Emergence and Mechanism in the Fractional Quantum Hall Effect

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1. Two Versions of Emergence
2. The Quantum Hall Effect
3. The Underdetermination of Mechanism in the FQHE
4. Law-Centric Emergence in the FQHE
5. Conclusion
1. Two Versions of Emergence

Emergence

Crowther (2015)

(i) Dependence. Emergent system is "ontologically determined" by the fundamental system.

(ii) Independence. Emergent system is novel with respect to fundamental system.

Task: Resolve tension between Dependence and Independence.
1. Two Versions of Emergence

**Mechanism-Centric Emergence**

"...emergent properties are not a panacea, to be appealed to whenever we are puzzled by the properties of large systems. In each case, we must produce a **detailed physical mechanism** for emergence, which rigorously explains the qualitative difference that we see with the microphysical."  (Mainwood 2006)

"...understanding emergent phenomena in terms of **symmetry breaking**—a structural dynamical feature of physical systems...—clarifies both how and why emergent phenomena are independent of any specific configuration of their microphysical base."  (Morrison 2012)

"...the electrons in FQHE states cannot just be correlated over small distances; they **must have a mechanism** of achieving long-range correlations. This mechanism is Chen *et al.*'s conception of long-range entanglement."  (Lancaster & Pexton 2015)
1. Two Versions of Emergence

Mechanism-Centric Emergence

Claim: To avoid triviality, novelty must be underwritten by a mechanism.

- **Microphysical mechanism**: a particular collection of entities and activities that are organized in such a way that they realize a regularity or principle (Weber et al. 2013).

- **High-level mechanism**: a general physical process that can be instantiated by any number of microphysical processes.
  - "a structural/dynamical feature of physical systems" (Morrison 2012).
  - "a higher organizing principle" (Laughlin & Pines 2000).
1. Two Versions of Emergence

Law-Centric Emergence

Claim: Novelty is underwritten by distinct dynamical laws.

- Exemplified by effective field theories (EFTs):

\[
\frac{\partial \mathcal{L}}{\partial \phi} - \frac{\partial}{\partial x} \frac{\partial \mathcal{L}}{\partial (\partial \phi)} = 0 \quad (1) \quad \text{high-energy (fundamental) system}
\]

\[
\frac{\partial \mathcal{L}_{\text{eff}}}{\partial \theta} - \frac{\partial}{\partial x} \frac{\partial \mathcal{L}_{\text{eff}}}{\partial (\partial \theta)} = 0 \quad (2) \quad \text{low-energy (emergent) system}
\]

- \( \mathcal{L}[\phi] \neq \mathcal{L}_{\text{eff}}[\theta] \)
- Low-energy behavior (2) is dynamically independent of, and dynamically robust with respect to, high-energy behavior (1).
1. Two Versions of Emergence

Story to come:
Mechanism-centric views of emergence fail to adequately account for emergence in the FQHE.
2. The Quantum Hall Effect

Classical Hall Effect:

\[ R_H = \frac{V_H}{I} = \frac{B}{eN} \]

Hall resistance

\[ N = \# \text{ electrons per area} \]

Longitudinal resistance

\[ R = \frac{V}{I} \]
2. The Quantum Hall Effect

- Low temperature (~0.02 K)
- Large magnetic field (~30 T)

Quantum Hall Effect:

Hall resistance
"Plateaus" at
\[ R_H = \frac{h}{e^2 n} \]

\( n = \text{integer or fraction} \)

Longitudinal resistance
\( R = 0 \) at plateaus in \( R_H \).
2. The Quantum Hall Effect

Integer Quantum Hall Effect (IQHE)

2-dim one-body Hamiltonian for electron in mag field:
- Discrete "Landau levels", with degeneracy $D = eB/h$.
- "Filling factor" $\nu \equiv N/D = Nh/eB = \# \text{ filled levels}$.

When $\nu = n = \text{integer}$:
1. System becomes incompressible.
   $\Rightarrow R = 0$

2. $B = B_n = Nh/en$
   $\Rightarrow R_H = B/eN$
   $= \frac{h}{e^2} \frac{1}{n}$

![Diagram of energy levels with $B < B_1$, $B = B_1$, and $B > B_1$](image)
2. The Quantum Hall Effect

**Integer Quantum Hall Effect (IQHE)**

1. Why is the system incompressible at integer values of ν?  
   (Or: Why does \( R \) vanish at plateaus in \( R_H \)?)

   **Ans.** Gapped structure of energy spectrum; Pauli Exclusion Principle (PEP) for electrons.

2. Why does incompressibility persist for small changes in \( B \)?  
   (Or: Why does the plot of \( R_H \) exhibit plateaus?)

