10. Maxwell.

Darrigol (2000), Chap 4.

1856. "On Faraday's Lines of Force".

- *Intended to obtain mathematical expression of Faraday's field concept.*
- *Distinction between "intensity" (force) and "quantity" (flux).*
- *Two circuit laws:* $\nabla\times \mathbf{H} = \mathbf{j}$ *and* $\mathbf{E} = -\partial \mathbf{A}/\partial t$.

James Clerk Maxwell
(1831-1879)

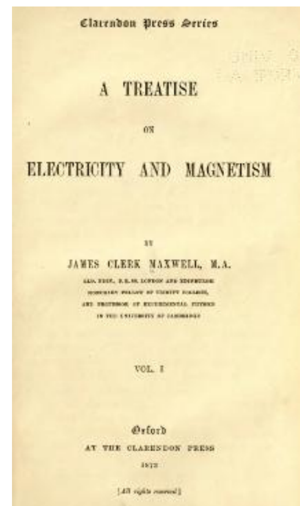
1861. "On Physical Lines of Force".

- *Mechanical model based on vortical nature of magnetism.*
- *Current = motion of idle wheels; charge = accumulation of idle wheels.*
- *Displacement current, full Maxwell's equations, velocity of light.*

1865. "Dynamical Theory of the Electromagnetic Field".

- *Replaced vortex model with dynamical justification of field equations.*
- *Potential A as reduced momentum of magnetic field.*
- *Light as electromagnetic wave.*

1873. *Treatise on Electricity and Magnetism.*



Clarendon Press Series

A TREATISE

ON

ELECTRICITY AND MAGNETISM

BY

JAMES CLERK MAXWELL, M.A.

VOL. I

Oxford

AT THE CLARENDON PRESS

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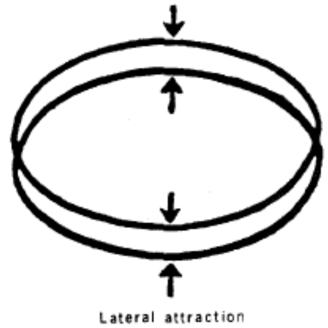
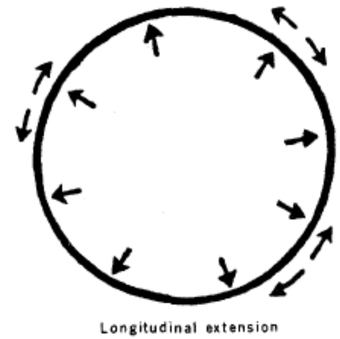
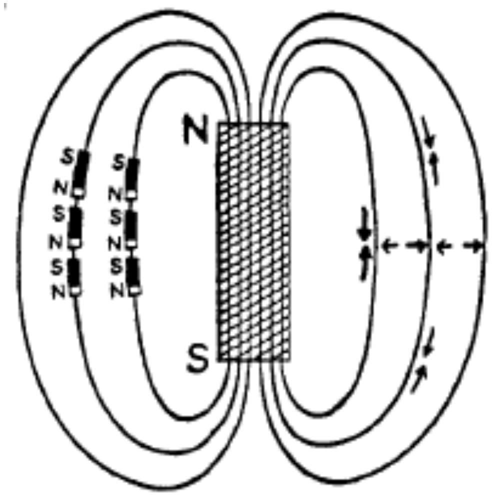
A. Letter to Thomson (1854).*



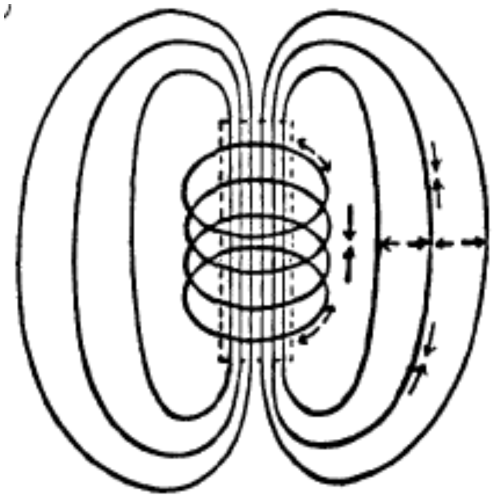


- According to Faraday...
  - *Electric current lines (in same direction) attract laterally and extend longitudinally.*
  - *Magnetic lines repel laterally and contract longitudinally.*
- "Quantity" = *lateral* measure of power, call it **a**.
- "Intensity" = *longitudinal* measure of power, call it $\boldsymbol{\alpha}$.



> *Observed Symmetry*:
>
> (1) Lateral attraction between current lines has same effect as longitudinal contract in magnetic lines.
>
> (2) Lateral repulsion between magnetic lines has same effect as longitudinal extension in current lines.

- *Now*: Express (1) and (2) in terms of quantities and intensities...



Replace magnet with current coil.

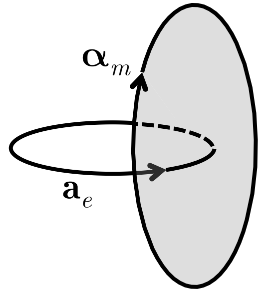
*N. Wise (1979) "The Mutual Embrace of Electricity and Magnetism", *Science* 203, 1310-1318.

