02. Malus and Polarization.

Buchwald (1989), Chaps 1, 2.

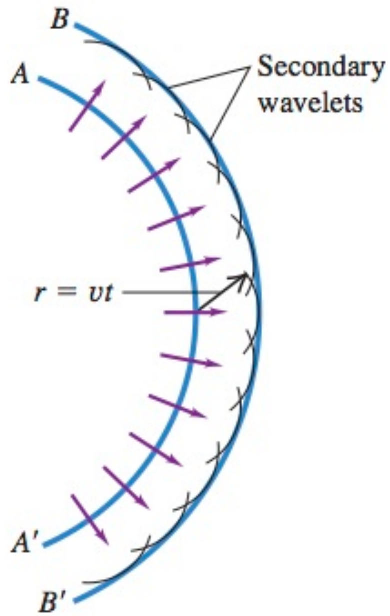
1. The Optical Ray.

- *Pre-17th Century*: The ray as the physical foundation of light.
    - *Euclid: "visual rays" proceed from the eye to object of sight.*
- 1690. Huygens's *Traite de la lumiere.*

Huygens's Principle:

"That each particle of matter in which a wave spreads, ought not to communicate its motion only to the next particle which is in the straight line drawn from the luminous point, but that it also imparts some of it necessarily to all the others which touch it and which oppose themselves to its movement. So it arises that around each particle there is made a wave of which that particle is the centre."

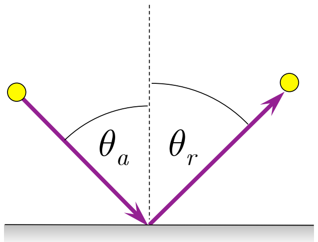
Christiaan Huygens
(1629-1695)

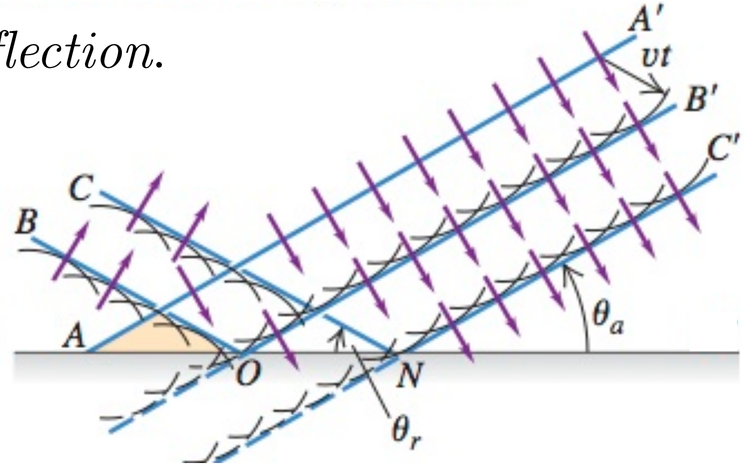


- *Or*: Every point of a wave front may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave.
    - "To accept Huygens's principle requires abandoning the idea that a ray has much intrinsic physical significance..." (pg. 5.)

Law of Reflection: $\theta_a = \theta_r$

- θ_a = *angle of incidence*, θ_r = *angle of reflection.*



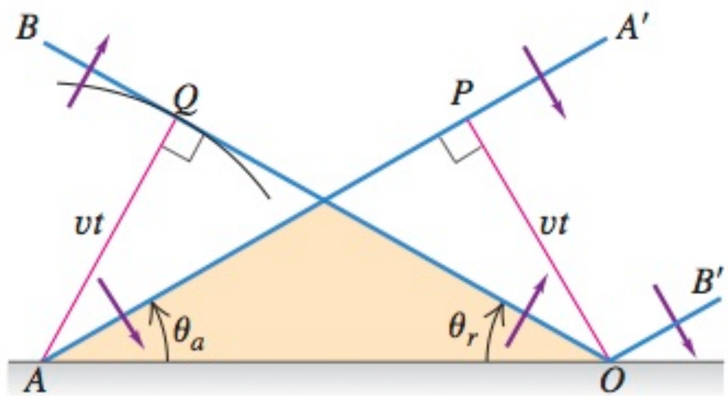


Emission Theory:

- Light particles scatter elastically.

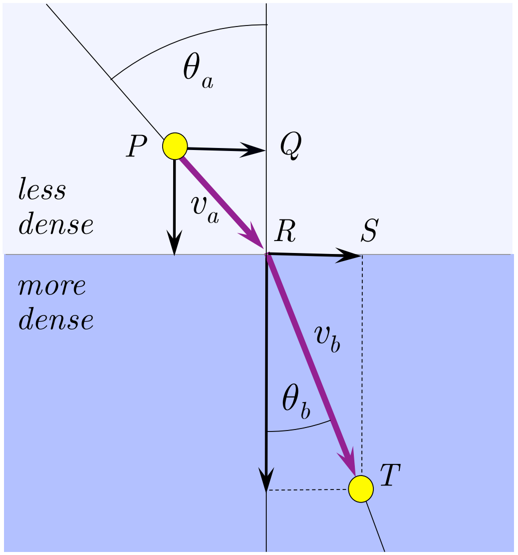
Wave Theory:

- AA', OB', NC' are successive positions of *incident* wave front.
- OB, NC are positions of *reflected* wave front.
- $\triangle APO$ and $\triangle OQA$ are congruent.

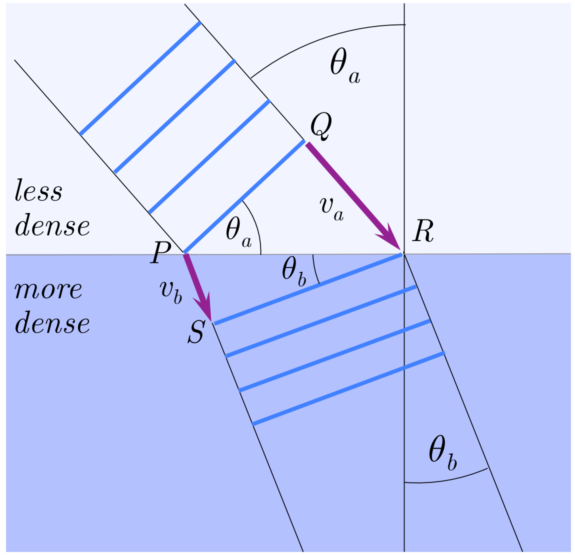


Law of Refraction ("Snell's Law"): $\sin\theta_a/\sin\theta_b = n$ (*const.*)

- θ_a = *angle of incidence*, θ_b = *angle of refraction.*



- n = "index of refraction"
- For *less dense to more dense*, $n > 1$ (or $\theta_b < \theta_a$).



