11. The Maxwellians: Fitzgerald and Lodge.

Hunt (1991), Chaps 1 & 2.

A. Introduction

- Main Characters:
  - George Francis FitzGerald
  - Oliver Lodge
  - Oliver Heaviside
  - Heinrich Hertz

- 1879-1894: "... transformed the rich but confusing raw material of the *Treatise* into a solid, concise, and well-confirmed theory". (Hunt, pg. 2.)
  - "explored the possibility of generating electromagnetic waves and then actually demonstrated their existence".
  - "delineated the paths of energy flow in the electromagnetic field".
  - "recast the long list of equations Maxwell had given into the compact set now known as 'Maxwell's Equations'".
  - "initiated the application of Maxwell's theory to problems of electrical communciations".

"Maxwell was only ½ a Maxwellian."

B. FitzGerald and Maxwell's Theory

George FitzGerald
(1851-1901)

- 1881. FitzGerald becomes Erasmus Smith's Professor of Natural and Experimental Philosophy at Trinity College Dublin.

Mathematical tradition at Trinity College Dublin

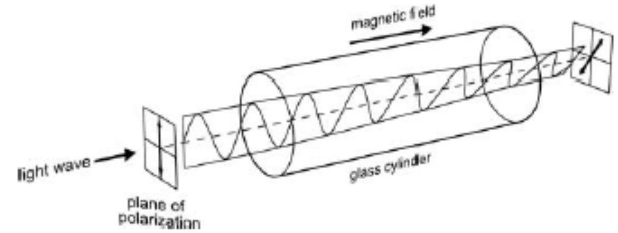
- 1813. Introduction of French analytic mathematical texts in instruction. (Similar "analytic revolution" at Cambridge ~1815-25.)
- 1839. MacCullagh (fellow of Trinity College) derives laws of reflection and refraction as mechanical effects of a particular type of ether.

James MacCullagh
(1809-1847)

- *Cambridge critiques of MacCullagh's ether*:
  - Unphysical: possesses negative compressibility (Green 1838).
  - Violates law of action and reaction (Stokes 1862).
- Fellow Cambridgeans reject MacCullagh (Thomson).
- FitzGerald stands by Dublin:

> "...the most recent theories of Maxwell, while attaching new meanings to MacCullagh's symbols, entirely confirm his results."

Reflection and Refraction in Maxwell's Theory



- *Faraday Effect*: Polarization of light is altered by *refraction* through a magnetized material.
- *Kerr Effect (1876)*: Polarization of light is altered by *reflection* off of a magnetized material.

> *Maxwell's explanation of Faraday Effect (a return to vortex theory)*:
> - Optical motion of ether is coupled to vortical motion of external magnetic field.
> - Include this coupling as a *new term* in kinetic energy.
> - *But*: Max offers no interpretation of optical ether motion in terms of electromagnetism.

- *1878. FitzGerald's intuition*: Can Maxwell's explanation of Faraday Effect be extended to Kerr Effect?
- Bridge = MacCullagh's optical ether theory.
- *In particular*: Maxwell's expressions for kinetic and potential energies of his ether have the same general form as kinetic and potential energies of MacCullagh's ether.

MacCullagh's Ether

$$KE_0 = \frac{1}{2}\rho\left(\frac{d\mathbf{R}}{dt}\right)^2 \quad \text{(flow)}$$

$$PE_0 = \frac{1}{2}a\mathbf{T}^2 \quad \text{(twist)}$$

$\mathbf{R}$ = spatial displacement of ether; ρ = density; a = rotational elasticity; $\mathbf{T}$ = amount of twist = $\nabla \times \mathbf{R}$.

Maxwell's ether (vortex model)

$$KE_1 = \frac{1}{2}\mu\mathbf{H}^2 \quad \text{(spin)}$$

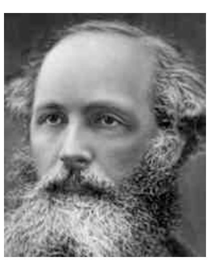
$$PE_2 = \frac{1}{2}\varepsilon\mathbf{E}^2 \quad \text{(displacement)}$$

$\mathbf{H}$ = magnetic force; ρ, ε = magnetic and electric constants; $\mathbf{E}$ = electric force; $\mathbf{E} = -\dfrac{d\mathbf{A}}{dt}, \mu\mathbf{H} = \nabla \times \mathbf{A}$.

- *Suppose*: $\mathbf{H} = d\mathbf{R}/dt$ (*magnetism as flow of MacCullagh's ether*).
- *Then*: $\varepsilon d\mathbf{E}/dt = \nabla \times \mathbf{H} = \nabla \times d\mathbf{R}/dt$.
- *So*: $\varepsilon\mathbf{E} = \nabla \times \mathbf{R} = \mathbf{T}$ (*electric displacement as a twist in MacCullagh's ether*).
- *And*: $KE_0 = KE_1$, $PE_0 = PE_2$.
- *Thus*: All of MacCullagh's results for reflection and refraction can be translated into Maxwell's theory.