<u>*Suppose*</u>:

- *Measure of longitudinal contraction of mag line = sum of mag intensity* $\boldsymbol{\alpha}_m$ *along line.*
- *Measure of lateral attraction of electric lines* ($\mathbf{a}_e$) = # *of elec lines that cross unit area.*

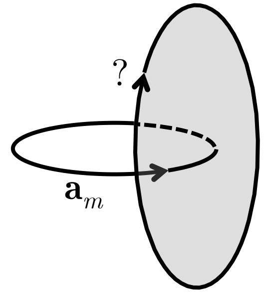
- (1) can now be expressed by ("Ampère's Law"):

> <u>*Theorem 1.*</u> *The magnetic intensity summed around a* <u>***closed loop***</u> *is measured by the total quantity of current through the loop.*
>
> $$\oint \boldsymbol{\alpha}_m \cdot d\mathbf{l} = \iint \mathbf{a}_e \cdot d\mathbf{S}$$



- What about (2)? Analog of Ampère's Law would be:

> <u>*Theorem 2.*</u> *The magnetic quantity through any surface is measured by the "current around its edge".*
>
> $$\oint (\mathit{current?}) \cdot d\mathbf{l} = \iint \mathbf{a}_m \cdot d\mathbf{S}$$



- <u>*But (Wise* 1979*)*</u>:
  - *Mag quantity* $\mathbf{a}_m$ *(lateral repulsion of mag lines) should depend on size and shape of current loop, not just current.*
  - *"Current around the edge" can't be current intensity* $\boldsymbol{\alpha}_e$ *(longitudinal extension) summed around edge, since* $\mathbf{a}_m$ *has no relation to* $\boldsymbol{\alpha}_e$*.*

B. "On Faraday's Lines of Force" (1856).

- *Assume*: Some intensity of the appropriate kind exists to complete the symmetry for Theorem 2; call it the "electro-tonic intensity".

"We may conceive of the electro-tonic state at any point of space as a quantity determinate in magnitude and direction, and we may represent the electro-tonic condition of a portion of space by any mechanical system which has at every point some quantity, which may be a velocity, a displacement, or a force, whose direction and magnitude correspond to those of the supposed electro-tonic state. This representation involves no physical theory, it is only a kind of artificial notation."

- *Now*: Re-express Theorem 2 as Faraday's Law: Electromotive force is the variation in time of "electro-tonic intensity".

Details:

- Adopt Thomson's heat analogy; replace heat with "imaginary incompressible fluid".
- $\mathbf{a} \cdot d\mathbf{S}$ = fluid *quantity* across surface element $d\mathbf{S}$, $\mathbf{a}$ = fluid current.
- $\boldsymbol{\alpha} \cdot d\mathbf{l}$ = fluid *intensity* along line element $d\mathbf{l}$, $\boldsymbol{\alpha}$ = moving force.

- *Then*: Ampère's Law can be expressed by $\nabla \times \boldsymbol{\alpha}_m = \mathbf{a}_e$.

How?

- According to Stokes's theorem, $\oint \boldsymbol{\alpha} \cdot d\mathbf{l} = \iint (\nabla \times \boldsymbol{\alpha}) \cdot d\mathbf{S}$.
- *So*: $\oint \boldsymbol{\alpha}_m \cdot d\mathbf{l} = \iint (\nabla \times \boldsymbol{\alpha}_m) \cdot d\mathbf{S} = \iint \mathbf{a}_e \cdot d\mathbf{S}$.

$\nabla \times \mathbf{H} = \mathbf{J}$

$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{S}$

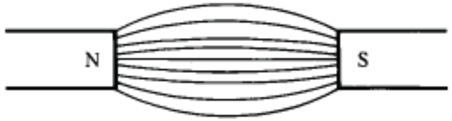
$\mathbf{B} = \nabla \times \mathbf{A}$

$\mathbf{E} = -\partial \mathbf{A}/\partial t$

- *And*: Faraday's Law can be expressed by $\oint \boldsymbol{\alpha}_e \cdot d\mathbf{l} = -\frac{d}{dt} \iint \mathbf{a}_m \cdot d\mathbf{S}$.
- *Now*: Since the fluid is incompressible, $\nabla \cdot \mathbf{a}_m = 0$.
- *Which means*: $\mathbf{a}_m = \nabla \times \boldsymbol{\alpha}_0$, for some $\boldsymbol{\alpha}_0$. Call it the *electrotonic intensity!*
- *Thus*: $\oint \boldsymbol{\alpha}_0 \cdot d\mathbf{l} = \iint (\nabla \times \boldsymbol{\alpha}_0) \cdot d\mathbf{S} = \iint \mathbf{a}_m \cdot d\mathbf{S}$.
- *And*: Faraday's Law becomes $\boldsymbol{\alpha}_e = -\frac{\partial \boldsymbol{\alpha}_0}{\partial t}$.

The symmetric analog of Ampère's Law: The magnetic quantity through any surface is measured by the electrotonic intensity around its edge.

C. "On Physical Lines of Force" (1861).



- *Recall*: Magnetic lines repel laterally and contract longitudinally.
  - *Suppose*: There are *vortices* along magnetic lines.
  - *Then*: Centrifugal forces would cause vortices to expand laterally and contract longitudinally.