   **Ans.** Impurities trap conduction electrons ("localization").

- IQHE is a *one-body* effect: single electron coupled to mag field.
- FQHE is a *one-body* or *many-body* effect: depending on the type of particle invoked.
  - *Four different mechanistic accounts...*
3. The Underdetermination of Mechanism in the FQHE

(i) **Laughlin Ground State Account** (Laughlin 1983)

- Particles = electrons.
- *Many-body* electron–electron interactions prevent electrons in partially filled highest Landau level from moving to unoccupied states.

$$\Psi_m(r_1, \ldots, r_N) = \prod_{1 \leq j < k \leq N} (z_j - z_k)^{1/\nu} e^{\sum |z_i|^2/4}$$

1. Why is the system incompressible at fractional values of $\nu$? (Or: Why does $R$ vanish at plateaus in $R_H$?)

**Ans.** Electron–electron interactions; PEP for electrons.

2. Why does incompressibility persist for small changes in $B$? (Or: Why does the plot of $R_H$ exhibit plateaus?)

**Ans.** Quasiparticle–impurity interactions ("localization").
3. The Underdetermination of Mechanism in the FQHE

(ii) Composite Fermion Account (Jain 1989)

- Particles = electrons with even attached Chern–Simons fluxes.
- Fluxes reduce $B$ field to IQHE values.
- FQHE as 1-body IQHE of composite fermions.

1. Why is the system incompressible at fractional values of $\nu$? (Or: Why does $R$ vanish at plateaus in $R_H$?)

   **Ans.** Electron–CS field interaction; gapped structure of energy spectrum; PEP for composite fermions.

2. Why does incompressibility persist for small changes in $B$? (Or: Why does the plot of $R_H$ exhibit plateaus?)

   **Ans.** Composite fermion–impurity interactions ("localization").
3. The Underdetermination of Mechanism in the FQHE

(iii) **Composite Boson Account** (Girvin & MacDonald 1987)

- Particles = electrons with *odd* attached Chern–Simons fluxes.
- Fluxes *cancel* $B$ field.
- FQHE as *1-body* effect of condensing composite bosons.

1. Why is the system incompressible at fractional values of $\nu$?
   (Or: Why does $R$ vanish at plateaus in $R_H$?)

   **Ans.** Electron–CS field interaction; BEC formation via spontaneous symmetry breaking; Meissner effect.

2. Why does incompressibility persist for small changes in $B$?
   (Or: Why does the plot of $R_H$ exhibit plateaus?)

   **Ans.** Vortex–impurity interactions ("localization").
1. Why is the system incompressible at fractional values of $\nu$? (Or: Why does $R$ vanish at plateaus in $R_H$?)

Ans. Long-range correlations.

2. Why does incompressibility persist for small changes in $B$? (Or: Why does the plot of $R_H$ exhibit plateaus?)

Ans. Quasiparticle–impurity interactions ("localization").
### 3. The Underdetermination of Mechanism in the FQHE

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>$R = 0$</th>
<th>$\textbf{Plateaus in } R_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Laughlin ground state</strong></td>
<td>Many-body Coulomb effect of strongly interacting electrons.</td>
<td><strong>Localization:</strong> quasiparticle–impurity interactions.</td>
</tr>
<tr>
<td><strong>Composite fermion</strong></td>
<td>One-body IQHE effect of non-interacting composite fermions.</td>
<td><strong>Localization:</strong> composite fermion–impurity interactions.</td>
</tr>
<tr>
<td><strong>Composite boson</strong></td>
<td>One-body effect of weakly-interacting composite bosons undergoing SSB.</td>
<td><strong>Localization:</strong> vortex–impurity interactions.</td>
</tr>
<tr>
<td><strong>Topological order</strong></td>
<td>Many-body long-range entangled effect of electrons.</td>
<td><strong>Localization:</strong> quasiparticle–impurity interactions.</td>
</tr>
</tbody>
</table>

- Four distinct microphysical mechanist accounts.
- Three distinct high-level mechanist accounts.
- **Concern:** Shouldn't there be just a single ontological mechanistic account of the emergence (and persistence) of incompressibility in the FQHE?
Claim: The novelty that characterizes incompressibility in an FQH system is underwritten by the distinct dynamical laws that govern it, compared to those that govern the fundamental 2-dim conductor.

\[ \mathcal{L}[\psi, A_\mu] = \psi^\dagger i(\partial_t - iA_t)\psi + \frac{1}{2m} \psi^\dagger (\partial_i - iA_i)^2 \psi + V(\psi^\dagger, \psi) \]  
\[ \mathcal{L}_{\text{eff}}[a_\mu, A_\mu, j^\mu] = -\frac{p}{4\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \frac{e}{2\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda + j^\mu a_\mu \]

- Low-energy behavior (4) of FQH system is dynamically independent of, and dynamically robust with respect to, high-energy behavior (3) of 2-dim conductor.
5. Conclusion

- There is an underdetermination of mechanistic accounts of the FQHE, at both the microphysical level and the level of higher organizing principles.

- This underdetermination is pernicious for mechanism-centric views of emergence.

- A law-centric view of emergence avoids underdetermination by avoiding reference to mechanisms.

- Under a law-centric view, the novelty exhibited by an FQH system, with respect to the fundamental electron system from which it emerges, is explained by appealing to the distinct dynamical laws that govern both systems.
References.