Emission Theory:

- Light particles attracted to denser medium, thus speed up.
- No change in horizontal velocity: $PQ = RS$.

$$\begin{aligned}\sin\theta_a/\sin\theta_b &= n\\ &= (PQ/v_a)/(RS/v_b)\\ &= v_b/v_a > 1.\end{aligned}$$

- *Thus*: Velocity of light is *greater* in denser medium.

⇐ Possible test! ⇒

Wave Theory:

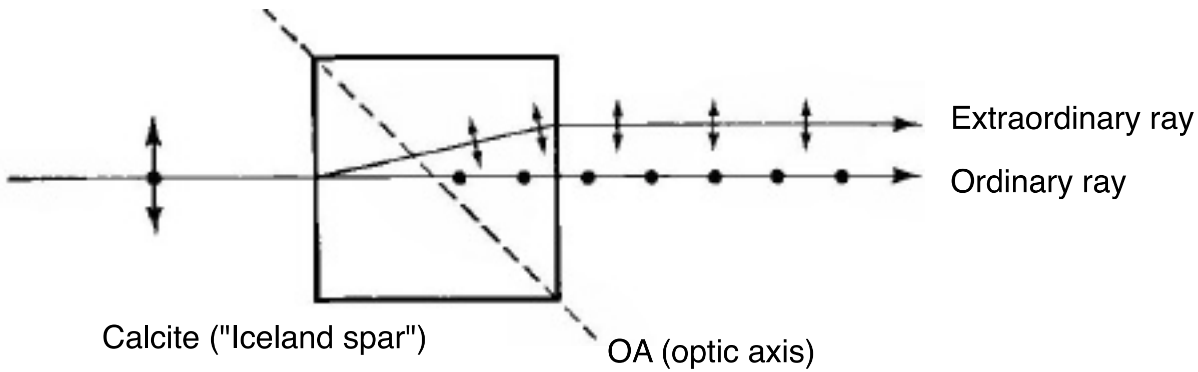
- Incident wave front PQ changes direction as it enters denser medium.
- For observed $\theta_b < \theta_a$, must have:

$$\begin{aligned}\sin\theta_a/\sin\theta_b &= n\\ &= (v_a/PR)/(v_b/PR)\\ &= v_a/v_b > 1.\end{aligned}$$

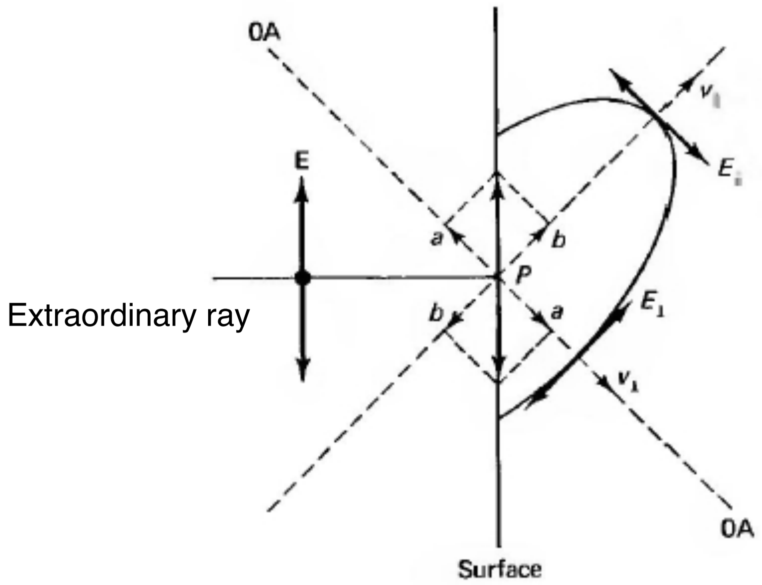
- *Thus*: Velocity of light is *less* in denser medium.

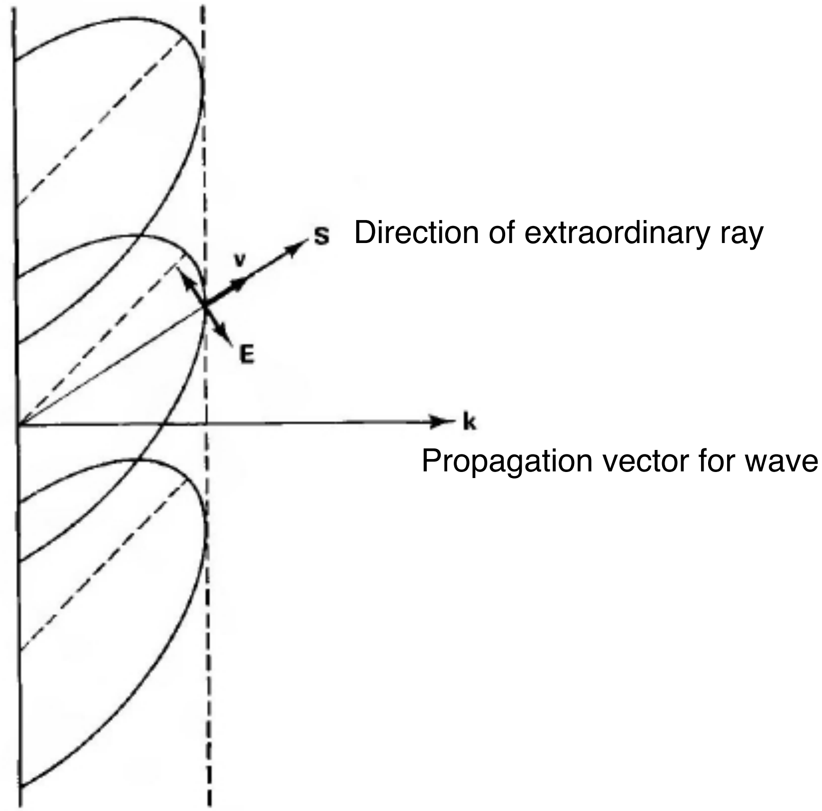
<u>*Significance of Huygens's Principle*</u>: Analysis of double refraction.