Task: Derive expression for stresses in a fluild that produce these effects

- *Let*: p = pressure; μ = density of medium; $\mathbf{H}$ = angular velocity of vortex.
- *Then*: Stresses (*force per area*) are encoded in stress tensor:

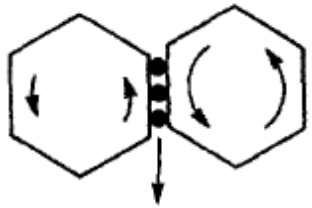
$$\sigma_{ij} = -p\delta_{ij} + \mu H_i H_j$$

- *And*: Force $\mathbf{f}$ due to stresses is divergence of stress tensor:

$$\mathbf{f} = (\nabla \cdot \mu\mathbf{H})\mathbf{H} + (\nabla \times \mathbf{H}) \times \mu\mathbf{H} + m\nabla(H^2/2) - \nabla p$$

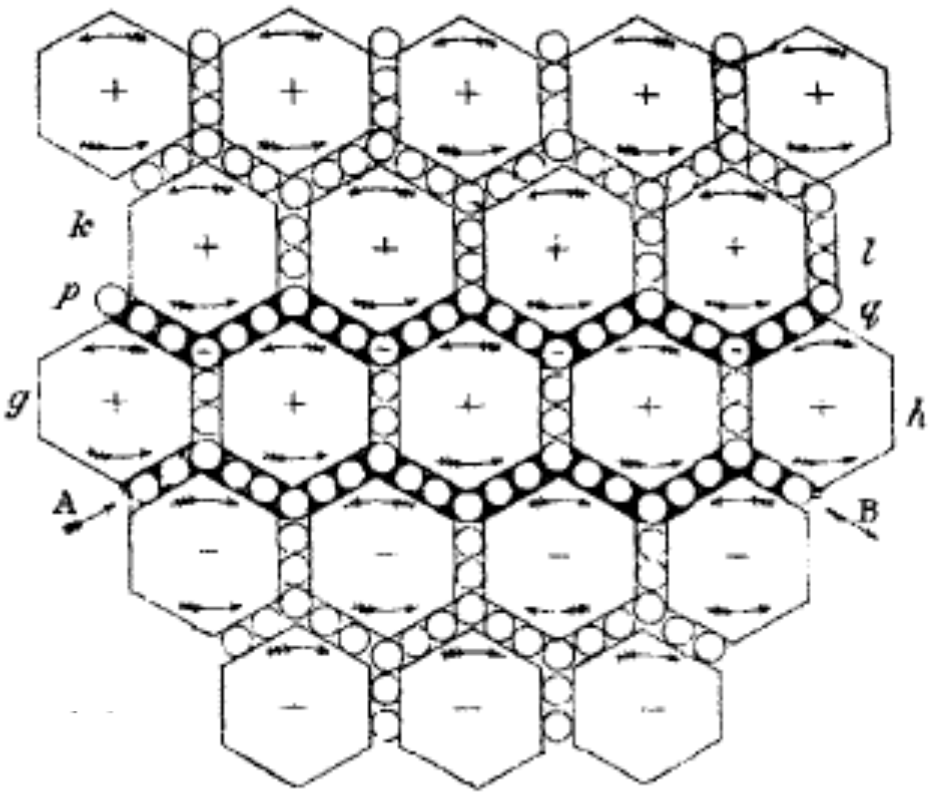
- *Maxwell identifies*:
  - $\mathbf{H}$, $\mu\mathbf{H}$ = magnetic intensity and magnetic quantity.
  - 1st term = force on magnetic poles (regions of non-zero $\nabla \cdot \mu\mathbf{H}$).
  - 2nd term = force on electric currents (regions of non-zero $\nabla \times \mathbf{H}$).

- *Question*: How can correspondence between non-zero $\nabla \times \mathbf{H}$ and existence of currents be represented?
- *Answer*: Idle wheels!



When two contiguous vortices do not rotate at same speed, idle-wheel particles must shift.

- *Maxwell shows*: $(Flux\ of\ idle\text{-}wheel\ particles) = \nabla \times (angular\ velocity\ of\ vortices)$.
- This *mechanical* result suggests *electromagnetic* result $\mathbf{j} = \nabla \times \mathbf{H}$ (Ampère's Law) if we identify current density $\mathbf{j}$ with idle-wheel particle flux.



- Electric current flows from A to B.
- Vortices gh set in anti-clockwise motion (+).
- Particles pq set in clockwise rotation (−) and move right-to-left, forming an induced current.
- If resistence of medium halts induced current, then rotating particles pq act on next row of vortices kl, which will rotate anti-clockwise (+).
- If primary current AB stops, vortices gh will stop rotating, but momentum of vortices kl will cause particles pq to move from left to right, in direction of primary current.
- If this current is resisted by medium, the motion of vortices beyond pq will stop.

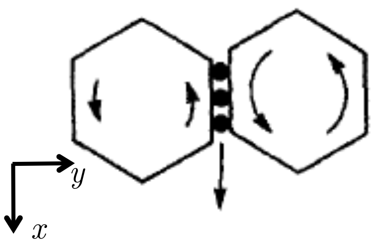
How to understand Maxwell's mechanical result

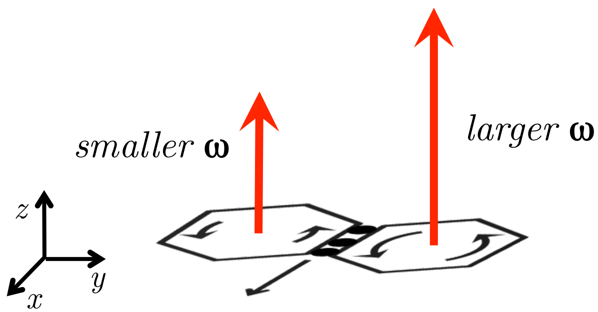
$$(\textit{Flux of idle-wheel particles}) = \nabla \times (\textit{angular velocity of vortices})$$

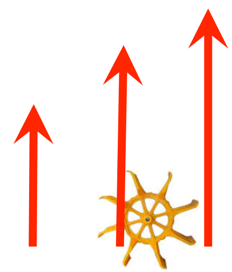
- *Recall*: The curl $\nabla \times \mathbf{F}$ of a vector field $\mathbf{F}(x, y, z)$ measures the *rate of change* of $\mathbf{F}(x, y, z)$ in a direction transverse (perpendicular) to the direction of $\mathbf{F}(x, y, z)$.

