CPT Invariance, the Spin-Statistics Connection, and the Ontology of RQFTs

Jonathan Bain

Polytechnic Institute of NYU
Brooklyn, New York

1. Two Ontological Theses about RQFTs.
2. The CPT and Spin-Statistics Theorems.
3. Options for the Received View.
4. Conclusion.

1. Two Ontological Theses about the Ontology of RQFTs.

CPT invariance:
Invariance under charge conjugation (C), space inversion (P), and time reversal (T).

Spin-statistics connection:
i. States that obey FD statistics possess half-integer spin.
ii. States that obey EB statistics possess integer spin.

- Properties of (collections of) *states* of an RQFT.
- *Essential* or *accidental* properties?
- States of *what*?

1. Two Ontological Theses about the Ontology of RQFTs.

- "This conclusion is part of a more general result, first derived by Pauli: ...particles of integer spin obey Bose-Einstein statistics, while particles of half-odd-integer spin obey Fermi-Dirac statistics" (Peskin & Schroeder 1995, pp. 57-58).
- "At the same time that we discuss P and T, it will be convenient to discuss a third (non-spacetime) discrete operation: *charge conjugation*, denoted by C. Under this operation, particles and antiparticles are interchanged" (Peskin & Schroeder 1995, pg. 64).
- Weinberg (1995, pp. 191, 238), Sterman (1993, pg. 523), Jost (1964, pp. 100, 106)

1. Two Ontological Theses about the Ontology of RQFTs.

Thesis I
CPT invariance and the spin-statistics connection are *essential properties* of *fundamental* states in RQFTs.

Thesis II
CPT invariance and the spin-statistics connection are properties of *particle* states.

- *Received View*: (Halvorson & Clifton 2002; Fraser 2008)
  RQFTs cannot be fundamentally about particles.

- Received View must either:
  Deny (I), or deny (II), or deny (I) and (II).

2. The CPT and Spin-Statistics Theorems.

Two ways to encode statistics in a QFT

(1) *On creation/annihilation operators that act on multi-particle states in a Fock space.*

$$[a(\mathbf{p}),\ a^\dagger(\mathbf{q})]_\mp = \delta(\mathbf{p}-\mathbf{q}).$$

(2) *On field operators.* ("Local Commutativity")

$$[\phi^\dagger(x),\ \phi(y)]_\mp = 0, \text{ for spacelike } (x-y).$$

2. The CPT and Spin-Statistics Theorems.

I. Textbook Lagrangian Approach (Fierz 1939; Pauli 1940)

Restricted Lorentz Invariance (RLI):
Invariance under $L^{\uparrow}{}_{+}$ (Lorentz boosts, no parity and time reversal transformations)

Spectrum Condition (SC):
The energy of all states is positive.

Causality:
The observable quantities associated with an RQFT commute at spacelike distances.

- "Observable quantities" = bosonic fields and bilinears in fermionic fields.

2. The CPT and Spin-Statistics Theorems.

I. Textbook Lagrangian Approach (Fierz 1939; Pauli 1940)

- *Claim*: Imposing wrong statistics on a, $a^\dagger$ for:
  - an integer spin field violates *Causality*.
  - a half-integer spin field violates either *Causality* or SC.

C1.	(RLI & *Causality* & SC) $\Rightarrow$ (spin-statistics connection for fermions)
C2.	(RLI & *Causality*) $\Rightarrow$ (spin-statistics connection for bosons)
C3.	[(spin-stats connection) & RLI & (local Hermitian Lagrangian)] $\Rightarrow$ (CPT invariance of Hamiltonian)

2. The CPT and Spin-Statistics Theorems.

I. Textbook Lagrangian Approach (Fierz 1939; Pauli 1940)

States of what?

- Bearers of spin-statistics connection are particle states:
  - Statistics is encoded in (anti-)commutation relations of a, $a^\dagger$ that act on *particle states* in a Fock space.

- Fundamental bearers of CPT invariance are particle states:
  - CPT invariance of Hamiltonian ultimately derived from invariance of a, $a^\dagger$ under C, P, and T.

2. The CPT and Spin-Statistics Theorems.

II. The Wightman Axiomatic Approach

- *Wightman function* $F_n(x_1, ..., x_n) = \langle \boldsymbol{\Omega}|\phi(x_1)...\phi(x_n)|\boldsymbol{\Omega}\rangle$.
- Restricted Lorentz Invariance (RLI).
- Spectrum Condition (SC).
- Local Commutativity (LC).

Weak Local Commutativity (WLC):

$$\langle \boldsymbol{\Omega}|\phi(x_1)...\phi(x_n)|\boldsymbol{\Omega}\rangle = i^K \langle \boldsymbol{\Omega}|\phi(x_n)...\phi(x_1)|\boldsymbol{\Omega}\rangle,$$

K = number of anti-commuting fields.

2. The CPT and Spin-Statistics Theorems.

II. The Wightman Axiomatic Approach

A1. (RLI & SC & LC) $\Rightarrow$ (spin-statistics connection for non-trivial fields)

A2. (RLI & SC & WLC) $\Rightarrow$ (CPT invariance of fields)

- Lüders & Zumino (1958); Burgoyne (1958).
- Jost (1957).