- *Double refraction*: Decomposition of a ray into two rays when it passes through a material that has a refractive index that depends on the polarization of the ray.



- *Ordinary ray* = ray that obeys Snell's Law.
- *Extraordiary ray* = ray that does not obey Snell's Law.
- <u>*Huygens*</u>: Path of extraordinary ray can be obtained by assuming secondary wavelets are non-spherical.





- Speeds $v_{||}$, $v_{\perp}$ of components parallel and perpendicular to OA are unequal.
- *So*: Wavelet with center P is ellipsoidal (not spherical).
- *And*: Resulting wave front is not perpendicular to refracted ray with velocity velocity $\mathbf{v} = \mathbf{v}_{||} + \mathbf{v}_{\perp}$.

- *Newton on a ray as a discrete thing*:

Issac Newton
(1643-1727)

> "By the Rays of Light I understand its least Parts, and those as well Successive in the same Lines, as Contemporary in several Lines. For it is manifest that Light consists of parts, both Successive and Contemporary; because in the same place you may stop that which comes one moment, and let pass that which comes presently after; and in the same time you may stop it in any one place, and let it pass in any other. For that part of Light which is stopp'd cannot be the same with that which is let pass. The least Light or part of Light, which may be stopp'd alone without the rest of the Light, or propagated alone, or do suffer any thing alone, which the rest of the Light doth not or suffers not, I call a Ray of Light." (*Optiks* 1704.)

- *1790s*: Newton's ray synonymous with emission theory among French.
- *18th Century*: Criticism of emission theory in form of Newtonian forces acting on particles, but retention of physical identity of ray maintained.
- *1727-1790s*. Big changes: quantitative expressions, precise experimental data.
    - *Foundation of Ecole Polytechnique forms group of men with common math background and concern with accurate experiment.*

- *1807*. Malus translates Huygens's explanation of double refraction into algebra.
  - *Advocate of emission theory (but wants to separate claim that rays exist from claim that rays are composed of particles).*

Etienne-Louis Malus
(1775-1812)

- *But*: Is Huygens's Principle compatible with emission theory?

- *Fermat's Principle of Least Time* = The path of a ray must be a minimum.

Huygens's Principle entails Fermat's Principle

- Secondary wavelets with minimal paths contribute the most at a given point due to constructive interference.

Pierre-Simon Laplace
(1749-1827)

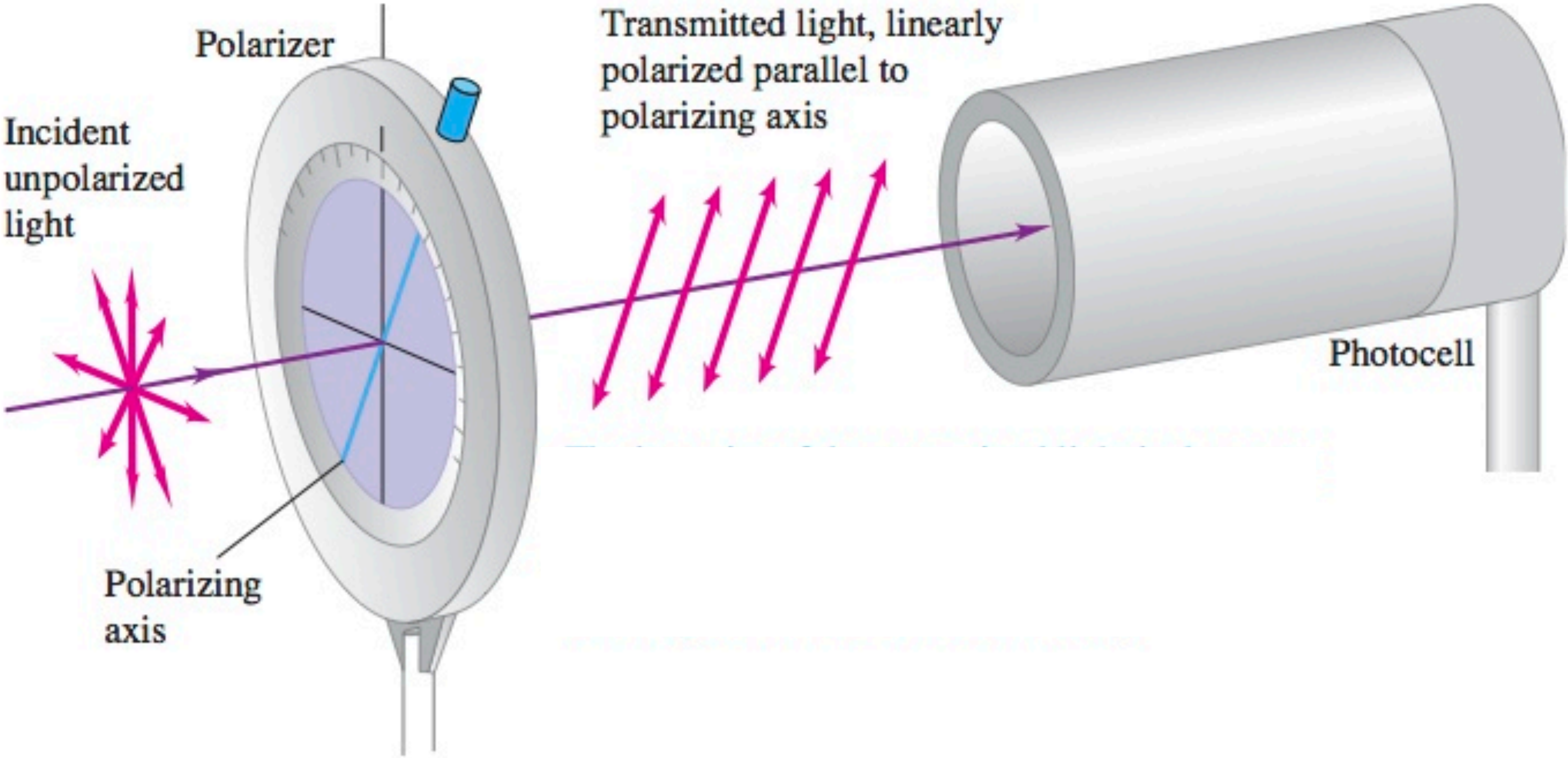
- *Laplace (emissionist)*: Anything that can be deduced from Huygens's Principle can be obtained from Fermat's Principle if the velocity of the rays as functions of direction are known.