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\mathbf{i} - \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right)\mathbf{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\mathbf{k}$$

Rate of change of F in yz-plane. *Rate of change of F in xz-plane.* *Rate of change of F in xy-plane.*





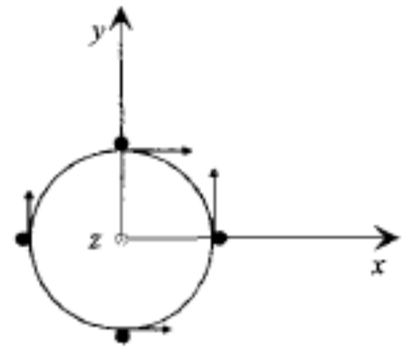


- *Angular velocity vector field changes in transverse direction.*
- *So*: $\nabla \times \boldsymbol{\omega}(x, y, z) \neq 0$.

- *Non-zero curl indicates a "rotation" in the vector field* $\boldsymbol{\omega}(x, y, z)$.
- *Paddle wheel test: Paddle wheel spins if and only if field has non-zero change in transverse direction.*

- *Important*: The "rotation" in $\boldsymbol{\omega}(x, y, z)$ encoded in its curl is *not* responsible for the rotation of the vortices!
  - Rotation of a vortex is encoded in a *particular value* of $\boldsymbol{\omega}(x, y, z)$ (*i.e.*, one particular vector $\boldsymbol{\omega}$ located at some point).

What about Faraday's Law?



- *Suppose*: Tangential force $\mathbf{T}$ of *particles* on *vortices* results in torque.
- *Maxwell shows*: $(\textit{torque}) = \nabla \times (\textit{tangential force}\ \mathbf{T})$
- *Recall*: $(\textit{torque}) = d/dt(\textit{angular momentum})$
- *And*: angular momentum of vortices $= \mu\mathbf{H}$.
- *Now*: Tangential force of *vortices* on *particles* is electromotive force $\mathbf{E} = -\mathbf{T}$.
- *So*: $\nabla \times \mathbf{E} = -\partial\mu\mathbf{H}/\partial t$.
- *And*: This is Faraday's Law (*via* Stokes' Theorem):

$$\oint \mathbf{E}\cdot d\mathbf{l} = \iint (\nabla\times\mathbf{E})\cdot d\mathbf{S} = -\frac{d}{dt}\iint \mu\mathbf{H}\cdot d\mathbf{S}$$

Recap #1:

- *Ampère's Law:* $\mathbf{j} = \nabla \times \mathbf{H}$. The current flux of idle wheels causes spatial changes in transverse directions of rotational velocities of vortices.
- *Faraday's Law:* $-\partial \mu \mathbf{H}/\partial t = \nabla \times \mathbf{E}$. Changes in angular momentum of vortices cause spatial changes in transverse directions of the tangential forces that vortices impart on idle-wheels.

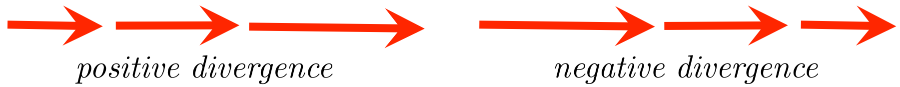
- *Problem*: Limited to *closed currents*: $\nabla \cdot \mathbf{j} = \nabla \cdot (\nabla \times \mathbf{H}) = 0$.
- *Now*: For ρ = charge density, conservation of charge requires (*continuity equation*): $\nabla \cdot \mathbf{j} + \partial \rho/\partial t = 0$.
- *And this reduces to*: $\partial \rho/\partial t = 0$.
- *Which entails*: No charge accumulation.
- *So*: Vortex model, as it stands, can't be applied to electrostatics.

How to understand Maxwell's continuity equation

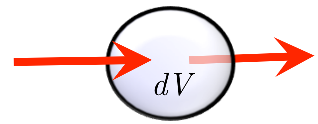
$$\nabla \cdot \mathbf{j} + \partial\rho/\partial t = 0.$$

- *Recall*: The divergence $\nabla \cdot \mathbf{F}$ of a vector field $\mathbf{F}(x, y, z)$ measures the *rate of change* of $\mathbf{F}(x, y, z)$ in the direction of $\mathbf{F}(x, y, z)$.

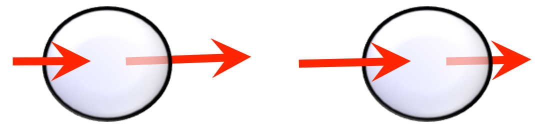
$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$



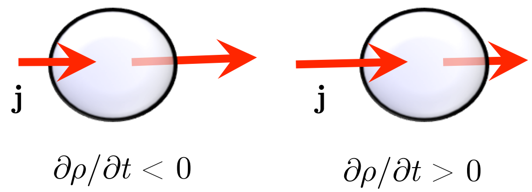
- Zero divergence means same amount of $\mathbf{F}$ leaves an infinitesimal volume dV as amount that enters it.