2. The CPT and Spin-Statistics Theorems.

II. The Wightman Axiomatic Approach

States of... fields?

- Distinguish two theorems: (Greenberg 1998)
  - *Spin-Locality Theorem (LC)*: Fields that (anti-)commute at spacelike distances must have (half-)integer spin.
  - *Spin-Statistics Theorem (Causality)*: Particles that obey (FD)EB stats must have (half-)integer spin.

- *Claim*: A field can violate Spin-Statistics while satisfying Spin-Locality.

2. The CPT and Spin-Statistics Theorems.

II. The Wightman Axiomatic Approach

- Consider a free relativistic neutral scalar field $\phi(x)$:

$$\phi(x) = \frac{1}{(2\pi)^3}\int \frac{d^3\mathbf{p}}{2E_{\mathbf{p}}}\left(a(\mathbf{p})e^{-i\mathbf{p}\cdot\mathbf{x}} + a^\dagger(\mathbf{p})e^{i\mathbf{p}\cdot\mathbf{x}}\right)$$

- Encode Fermi-Dirac statistics on a, $a^\dagger$:

$$[a(\mathbf{p}),\ a^\dagger(\mathbf{q})]_+ = 2E_{\mathbf{p}}\delta(\mathbf{p}-\mathbf{q})$$

$$[a(\mathbf{p}),\ a(\mathbf{q})]_+ = [a^\dagger(\mathbf{p}),\ a^\dagger(\mathbf{q})]_+ = 0$$

- Then the field violates *Local Commutativity*:

$$[\phi(x),\ \phi(y)]_+ = \Delta^{(1)}(x-y)$$

- *Thus*: A theory of free neutral spin-0 fermions with nonlocal observables.

2. The CPT and Spin-Statistics Theorems.

II. The Wightman Axiomatic Approach

- Distinguish three properties:
  (a) spin-statistics connection (property of particles)
  (b) spin-locality connection (property of fields).
  (c) CPT invariance (property of either particles or fields).
- Which of (a) or (b) is fundamental?

2. The CPT and Spin-Statistics Theorems.

III. The Algebraic Approach (Guido & Longo 1995)

- Net of von Neumann algebras $\mathcal{O} \mapsto \mathfrak{R}(\mathcal{O})$.
- Vacuum representation $(\mathcal{H}_0, \pi_0)$, for separable $\mathcal{H}_0$ with cyclic and separating vacuum vector $\mathbf{\Omega}$.

Microcausality:

For $A_1 \in \mathfrak{R}(\mathcal{O}_1)$, $A_2 \in \mathfrak{R}(\mathcal{O}_2)$, *and* $\mathcal{O}_1$, $\mathcal{O}_2$ *spacelike separated*, $[A_1, A_2] = 0$.

Weak Additivity:

$$\mathfrak{R} = \bigcup_x \mathfrak{R}(\mathcal{O} + x)$$

2. The CPT and Spin-Statistics Theorems.

III. The Algebraic Approach (Guido & Longo 1995)

Modular Covariance (MC):

$$\Delta^{it}{}_W \mathfrak{R}(\mathcal{O}) \Delta^{-it}{}_W = \mathfrak{R}(\Lambda_W(t)\mathcal{O})$$

- *For any wedge W in Minkowski spacetime, the modular operator $\Delta^{it}{}_W$ of $\mathfrak{R}(W)$ implements Lorentz boosts on $\mathfrak{R}$.*

Tomita-Takasaki Theorem:

- Let $\mathfrak{R}$ be a von Neumann algebra with cyclic and separating Ω.
- *Then*: $\mathfrak{R}$ possesses a *modular operator* Δ and a *modular conjugate operator* J such that $J\Omega = \Omega = \Delta\Omega$, $\Delta^{it}\mathfrak{R}\Delta^{-it} = \mathfrak{R}$, and $J\mathfrak{R}J = \mathfrak{R}'$.

2. The CPT and Spin-Statistics Theorems.

III. The Algebraic Approach (Guido & Longo 1995)

DHR representation: (Doplicher, Haag & Roberts 1971, 1974)

A representation $(\mathcal{H}, \pi)$ such that, for any region $\mathcal{O}$, $\pi|_{\mathfrak{R}(\mathcal{O}')}$ is unitarily equivalent to $\pi_0|_{\mathfrak{R}(\mathcal{O}')}$.

- DHR state = localized state = state that differs from the vacuum only in $\mathcal{O}$.
- Characteristics of DHR representations:
  - possess conjugates.
  - admit representations of the permutation group.
  - "finite statistics" = admits finite rep. of perm. group.

2. The CPT and Spin-Statistics Theorems.

III. The Algebraic Approach (Guido & Longo 1995)

D1. [(*Microcausality*) & (*Weak Additivity*) & MC] $\Rightarrow$ (CPT invariance of *DHR representations*)

D2. [(*Microcausality*) & (*Weak Additivity*) & MC] $\Rightarrow$ (spin-statistics connection for irreducible Poincaré-invariant *DHR representations* with finite statistics and masses)

2. The CPT and Spin-Statistics Theorems.

III. The Algebraic Approach (Guido & Longo 1995)

States of what?

