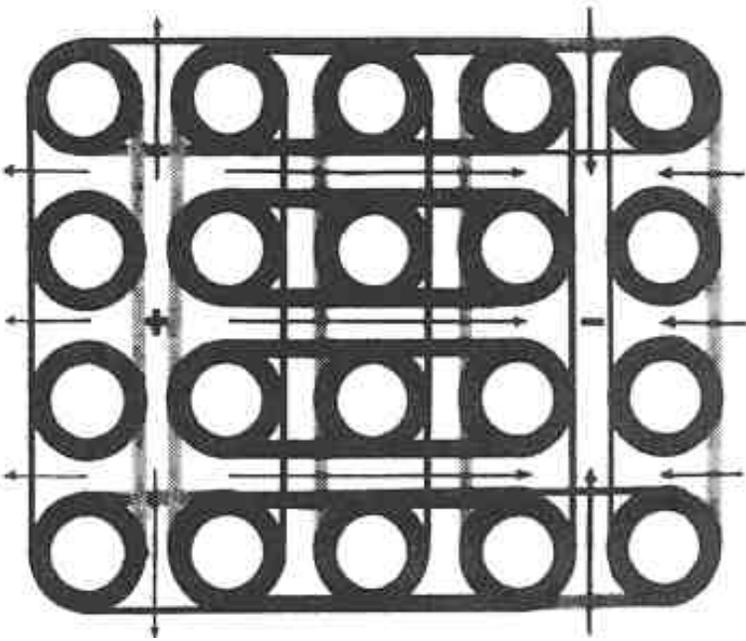
Moral: Electric displacement should not be taken literally as a displacement of something; only a rearrangement of tension.

Example 2: Discharging a condenser.



- Partly loosen bands around wire between charged plates.
 - Bands will slip slightly over wheels and dissipate energy in friction (corresponds to a conducting path with resistance).
- System is no longer self-locked:
 - Bands along conducting path slip (corresponds to a conduction current).
 - Wheels on either side turn in opposite directions (corresponds to accompanying magnetic field).
- As system returns to original, unstrained state, energy stored in strained bands dissipates as heat in conducting wire.
 - Energy comes in from the surrounding medium along the length of the strained bands and enters the 'wire' at its sides.
 - In particular: "Energy does not flow along the line of the current, but instead in paths perpendicular both to the axes of the wheels (in the direction of the magnetic force) and to the 'polarization' lines from the loose to the tight sides of the bands (the electric displacement vector)." (Hunt, pg. 82.)

Example 3: Electromagnetic waves generated by a discharging condenser.



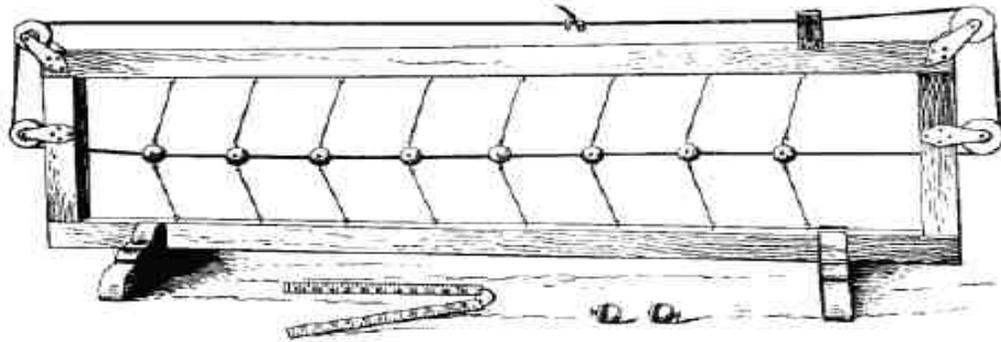
- If resistance along conducting path is not too great...
 - *Inertia of wheels carries them past equilibrium point and wheels and bands will "bounce" from one state of strain to its opposite.*
 - *Oscillations will gradually decrease in amplitude.*
 - *"Such an oscillatory discharge will cause the wheels in the surrounding field to bounce as well; ...and a wave of changing motion and strain will spread out from the discharging wire." (Hunt, pg. 82.)*

- Problem with wheel and band model: Only represents 2-dim system.
 - *FitzGerald proposes 3-dim paddle wheel and fluid-cell model.*
- Such models are only *analogies*, and not *likenesses*:

"I do not in the least intend to convey the impression that the actual structure of the aether is a bit like what I have described."