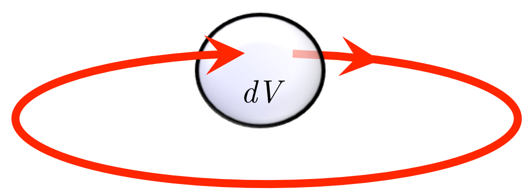
- Positive/negative divergence means more/less $\mathbf{F}$ leaves dV than enters it.



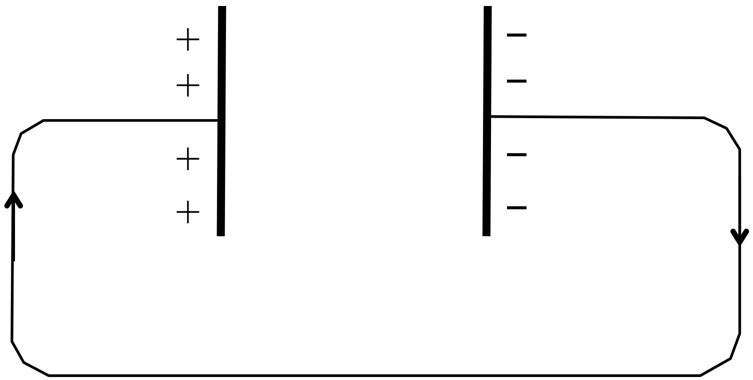
- *So*: Continuity equation means that if more current $\mathbf{j}$ leaves dV than enters it ($\nabla \cdot \mathbf{j} > 0$), then amount of charges within dV decreases. If more current enters dV than leaves it ($\nabla \cdot \mathbf{j} < 0$), then amount of charges inside dV increases.



- If $\nabla \cdot \mathbf{j} = 0$, then same amount of current enters dV as leaves it; which occurs if all currents are closed.



• *Example of electrostatics*: Charging a capacitor.

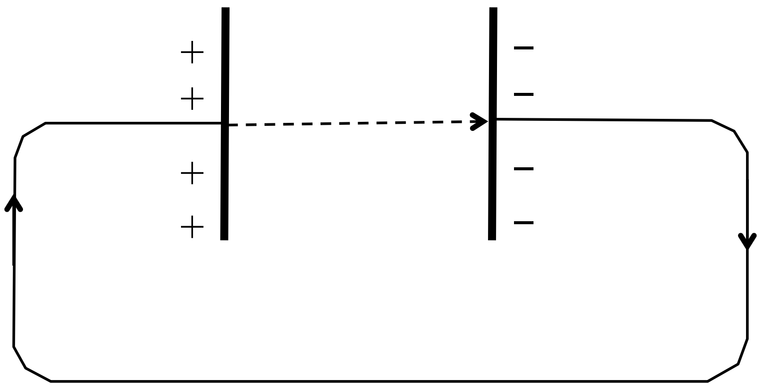


• Ampère's Law $\mathbf{j} = \nabla \times \mathbf{H}$ entails a closed circuit and no charge accumulation:

$$\nabla \cdot \mathbf{j} = 0, \quad \partial\rho/\partial t = 0.$$

• Can't apply to charging capacitor with open circuit and charge accumulation:

$$\nabla \cdot \mathbf{j} \neq 0, \quad \partial\rho/\partial t \neq 0.$$



• Need an additional term in Ampère's Law to "close the circuit":

$$\mathbf{j} = \nabla \times \mathbf{H} + X$$

• *Then*: $\nabla \cdot \mathbf{j} = \nabla \cdot (\nabla \times \mathbf{H}) + \nabla \cdot X$

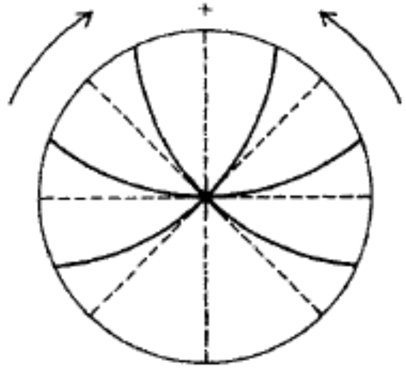
$$= \nabla \cdot X$$

• *So*: Need to find a form for X such that

$$\nabla \cdot X \neq 0.$$

Displacement Current: How to Apply Vortex Model to Electrostatics

- *Suppose*: Tangential action $\mathbf{T}$ of particles on vortices causes an elastic deformation in medium.