- Bearers of spin-stats connection and CPT invariance are (a subset of) DHR states...
- ...which represent localized particle states.
- "We did not treat in [DHR 1971] any of the particle aspects of the theory. This will be the essential objective of the present paper" (DHR 1974).
- Naiveté?

3. Options for the Received View.

Received View:

RQFTs cannot be fundamentally about particles.

Pre-theoretic intuitions:

- Particles must be localizable.
- Particles must be countable.

Choice of representational schemes:

- Localizability $\Rightarrow$ local number operators
- Countability $\Rightarrow$ unique total number operator

Technical results:

Formulations of interacting RQFTs do not admit local and unique total number operators.

3. Options for the Received View.

Three Options for Received View:

(A) Deny Thesis (I): CPT invariance and the spin-statistics connection are not essential properties of fundamental states.

(B) Deny Thesis (II): CPT invariance and the spin-statistics connection are not properties of particle states.

(C) Both (A) and (B).

3. Options for the Received View.

Option (A): CPT invariance and spin-stats connection are properties of particles; but particles are not fundamental.

- Wallace (2009):
  - C transforms particles into antiparticles.
  - "...particles are emergent phenomena, which emerge in domains where the underlying quantum field can be treated as approximately linear" (pg. 219).

3. Options for the Received View.

Option (A): CPT invariance and spin-stats connection are properties of particles; but particles are not fundamental.

- Wallace (2006, 2011):
  - Defends "cut-off quantum field theory" (CQFT).
  - CPT and Spin-Stats Theorems fail for CQFTs.
  - "QFTs as a whole are to be regarded only as approximate descriptions of some as-yet-unknown deeper theory, which gives a mathematically self-contained description of the short-distance physics" (2006, pg. 45).

3. Options for the Received View.

Option (A): CPT invariance and spin-stats connection are properties of particles; but particles are not fundamental.

Suggests:

> *Thesis I′*
> CPT invariance and the spin-statistics connection are *essential properties* of *fundamental* states in idealized, linear, non-interacting RQFTs.

Problem:
Evidence for CPT invariance and the spin-stats connection comes from interacting QFTs.

3. Options for the Received View.

Option (B): CPT inv. and spin-stats connection are not properties of particles; but are properties of fundamental states.

- Baker & Halvorson (2010):
  - DHR states and their conjugates represent matter and antimatter states.
  - DHR states are more general than particle states.

Suggests:

(i) DHR states are the (fundamental) physically possible states in RQFTs.

(ii) DHR states possess CPT inv. and spin-stats connection.

(iii) DHR states are more general than particle states.

3. Options for the Received View.

Option (B): CPT inv. and spin-stats connection are not properties of particles; but are properties of fundamental states.

Problems:

(a) Should massive, Poincaré-invariant DHR states with finite statistics be identified as the (fundamental) physically possible states in RQFTs?

- *Nonlocal electromagnetic states do not satisfy DHR criterion.*
- *What about massless gauge bosons?*

3. Options for the Received View.

Option (B): CPT inv. and spin-stats connection are not properties of particles; but are properties of fundamental states.

Problems:

(b) Is *Modular Covariance* physically reasonable?

- ($\mathfrak{R}$ *generated by Wightman fields*) ⇒ (MC). (Bisognano & Wichmann 1976)
- [(*Microcausality*) & (*Additivity*) & (*Conformal Invariance*)] ⇒ (MC). (Brunetti *et al.* 1993)
- [(*Microcausality*) & (*Weak Additivity*) & (MC)] ⇒ (*Poincaré Invariance*). (Guido & Longo 1995)
- MC explains Unruh effect (?). (Guido & Longo 1995)

3. Options for the Received View.

Option (B): CPT inv. and spin-stats connection are not properties of particles; but are properties of fundamental states.

Axiomatic B'er: CPT inv. and spin-stats connection are properties of fundamental fields.

Problem: (Baker 2009)

Standard field interpretations are not consistent with the Received View's denial of a particle interpretation.

- *Hilbert space of wavefunctional states is unitarily equivalent to the Fock space of particle states required by the Recieved View's representations of particles.*

3. Options for the Received View.

Option (C): CPT inv. and spin-stats connection are not properties of particles; nor are they properties of fundamental states in RQFTs.

- Perhaps CPT invariance and the spin-statistics connection are properties not unique to RQFTs.
  - (a) Non-relativistic derivations of spin-stats connection.
  - (b) Classical derivations of CPT invariance.
- *Task of Option C′er*: Sort through literature on (a) and (b) to determine extent to which it justifies denial of Theses I and II.

4. Conclusion.

Suggestions:

- What we take to be the ontology of RQFTs should depend, in part, on what we take the essential properties of RQFTs to be.
- What we take the essential properties of RQFTs to be should be determined, in part, on results internal to these theories (*viz.*, the CPT and Spin-Statistics Theorems).
- Pre-theoretic intutions may be misleading.