II. Lodge's string and beads model (1876).



- A loop of string running through a series of buttons attached by elastic bands to an outer frame.

- *Dielectric* = Buttons attached firmly to string so that they elastically resist any displacement.
 - *Elasticity of bands = inductive capacity of dielectric.*
- *Conductor* = Buttons attached loosely to string, allowing it to pass through while generating heat by friction.
 - *Coefficient of friction = electrical resistivity.*



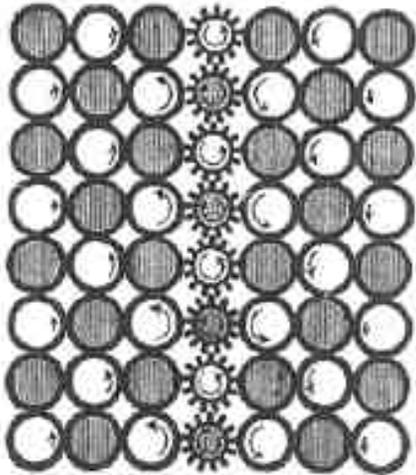
Pierre Duhem
(1861-1916)

"Here is a book intended to expound the modern theories of electricity and to expound a new theory. In it there are nothing but strings which move around pulleys, which roll around drums, which go through pearl beads, which carry weights; and tubes which pump water while others swell and contract; toothed wheels which are geared to one another and engage hooks. We thought we were entering the tranquil and neatly ordered abode of reason, but we find ourselves in a factory."

III. Lodge's cogwheel model (1889).



- Toothed wheels geared directly together.
 - *But then:* Adjoining wheels spin in opposite directions; how can they represent a single magnetic field?
- *Solution:* Divide ether into alternating layers of positive and negative wheels and take positive wheel spinning in one direction as equivalent to negative wheel spinning in opposite direction.



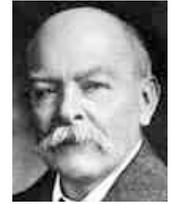
- *Electric current* = rack moving between wheels.
- *Electric displacement* = elastic yielding of wheels along direction of electric force.
- *Conductor* = toothless wheels that partially slip against each other generating heat by friction.
 - Like FitzGerald's wheels slipping on bands.

Criticism:



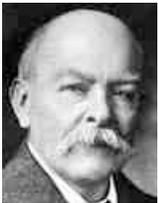
"It continues the dualistic view of electricity, making another distinction than *mere* difference of displacement between positive and negative electricity, while we have no evidence of anything of the kind..."

"It appears to me that Lodge has grafted on to his model the representation of current in the entirely different model of FitzGerald, and so obtains something quite inconsistent with his previous ideas."



John Poynting
(1852-1914)

- Lodge takes displacement from Maxwell (idle-wheels literally displaced).
 - So: *Shouldn't he also represent current, like Maxwell, as a real flow of particles (positive wheels in one direction, negative wheels in the other)?*
 - But: *He adopts FitzGerald's concept of current as a line of "slip" with no real displacement.*



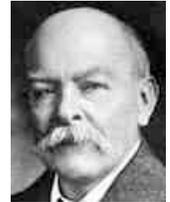
[To accept Lodge's hybrid would be to accept] "...a difference in kind between the process of displacing in a dielectric, the equal and opposite motions of the two fluids which are leading to an electrostatic strain, and the current in a conductor."

- And: The keystone of Maxwell's theory was his intimate connection between displacement and conduction.

A deeper criticism (Poynting 1893):

- Models may be useful aids in understanding, but should not be mistaken for likenesses of reality.

"I believe that the time has hardly come for ultimate mechanical construction, and that, at present, progress is more likely to be made if we are content with an electromagnetic explanation... and leave the ether out of account."

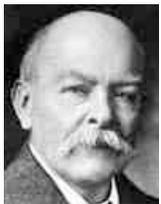


Two general types of explanation:

- *Causal/mechanical explanation* = explanation based on an appeal to causes/mechanisms.
- *Covering law explanation* = explanation based on an appeal to laws.

- Mechanical models are perhaps popular due to the nature of our minds:

"Probably because we are able to picture mechanical processes, able to think of ourselves as seeing what goes on, seeing kinetic energy manifested in the moving parts, able to think of ourselves as part of the connecting machinery, feeling the stresses, and helping to make the strains, we have come to regard mechanical explanations as the inevitable and ultimate ones."