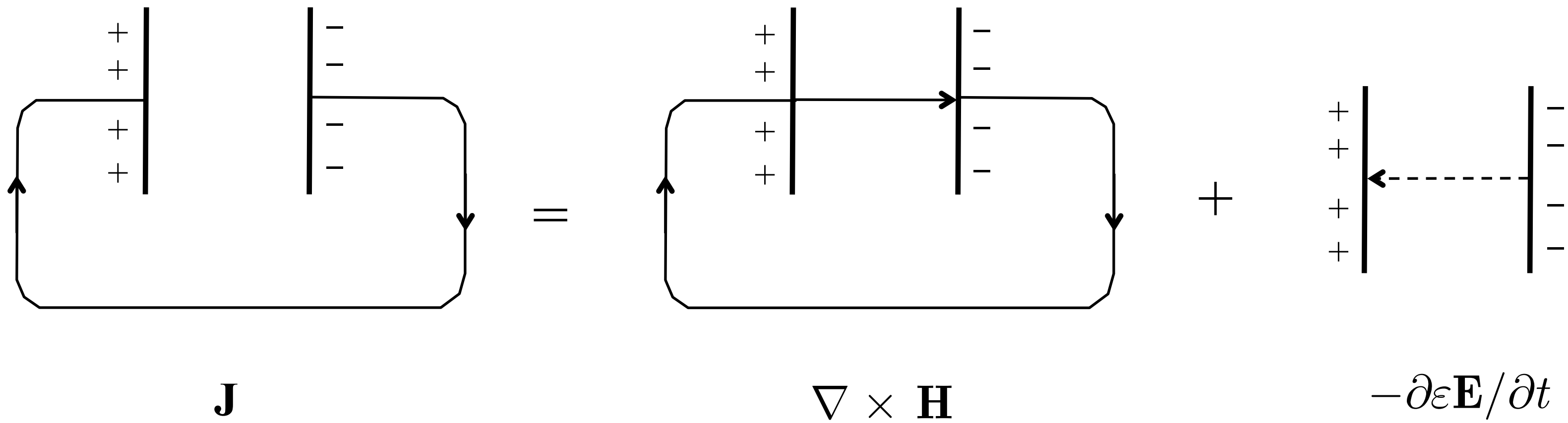
> "The undulatory theory of light requires us to admit this kind of elasticity in the luminiferous medium, in order to account for transverse vibrations. We need not be surprised if the magneto-electric medium possesses the same property."

- *Then*: Particles in contact with vortices will be *displaced* in a direction opposite to electromotive force $\mathbf{E} = -\mathbf{T}$.
- *Let*: Average displacement $\boldsymbol{\delta} = -\varepsilon\mathbf{E}$, $\quad \varepsilon =$ constant.
- *Then*: Relation between flux of particles and angular velocity of vortices is

$$\mathbf{J} = \nabla \times \mathbf{H} + \partial\boldsymbol{\delta}/\partial t$$

- *So*: $\nabla \cdot \mathbf{J} = \partial/\partial t(\nabla \cdot \boldsymbol{\delta})$.
- *And*: This entails Maxwell's continuity equation, provided $\nabla \cdot \boldsymbol{\delta} = -\rho$.

- The charging capacitor example again:



- The role of the displacement current term $(-\partial \varepsilon \mathbf{E}/\partial t)$ is to cancel out the part of the $\nabla \times \mathbf{H}$ term in the space between the plates.
- *Idea*:
  (a) Magentic field $\mathbf{H}$ gives rise to a complete (closed) current through the wire and the space between the plates.
  (b) Electric field $\mathbf{E}$ gives rise to a reverse current between the places.
  (c) The actual conduction current is the sum of (a) and (b).

- *Moreover*: Speed of transverse waves in the medium = $(k/m)^{1/2}$.
  - k = *transverse elasticity* = $1/(4\pi^2\varepsilon)$.
  - m = *density of medium* = $\mu/4\pi^2$.
- *So*: Speed of transverse waves in vacuum = $(1/\varepsilon_0\mu_0)^{1/2}$.
- *And*: This is equal to (rough) estimates of the speed of light in vacuum!

"We can scarcely avoid the inference that *light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena.*"

Recap #2:

- *Ampère's Law:* $\mathbf{J} = \nabla \times \mathbf{H} + \partial\boldsymbol{\delta}/\partial t$. The current flux of idle wheels causes spatial changes in transverse directions of rotational velocities of vortices, and displacements of idle wheels due to elastic deformations of medium.
- *Faraday's Law:* $-\partial\mu\mathbf{H}/\partial t = \nabla \times \mathbf{E}$. Changes in angular momentum of vortices cause spatial changes in transverse directions of the tangential forces that vortices impart on idle-wheels.

How seriously did Maxwell take the vortex model?

"The conception of a particle having its motion connected with that of a vortex by perfect rolling contact may appear somewhat awkward. I do not bring it forward as a mode of connexion existing in nature, or even as that which I would willingly assent to as an electrical hypothesis. It is however a mode of connexion which is mechanically conceivable, and easily investigated, and it serves to bring out the actual mechanical connexions between the known electro-magnetic phenomena; so that I venture to say that any one who understands the provisional and temporary character of this hypothesis, will find himself rather helped than hindered by it in his search after the true interpretation of the phenomena."

"The nature of this mechanism is to the true mechanism what an orrery is to the solar system."

D. "Dynamical Theory of the Electromagnetic Field" (1865).

- *Recall*: Faraday's Law $\mathbf{E} = -\partial\mathbf{A}/\partial t$, ($\mathbf{A}$ = *electrotonic intensity*).

- *Suggests*: $\mathbf{A}$ is the *momentum* associated with the flow of idle-wheel particles (in analogy with Newton's 2nd Law: $\mathbf{F} = d\mathbf{p}/dt$).