- *Task*: Use MacCullagh translation to provide an account of Kerr Effect in Maxwell's theory.
  - FitzGerald takes Maxwell's term for Faraday Effect (describes coupling of ether motion to magnetic vortices) and adds it to MacCullagh's expression for kinetic energy.

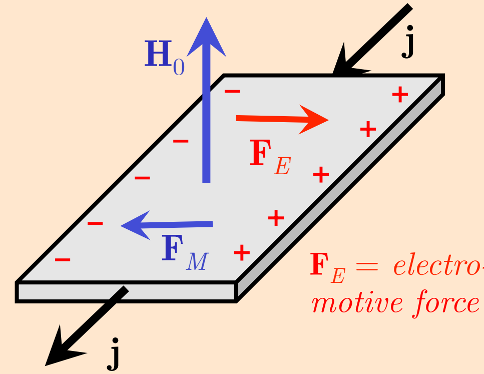
- This gives accurate description of Kerr Effect.
- *But*: What is physical meaning of added term?
  - Maxwell treats magnetism as ether *spin*, FitzGerald treats it as ether *flow*.
  - *So*: FitzGerald's term is *not* a consequence of Maxwell's vortex hypothesis.



Max

- *In other words*: FitzGerald's new term is added in a purely *ad hoc* manner without independent electromagnetic justification.

Hall Effect (1879): A magnetic field $\mathbf{H}_0$ applied perpendicular to an electric current $\mathbf{j}$ induces an electromotive force $h\mathbf{H}_0 \times \mathbf{j}$ in a direction perpendicular to both the magnetic field and the current (where h is a constant).




Edwin Hall
(1855-1938)

- 1880. Henry Rowland shows that if Hall Effect applies to displacement currents, then it gives rise to an added term $h\mathbf{H}_0 \times \partial\mathbf{D}/\partial t$ in the electromotive force for a magnetic field that accounts for Faraday Effect.
  - *Pure EM account of Faraday Effect.*

- 1881. R. T. Glazebrook shows that the corresponding term in the kinetic energy is equivalent to FitzGerald's added term!
  - *FitzGerald's term is* $-2k(\mathbf{H}_0 \cdot \nabla)\mathbf{H}$, *which differs from Rowland's term* $k\mathbf{H}_0 \times (\nabla \times \mathbf{H})$ *by a gradient, which can be absorbed into scalar potential.*

- *Upshot*: Hall Effect provides the electromagnetic justification for FitzGerald's added term.

- *Significance of FitzGerald's work*: Suggests the ether is not as physical as the elastic solid of Thomson and Maxwell.
  - *Perhaps need to take MacCullagh's non-physical ether seriously.*

C. FitzGerald, Lodge and Electromagnetic Waves

- Hunt: Maxwell's belief that light waves were electromagnetic does *not* imply that he believed they were *generated* electromagnetically.
    - *Max thought generation of light waves was mechanical, dependent on connection between matter and ether.*
- 1879. Lodge presents (unpublished) paper to British Association for the Advancement of Science (BAAS) on possibility of producing EM waves.
- 1902 reflection in letter to Larmor:

Oliver Lodge
(1851-1940)

> "So far as I recollect it, it was to the effect that as in my electrostatic models positive and negative electricity always did opposite things -- so that shear of ether explained electrostatic energy -- so this oppositeness of behavior would make them as regards rotation act as if they were geared up like cogwheels rotating in opposite directions: wherefore a disturbance would spread laterally when any part of it was twisted (magnetism); and hence that light could be got from oscillations."

- Refers to 1889 cogwheel model of the ether...

- *Vortices directly geared together.*
- *Two types of vortex: positive electricity and negative electricity, spinning in opposite directions.*

- Elements of this model already worked out in 1880 letter to FitzGerald:

> "My notion, which I think you don't like and which is certainly very crude, that Ether is + & − Electricity together & that when an EMF [electromotive force] is applied the ether is sheared but not moved bodily one way or the other... [I] fancied that light... was periodic electrical displacements viz. + up & − down, the restoring force being due to the electro-static strain instead of ordinary elasticity. And hence I imagined that light might be excited electrically."

- <u>*But note*</u>: Lodge is looking to generate *light*, not EM waves in general.
  - "He had not yet hit on the important idea, developed later by FitzGerald and Hertz, of producing and detecting relatively longer electromagnetic waves." (Hunt, pg. 31.)

- 1879. FitzGerald argues that waves cannot be produced electromagnetically.