IV. FitzGerald's vortex sponge model (1885).

- Claim (FitzGerald 1885): The ether is an enormous tangled sponge of vortex filaments with matter as closed vortex rings within it.
 - *Not a mechanical illustration or analogy, but a claim about true likeness.*
 - Motivation: "*Radical mechanism*" = *Only pure matter and motion are allowed in fundamental physical phenomena.* (Hunt, pg. 97.)

- A rejection of Thomson's (and Stoke's) elastic jelly ether:

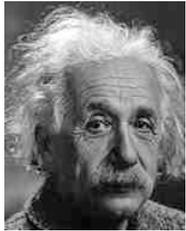


"The hypothesis that the ether is like a thin jelly in no way *explains* this property, as it is the possession of properties analogous to rigidity that requires explanation."

- Why a vortex sponge? It's the only purely mechanical theory that can be based on a continuous ether capable of supporting strong shearing stresses.
 - 1887. Thomson shows that spinning vortices make a fluid rigid enough to resist deformation, and to support transverse waves!
- 1889. FitzGerald attempts to demonstrate that mechanics of a vortex sponge can be correlated with electric and magnetic vectors of Maxwell's theory.
 - But: Thomson shows that FitzGerald's sponge is unstable and will lose ability to support waves or exert forces.

E. Mathematical Models

- Late 1800's. Mechanical models fall out of fashion: "...Maxwell's equations of the electromagnetic field came to be regarded as themselves fundamental and self-sufficient". (Hunt, pg. 104.)



Albert Einstein
(1879-1955)

"One got used to operating with these fields as independent substances without finding it necessary to give one's self an account of their mechanical nature; thus mechanics as the basis of physics was being abandoned, almost unnoticeably, because its adaptability to the facts presented itself finally as hopeless." (1949)

"My experience of so-called 'models' is that they are harder to understand than the equations of motion!"



Heaviside's vector analysis.



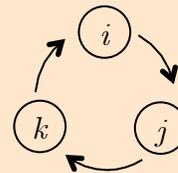
William Hamilton
(1805-1865)

- 1843. Hamilton invents quaternions.
 - Quaternion = a scalar magnitude combined with a vector.
 - A (3-dim) vector can be decomposed into three components (each a scalar magnitude and a direction).
 - So: A quaternion consists of four "parts".

Recall: Complex numbers. $\mathbb{C} = \{\text{all } p + iq, \text{ where } p, q \in \mathbb{R} \text{ and } i^2 = -1\}.$

- 2-dim number system.
- Complex multiplication: $(2 + i3) \times (1 + i6) = (2 \times 1) + (2 \times i6) + (i3 \times 1) + (i3 \times i6)$
 $= 2 + i12 + i3 - 18$
 $= -16 + i15$

Quaternions. $\mathbb{H} = \left\{ \begin{array}{l} \text{all } p + iq + jr + ks, \text{ where } p, q, r, s \in \mathbb{R} \text{ and} \\ i^2 = j^2 = k^2 = -1 \\ ij = k, \quad ji = -k \\ jk = i, \quad ik = -j \\ ki = j, \quad kj = -i \end{array} \right\}$



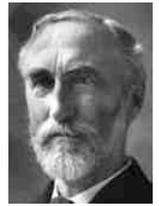
- 4-dim number system.
- Quaternionic multiplication: $(2 + i3 + j5 + k2) \times (1 + i2 + j2 + k5)$
 $= 2 + i4 + j4 + k10 + i3 + i^26 + ij6 + ik15 + j5 + ji10 + j^210$
 $+ jk25 + k2 + ki4 + kj4 + k^210$
 $= 2 + i4 + j4 + k10 + i3 - 6 + k6 - j15 + j5 - k10 - 10 + i25$
 $+ k2 + j4 - i4 - 10$
 $= -24 + i28 - j2 + k8$

- Heaviside learns quaternion algebra from Tait's (1867) *Elementary Treatise on Quaternions*.



"Without exception the hardest book to read I ever saw."

- Breaks quaternions into scalar and vector parts, drops the scalar part, and devises an algebra (rules for addition and multiplication) for the vector part.
- 1882-83. Series of articles by Heaviside in the *Electrician* on vector methods.
 - Gibbs devises similar system.
- "With Heaviside's vector methods, one could portray the electric and magnetic fields and their interactions in a direct and almost palpable way; it was no longer necessary to invoke a mechanical model to give the electromagnetic equations physical content." (Hunt, pg. 107.)



Willard Gibbs
(1838-1903)