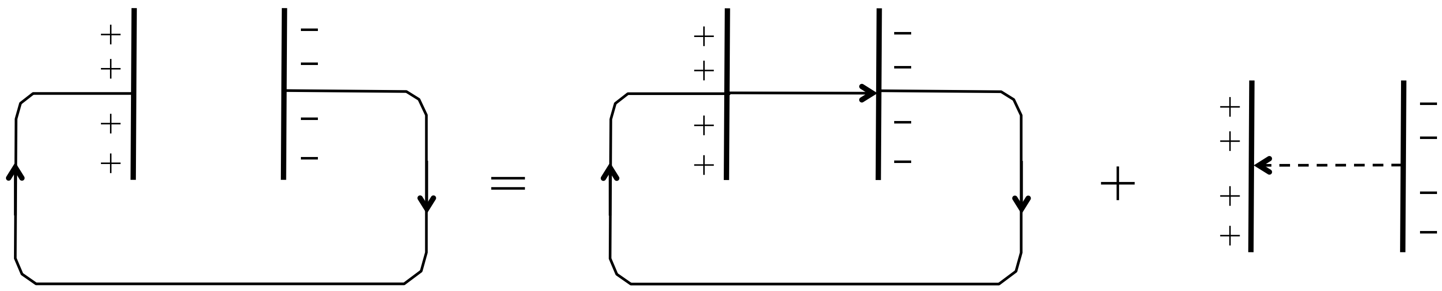
- *Now*: Generalize away from specific vortex model. Suppose "circuit momentum" is line integral of "electromagnetic momentum" $\mathbf{A}$.

- *Then*: Faraday's Law is $\int \mathbf{E}\cdot d\mathbf{l} = -\dfrac{d}{dt}\int \mathbf{A}\cdot d\mathbf{l}$.

- What about Ampère's Law?
  - *In vortex model*: Current = flow of idle-wheel particles.
  - *In new generalized theory*: Adopt Faraday's concept of current as a variation or transfer of *polarization*.

- *Let*: The polarization (electric displacement) $\mathbf{D}$ = the displacement of electricity in molecules of a dielectric.
- *Related to electromotive force by*: $\mathbf{E} = \mathbf{D}/\varepsilon$.
- *Total current then is*: $\mathbf{J} = \partial\mathbf{D}/\partial t + \mathbf{j}$
  - $\mathbf{j}$ = *rate at which electricity passes from one molecule to another*
- *And*: Ampère's Law becomes $\nabla \times \mathbf{H} = \mathbf{j} + \partial\varepsilon\mathbf{E}/\partial t$.
  - *Same as "On Physical Lines of Force", but*:
    "In the old theory, what Maxwell called the 'displacement current' was $-\partial\varepsilon\mathbf{E}/\partial t$ and contributed to the conduction current. In the new theory, the displacement current became a contribution to a divergenceless total current." (Darrigol, pg. 161.)

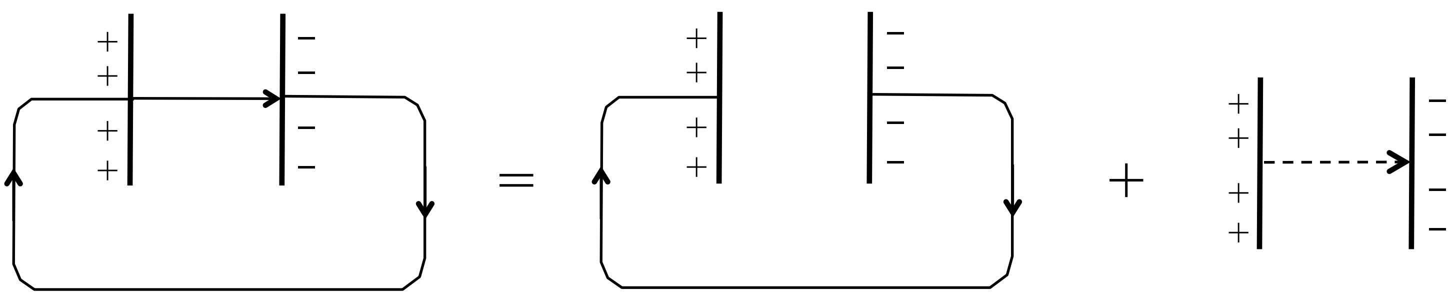
1861 Vortex model formulation of Ampère's Law: $\mathbf{J} = \nabla\times\mathbf{H} - \partial\varepsilon\mathbf{E}/\partial t$



$\mathbf{J}$ $\quad$ $\nabla\times\mathbf{H}$ $\quad$ $-\partial\varepsilon\mathbf{E}/\partial t$

- *Last term represents actual displacement of idle-wheels, in direction opposite of initial flow, due to elastic reaction of ether on vortices.*

New 1865 formulation of Ampère's Law: $\nabla\times\mathbf{H} = \mathbf{j} + \partial\varepsilon\mathbf{E}/\partial t$



$\nabla\times\mathbf{H}$ $\quad$ $\mathbf{j}$ $\quad$ $+\partial\varepsilon\mathbf{E}/\partial t$

- *Contemporary presentation*: A magnetic field $\mathbf{H}$ is induced by a current $\mathbf{j}$ *and/or* a changing electric field $\mathbf{E}$.