<u>*Two arguments:*</u>

1. <u>*Max claims*</u>: If $\mathbf{A}$ and $d\mathbf{A}/dt$ at some time are both zero except in a region, then waves will propagate out from that region.

- <u>*Fitz claims*</u>: This violates Gauss's theorem that it is impossible for a potential function to be zero in one region and nonzero in another.
- <u>*But*</u>: Maxwell's claim survives if displacement currents are allowed.

2. <u>*Max claims*</u>: His theory is equivalent to the action-at-a-distance theories of German researchers (Weber, Neumann).

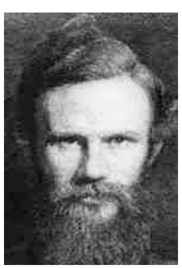
- <u>*But*</u>: No account in action-at-a-distance theories of how energy of an electrical system occupying a given region can be transferred into a non-conducting region surrounding it.
- <u>*In particular*</u>: Any closed system of conductors must conserve its energy.

- <u>*Basic problem*</u>: How can conserved energy of an electrical system be transferred to waves that would disperse it throughout space?

- ~1883. FitzGerald reads Rayleigh's (1877) *Theory of Sound*.
  - Rayleigh uses same differential equation to describe sound waves that FitzGerald considers for EM waves.
  - *But*: Rayleigh's solution corresponds to a train of progressive waves; FitzGerald's solution corresponds to a standing wave.
  - *And*: Rayleigh's solution allows for a gradual transferral of energy to medium that obeys conservation law.

John Strutt
(Lord Rayleigh)
(1842-1919)

- Convinces FitzGerald that oscillating currents can generate EM waves.



"It is most likely that all periodic electric and magnetic [oscillations] are accompanied by a loss of energy just like, and in fact teh same thing as, the radiation of heat. In fact it extends the dissipation of energy by radiation to the case of periodically working electric and electro-magnetic machines."

- *Question #1*: Which part of the energy of an oscillating current is radiated and which part merely sloshes back and forth between current and ether?
- *The Jelly Bubble Analogy*:

> "It is obvious that if a bubble in a big jelly expands or rotates it gives potential energy to the jelly which, when it contracts, is restored to it if the changes go on slowly; while if they go on fast, energy of vibration, *i.e.* kinetic energy, will be given to the jelly, and it will not in general be restored but will be radiated. Hence the energy of the radiation will depend on how fast the bubble changes, *i.e.* on the period of vibration."

- And same thing happens between oscillating current and ether:

> "...on each reversal [of the current], a large part of the energy of the ether reverts to the current and it is evidently only the part out of reach of the current during the opposite phase that is lost, i.e. the total energy that cannot send back a reverse wave in time to help the reversing current."

- Distinction between the near field and the far field of a radiating source, and a focus on the energy that escapes from the immediate vicinity of the source.

- *Question #2*: How does the energy of the near field depend on frequency?

FitzGerald's formula for energy at a distance r from source:

$$= (\pi a^2 i_0)^2 \frac{l^2 \mu}{6}\left(l^2 + \frac{1}{r^2}\right)$$

a = radius of current loop, i_0 = current amplitude, $l = 2\pi/\lambda$, λ = wavelength

"...one part of this is independent [of r], while the other part varies inversely as the square of the distance from the current. It is evidently only the first part of the energy that is really radiated..."

"[The $1/r^2$ term is]... the energy of the forced displacement currents produced directly by the variation of the primary current, and which start the radiating displacement currents."

- $1/r^2$-*term* (negligible far from source) = the energy that sloshes back and forth between the current and the field.
- *Term independent of r* = "...the energy of the additional displacement currents set up by these forced displacement currents; this energy is unable to return to the wire before the primary current reverses, and so is radiated out into space." (Hunt, pg. 43.)

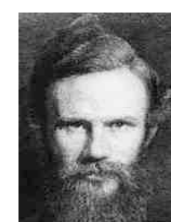
Energy radiated per second (i.e., power)

$$= (\pi a^2 i_0)^2 \frac{8\mu\pi^4}{3T^4c^3}$$ $a =$ *radius of current loop*, $i_0 =$ *current amplitude*, $T =$ *period of oscillation*, $c =$ *velocity of light.*

"If we calculate the amount of this energy we find that it is very small, except for very rapid vibrations indeed."

- For discharging capacitors, $T = 10$ million cycles/second, which might be detectable...
- *But*: Can detect short wavelength light (with eyes). What about high frequency/long wavelength radiation?
- *FitzGerald's suggestion*:

"By the method of interference you can find whether there are progressive or statinary waves."



- *In particular*: "One would simply reflect the waves from a wall and adjust the distance until a resonance was reached and then, by searching out the nodes and loops, proceed to measure the wavelength direclty." (Hunt, pg. 45.)