- *But Still*: Emissionists must assume velocity *a priori*, whereas wave theorists need only assume the wave front.

2. Polarization.

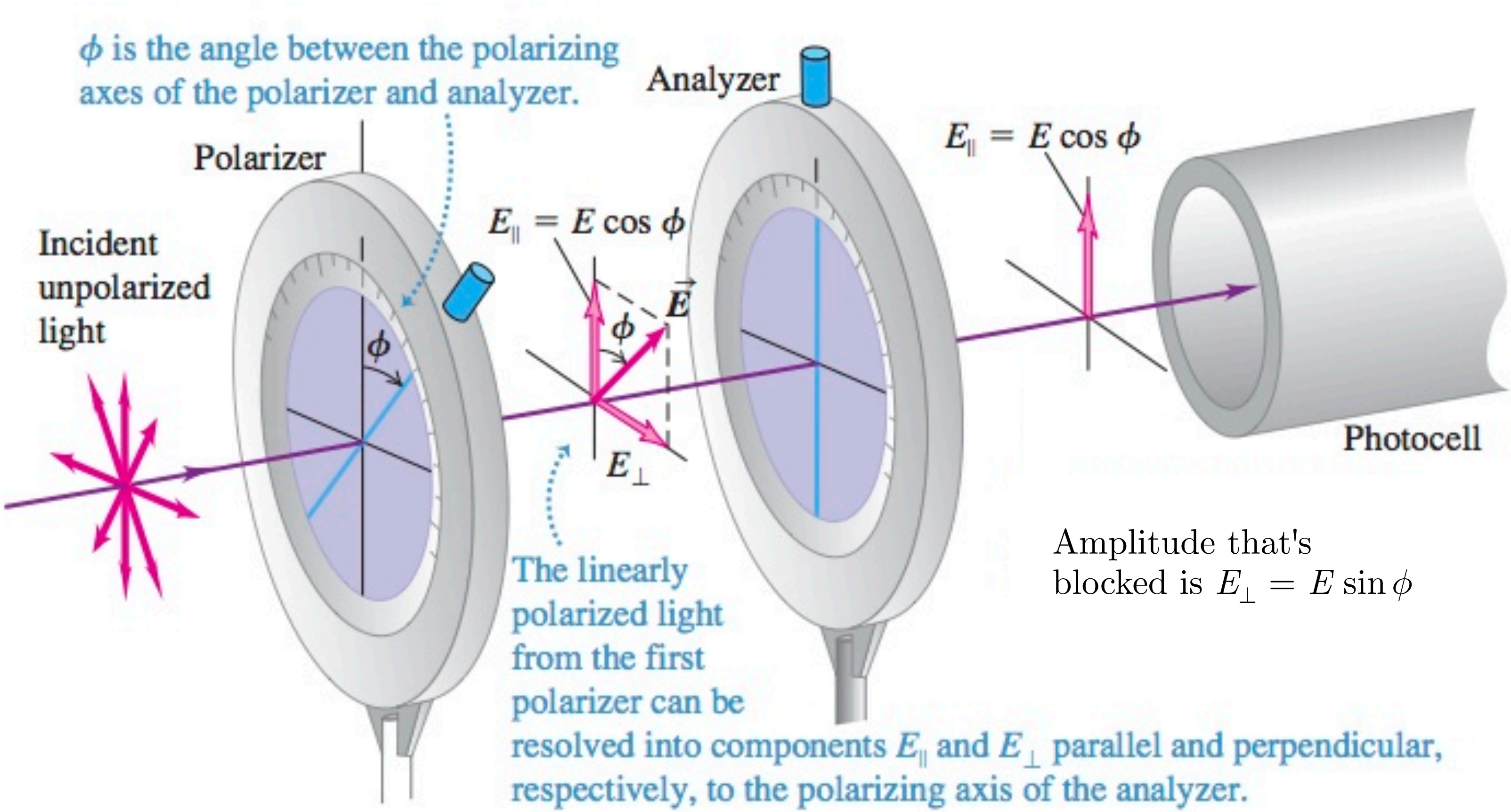
A. Contemporary Concepts.

- A wave is *linearly polarized in a given direction* just when it has displacements only in that direction.
- A *polarizing filter* permits waves only with a certain polarization direction.



- Intensity of transmitted light is half incident intensity.
  - *Incident light is random mixture of all states of polarization, so components perpendicular and parallel to axis of polarization will, on average, be equal.*

- Suppose transmitted light is passed through second polarizer (analyzer).