- *Let*: The polarization (electric displacement) $\mathbf{D}$ = the displacement of electricity in molecules of a dielectric.
- *Related to electromotive force by*: $\mathbf{E} = \mathbf{D}/\varepsilon$.
- *Total current then is*: $\mathbf{J} = \partial\mathbf{D}/\partial t + \mathbf{j}$
  - $\mathbf{j}$ = *rate at which electricity passes from one molecule to another*
- *And*: Ampère's Law becomes $\nabla \times \mathbf{H} = \mathbf{j} + \partial\varepsilon\mathbf{E}/\partial t$.
  - *Same as "On Physical Lines of Force", but*:
    "In the old theory, what Maxwell called the 'displacement current' was $-\partial\varepsilon\mathbf{E}/\partial t$ and contributed to the conduction current. In the new theory, the displacement current became a contribution to a divergenceless total current." (Darrigol, pg. 161.)
- *Now*: For conductor moving with velocity $\mathbf{v}$, Faraday's Law entails:

$$\mathbf{E} = -\partial\mathbf{A}/\partial t + \mathbf{v} \times (\nabla \times \mathbf{A}) - \nabla\psi, \qquad \psi = \text{"electric potential"}$$

- Combine with Ampere's Law to get:

$$\varepsilon\mu\frac{\partial^2\mathbf{B}}{\partial t^2} = \nabla^2\mathbf{B}$$

  - *Wave equation for "magnetic induction"* $\mathbf{B} = \mu\mathbf{H}$.
  - *Propagation speed* = $(1/\varepsilon\mu)^{1/2}$.

E. Treatise on Electricity and Magnetism (1873).

Key Concepts

- *Polarization* = a state of constraint of a dielectric such that each portion of it acquires equal and opposite properties on two opposite sides.
- *Electric charge* = spatial discontinuity of polarization; occurs at the limit between a polarized dielectric and a conductor.
- *Electric current* = rate of transfer of polarization (thus always closed).
- *Force* = *intensity*.
  - $\mathbf{H}$ = magnetic force.
  - $\mathbf{E}$ = electric force.
- *Flux* = *quantity*.
  - $\mathbf{B} = \mu\mathbf{H}$ = magnetic flux density (or "magnetic induction").
  - $\mathbf{D} = \varepsilon\mathbf{E}$ = electric flux density ("electric induction", "polarization", "displacement").

Misunderstandings

1. *Max says:* Polarization of a portion of a dielectric is "a displacement of electricity".
   - *Means*: A portion of a dielectric, if separate from the rest, presents opposite charges at two opposite extremes.
   - *Doesn't mean*: An electrically charged substance is displaced.

2. *Maxwell says:* "The motions of electricity are like those of an incompressible fluid."
   - *Means*: The closed nature of total current makes it analogous to the flow of an incompressible fluid.
   - *Doesn't mean*: Electricity is an incompressible fluid.

"General Equations for the Electromagnetic Field" (Treatise, Chap. IX, Vol. II)

$\mathbf{A}$ = *electromagnetic momentum*	$\mathbf{H}$ = *magnetic force*	ψ = *electric scalar potential*
$\mathbf{B}$ = *magnetic induction*	$\mathbf{M}$ = *intensity of magnetization*	Ψ = *magnetic scalar potential*
$\mathbf{J}$ = *total electric current*	$\mathbf{j}$ = *current of conduction*	ε = *dielectric inductive capacity*
$\mathbf{D}$ = *electric displacement*	ρ = *electric density*	μ = *magnetic inductive capacity*
$\mathbf{E}$ = *electromotive force*	m = *density of magnetic 'matter'*	$\mathbf{v}$ = *velocity*
$\mathbf{f}$ = *mechanical force*	σ = *conductivity for electric currents*	

1. $\mathbf{B} = \nabla \times \mathbf{A}$	Equations of magnetic induction (A).
2. $\nabla \cdot \mathbf{B} = 0$	Condition on $\mathbf{B}$, due to (1).
3. $\mathbf{E} = \partial \mathbf{A}/\partial t + \mathbf{v} \times (\nabla \times \mathbf{A}) - \nabla \psi$	Equations of electromotive force (B).
4. $\mathbf{f} = \mathbf{J} \times \mathbf{B} - \rho \nabla \psi - m \nabla \Psi$	Equations of mechanical force (C).
5. $\mathbf{B} = \mathbf{H} + \mathbf{M}$	Equations of magnetization (D).
6. $\nabla \times \mathbf{H} = \mathbf{J}$	Equations of electric currents (E).
7. $\mathbf{D} = \varepsilon \mathbf{E}$	Equation of electric displacement (F).
8. $\mathbf{j} = \sigma \mathbf{E}$	Equation for current of conduction (Ohm's Law) (G).
9. $\mathbf{J} = \mathbf{j} + \partial \mathbf{D}/\partial t$	Equation of the total current (H); which can also be written as $\mathbf{J} = (\sigma + \varepsilon \partial/\partial t)\mathbf{E}$ (I).
10. $\nabla \cdot \mathbf{D} = \rho$	Equation for the electric volume-density (J); and an equation for electric surface-density (K).
11. $\mathbf{B} = \mu \mathbf{H}$	Equation of induced magnetization (L).
12. $\nabla \cdot \mathbf{M} = m$	Equation for the magnetic volume-density.
13. $\mathbf{H} = -\nabla \Psi$	Equation for $\mathbf{H}$ when $\mathbf{J} = 0$.

"[These equations] may be combined so as to eliminate some of these quantities, but our objective at present is not to obtain compactness in the mathematical formulae, but to express every relation of which we have any knowledge. To eliminate a quantity which expresses a useful idea would be rather a loss than a gain in this stage of our enquiry."

Key Characteristics of Maxwell's Theory

- Essentially *macroscopic*: basic concepts of field, charge, and current have macroscopic meanings; treats matter and ether as single continuous medium with macroscopic properties.
- Recognizes that a more detailed *microscopic* picture of the connection between ether and matter is needed.

Maxwell's Innovations

- New geometrization of Faraday's and Thomson's field concepts.
- Distinction between quantity and intensity.
- Concept of displacement current.
- Unification of optics, electricity, and magnetism.