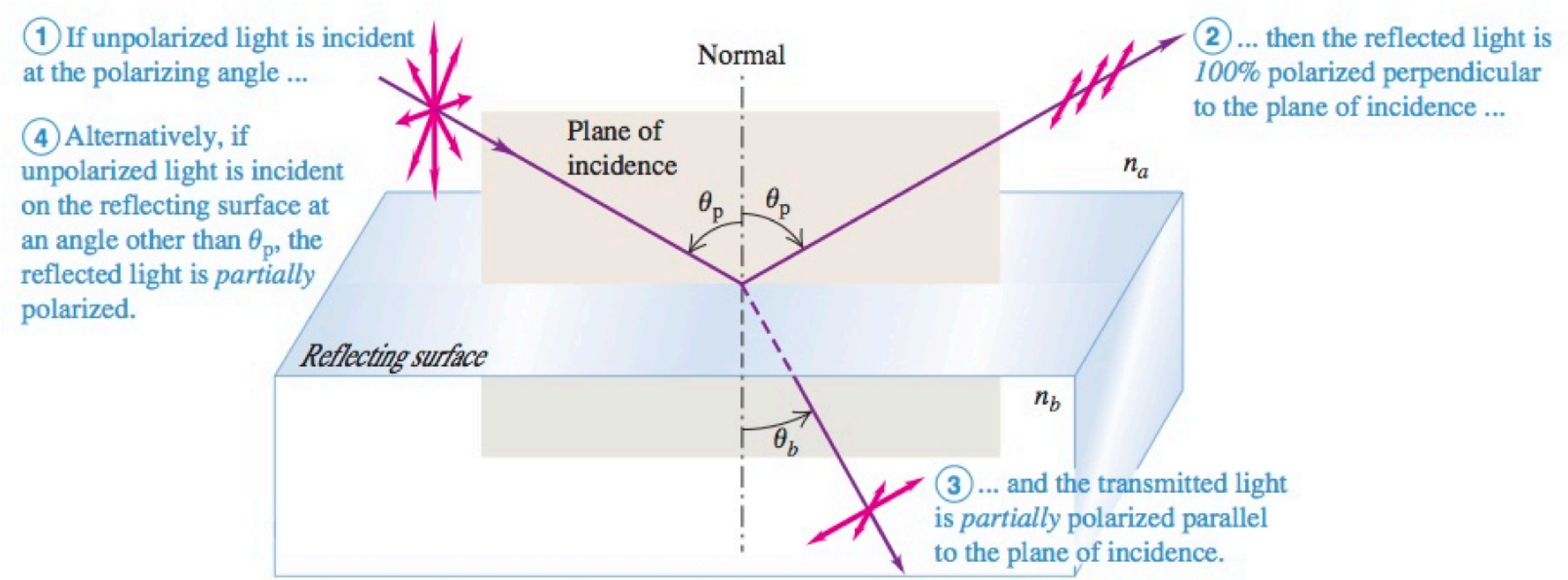
Malus's Law (for polarized incident light passing through analyzer)

The intensity I of light from analyzer is

$$I = I_{max}\cos^2\phi$$

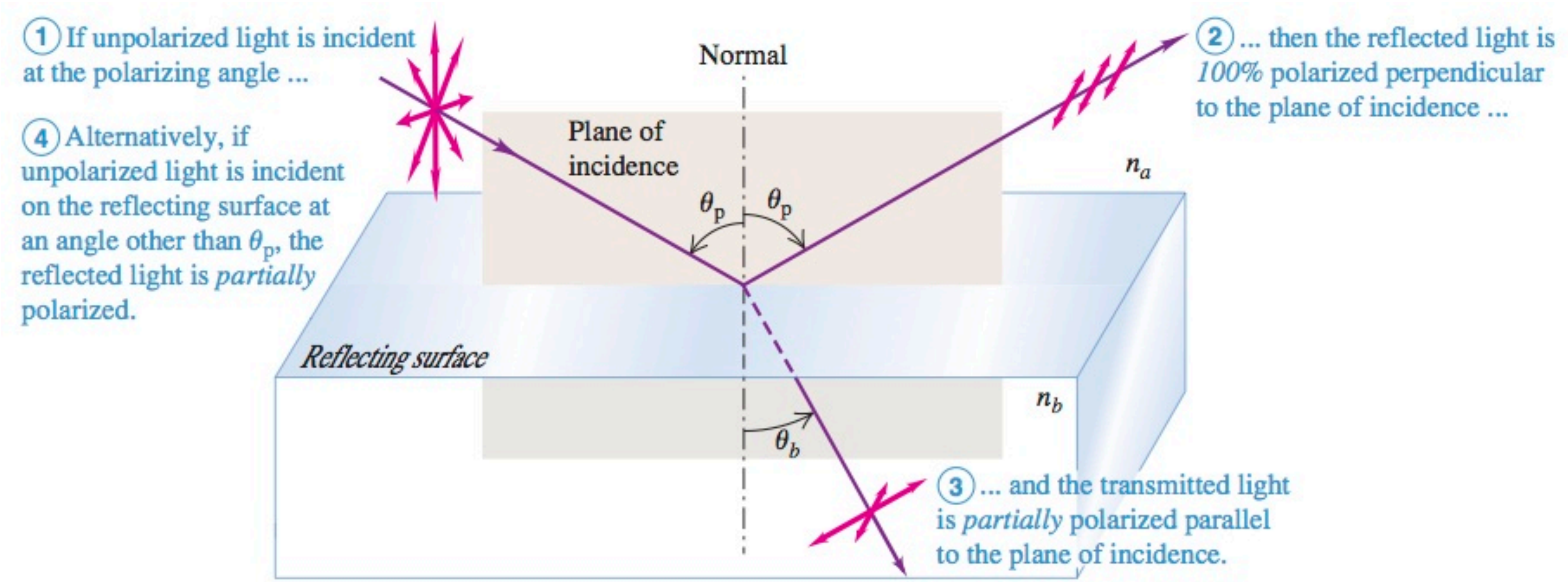
where I_{max} is the maximum intensity of light transmitted (at $\phi = 0$).

Polarization by reflection



- *Also*: When $\theta_a = \theta_p$, reflected and refracted rays are perpendicular.
- *So*: At the polarizing angle, light with components in (or parallel to) plane of incidence is not reflected, but is completely refracted. And light with components perpendicular to plane of incidence is partially reflected and partially refracted.
  - *Incident beam splits into three parts: one part is reflected, another part (with same polarization) is refracted; yet another part is refracted.*

Polarization by reflection

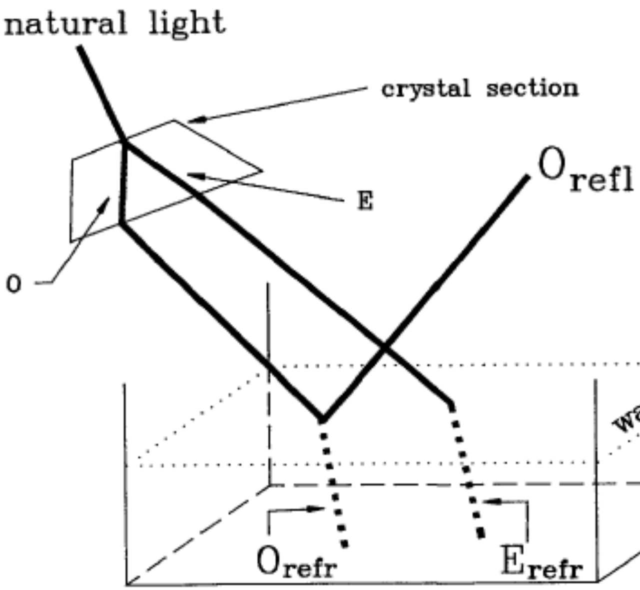


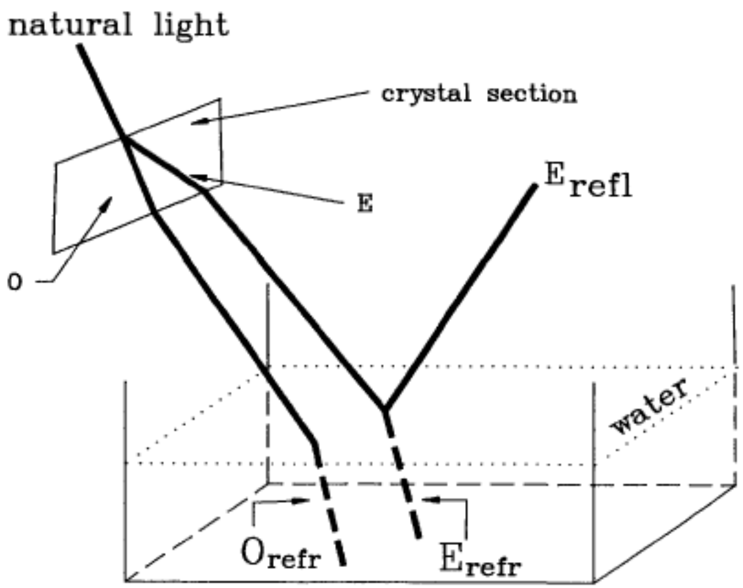
- *Now*: What happens when incident light is not unpolarized, but rather is linearly polarized in plane of incidence?
  - *No reflection! No component of incident light is perpendicular to the plane of incidence.*
  - *Thus: Light is completely refracted.*
- *Entails*: If there's no reflection, then the incident light must be polarized in the plane of incidence!

B. Malus's Theory of Partial Reflection.

- *Malus's 1808 observation*: When a reflected beam passes through a doubly refracting crystal, instead of two images, only one is observed, due to the ordinary or the extraordinary beam, depending on the position of the crystal.
- *And*: When O and E beams are incident on water at the "modifying angle"...

> "[When the crystal's principal section is parallel to the plane of reflection], the O ray, on being refracted, abandoned part of its molecules to partial reflection as would a bundle of direct light [*i.e.*, unmodified light], but the E ray entirely penetrated the liquid; none of its molecules escaped refraction. Conversely, when the principal section of the crystal was perpendicular to the plane of incidence, only the E ray produced a partial reflection, and the O ray was entirely refracted."





- "Beams of light... could be made to escape partial reflection -- to be entirely refracted -- by modifying them and then reflecting them in a plane 90° to the one in which they were modified." (Buchwald, pg. 44.)

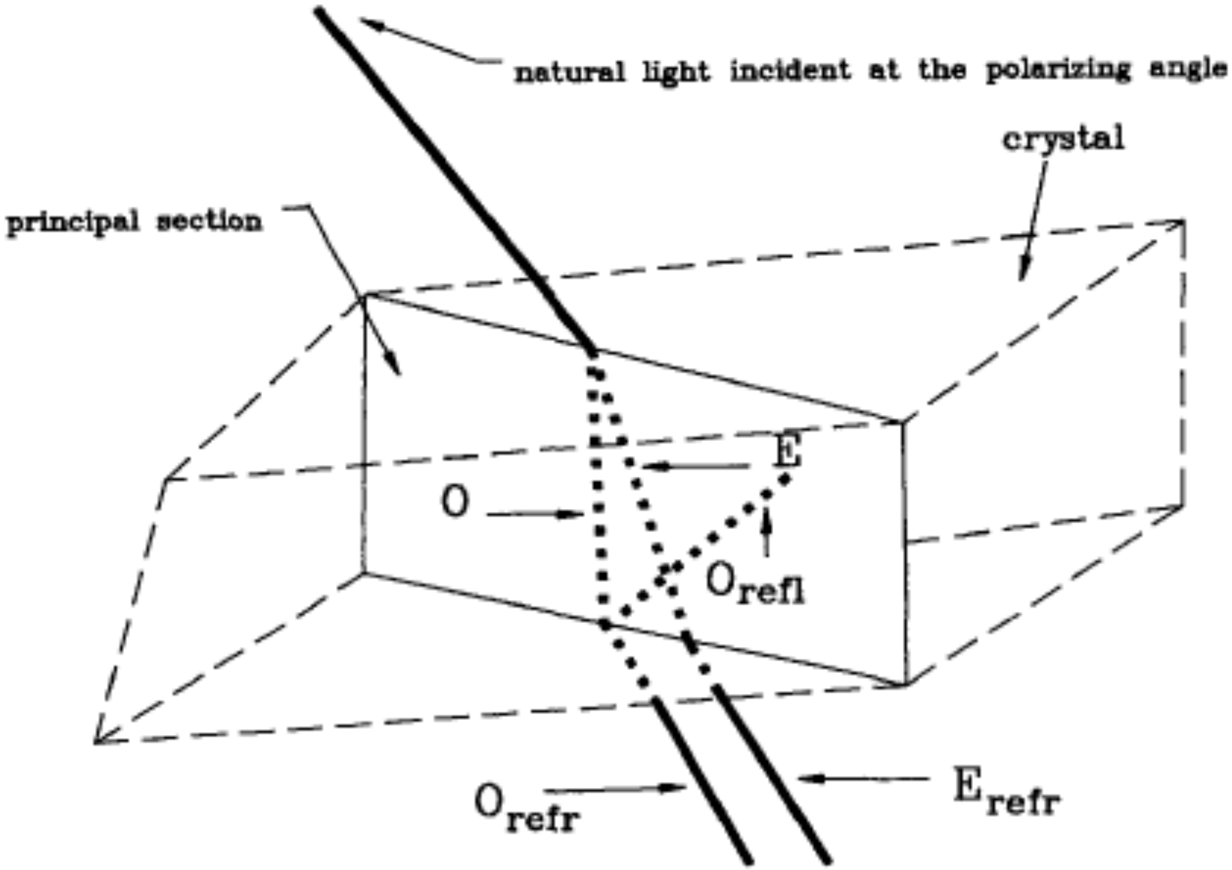


FIG. 2.3 Partial reflection within a crystal.

- Malus's (1811) "Theorie de la double refraction".
  - *Explains how a "modified" or "polarized" beam divides when it enters a doubly refracting crystal.*

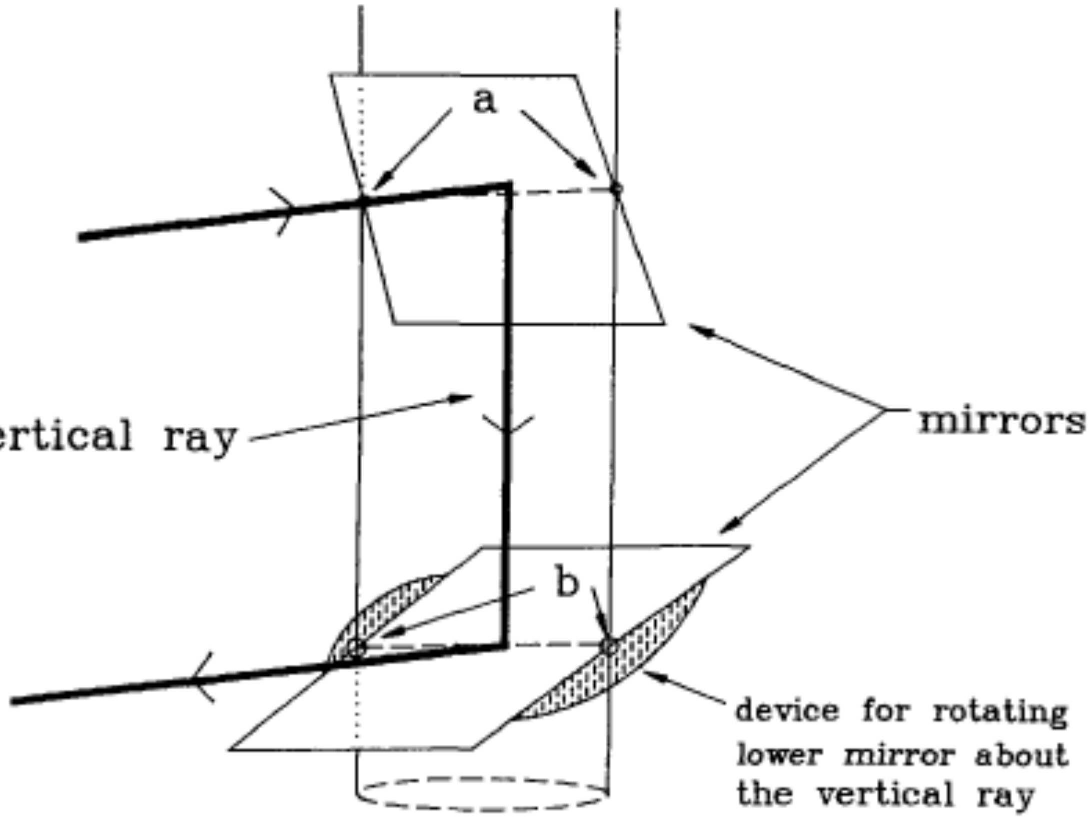
> *Malus's Law*: Number of rays in the ordinary (*resp*. extra-ordinary) beam is proportional to the square of the cosine (*resp*. sine) of the angle between the plane of polarization of the incident beam and the crystal's principle section.

- *Relation to contemporary account*: The angle Malus refers to above is the angle between the polarizing axis of the (assumed) polarizer that initially produced the incident beam, and the polarizing axis of the analyzer through which the beam passes (in this case, a doubly refracting crystal).

Malus's polarimeter:

- Investigates behavior of polarized beams when they are again reflected at the "polarizing" angle.
- Can vary angle between planes of reflection while holding angle of incidence fixed.

> *Plane of reflection of mirror = plane perpendicular to mirror's surface and containing incident and reflected rays.*



- *Malus observes*: Reflected light from lower mirror at polarizing incidence obeys cosine squared law; so refracted light in lower mirror should obey sine squared law (conservation of rays).

- Suggests analogy between double refraction and partial reflection.
  - *In partial reflection, reflected beam plays role of O beam in double refraction, while refracted beam plays role of E beam.*

- But: In partial reflection, refracted beam doesn't vanish, even at polarizing incidence:

Empirically, intensities for partial reflection should obey:

$$I_{refl} \propto \cos^2\varepsilon$$

$$I_{refr} \propto \sin^2\varepsilon + I_0$$

$$I = I_{refl} + I_{refr}$$

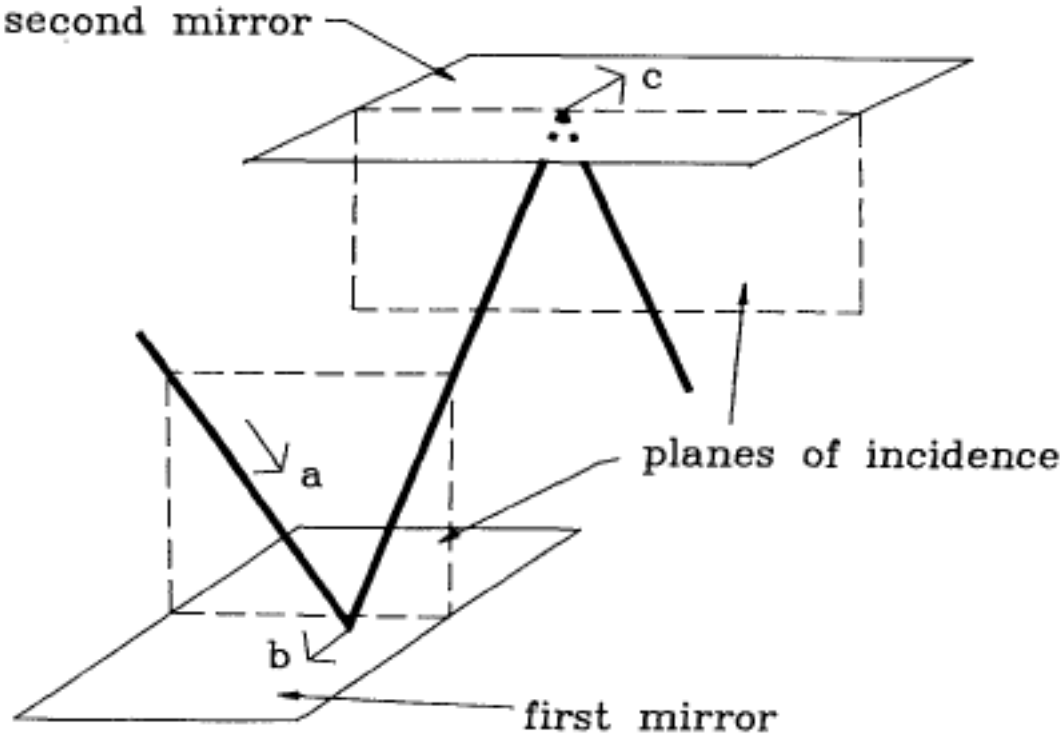
ε = angle between plane of reflection and plane of polarization
I = intensity of incident beam
I_0 = minimum intensity for refracted beam

- *Analogy to double refraction breaks down*:
  For double refraction, $I_{refr} \propto \sin^2\varepsilon$.

- So: "... if we wish to understand fully the meaning of the cosine law for the reflection of polarized light, we must inquire into the *composition* of the refracted beam, the one that seems to violate the analogy to double refraction." (Buchwald, pg. 48.)

Define 3 axes for polarimeter:

- *a* lies in direction of bundle incident on first mirror.
- *b* is perpendicular to plane of incidence on first mirror.
- *c* is perpendicular to plane of incidence on second mirror.



- *Now*: Suppose these axes are attached to the rays themselves.

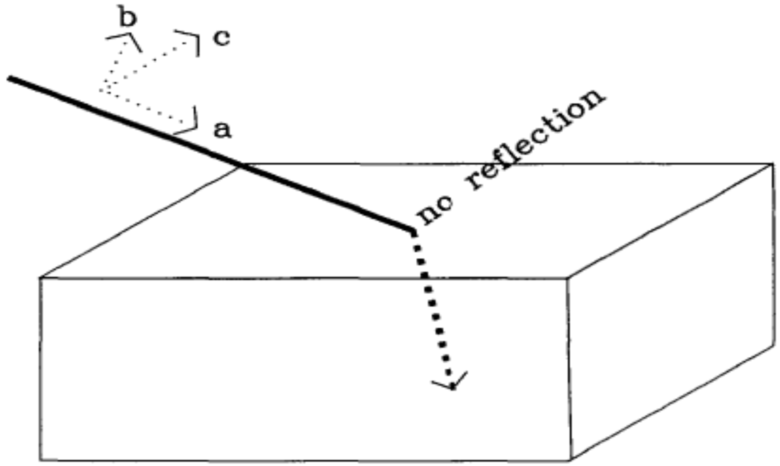
Two Assumptions:

- When a given set of rays is polarized by reflection, a proportional set becomes polarized by the refraction that occurs at the same moment, but in the opposite sense.
- A ray whose asymmetry (defined by *c* axis) is normal to the plane of incidence cannot have its *c* axis turned into that plane by reflection.

Malus's Account of Partial Reflection:

1. At first mirror, for natural (unpolarized) light:
   - *Portion of rays has c axes rotated into plane of incidence and reflects.*
   - *Portion of rays has b axes rotated into plane of incidence and refracts.*
   - *Remaining portion (with unspecified asymmetry) refracts.*

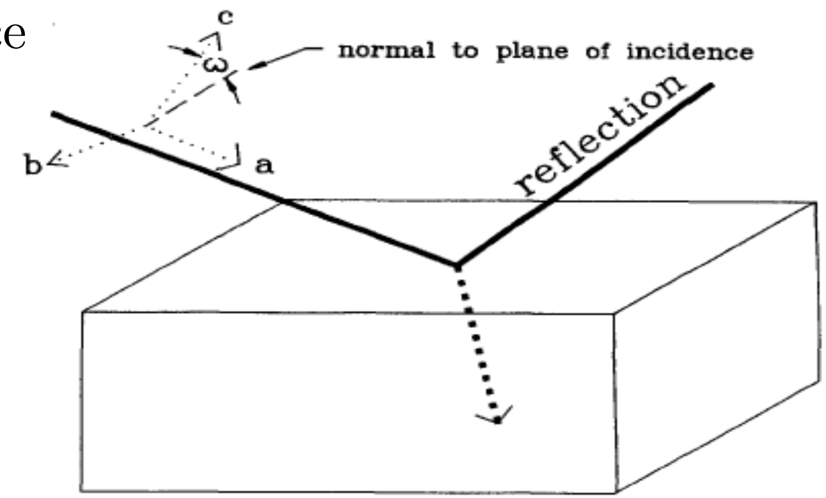
2. At second mirror, at polarizing incidence, two cases:



(a) Mirror is parallel to *c* axes of polarized rays. So *b* axes lie in plane of incidence: All rays refract.

(b) Mirror is not parallel to *c* axes of rays. So *b* axes do not lie in plane of incidence

- *Portion of rays reflect (so b axes are normal to plane of incidence).*
- *Portion of refracting rays have their b axes in plane of incidence.*
- *Portion of refracting rays have unspecified asymmetry.*



Malus's law for ray counts for 2nd mirror:

- # of reflected rays $= k_x N\sin^2\varepsilon$.
- # of refracted rays with b axes in plane of incidence $= k_x N\cos^2\varepsilon$.
- # of refracted rays with unspecified symmetry $= N(1 - k_x N\sin^2\varepsilon - k_x N\cos^2\varepsilon)$.
- *Where*: $N =$ # of incident rays; k_x, $k_y =$ constants; $\varepsilon =$ angle between c axis and normal to plane of incidence.

Task: To determine the values of the factors under the most general conditions.

> "These observations lead us to conclude that light acquires in these circumstances properties that are independent of its direction with respect to the reflecting surface and that are the same for the south and north sides [of the ray], and different for the east and west sides. In calling these sides poles, I will call polarization the modification that gives light properties relative to these poles."