Prospects for modeling spacetime as a phenomenon that emerges in the low-energy limit of a quantum liquid.

1. EFTs in Condensed Matter Systems
2. Superfluid $^4\text{He}$ and Superfluid $^3\text{He}$-$\text{A}$
3. Low-Energy Emergence
4. Universality, Dynamical Structure and Structural Realism
1. EFTs in Condensed Matter Systems

**Highly-correlated condensed matter system** = **Non-relativistic many-body quantum system that displays macroscopic quantum effects**

- superfluids
- superconductors
- Bose-Einstein condensates
- quantum Hall liquids

**Effective Field Theory of condensed matter system** = **Theory of low-energy dynamics of system: describes states with energy close to zero**

- bosonic collective modes of ground state
- fermionic excitations above ground state ("quasiparticles")
- topological defects ("vortices")
1. EFTs in Condensed Matter Systems

**How to Construct a Condensed Matter EFT**
Take Low-energy "limit":

- Expand Lagrangian in small fluctuations in field variables about ground state and integrate out high-energy fluctuations. \( (^4\text{He}) \)

**OR**

- Linearize the energy about the values where it vanishes and then construct the corresponding Hamiltonian. \( (^3\text{He-A}) \)

For fermionic liquids, the type of EFT that results is ultimately based on *topological* properties of the ground state of system, as opposed to its *symmetries.*
2a. "Acoustic" Spacetimes and Superfluid $^4\text{He}$

- liquid consisting of many $^4\text{He}$ atoms, all phases aligned
- model ground state as single quantum particle: $\varphi_0 = \rho_0 e^{i\theta}$

\[
\mathcal{L}_{^4\text{He}} = i\varphi^\dagger \partial_t \varphi - \frac{1}{2m} \partial_i \varphi^\dagger \partial_i \varphi + \mu \varphi^\dagger \varphi - \alpha^2 (\varphi^\dagger \varphi)^2
\]

**Non-relativistic Lagrangian for Superfluid $^4\text{He}$**

Low-energy limit:

- Let $\varphi = \rho e^{i\theta}$, $\rho = \rho_0 + \delta \rho$, $\theta = \theta_0 + \delta \theta$
- Integrate out high-energy fluctuations $\delta \rho$

\[
\mathcal{L}_{^4\text{He}} = \mathcal{L}_0[\rho_0, \theta_0] + \mathcal{L}'_{^4\text{He}}[\delta \theta]
\]

- ground state
- low-energy fluctuations above ground state (EFT)
2a. "Acoustic" Spacetimes and Superfluid $^4$He

\[ \mathcal{L}'_{^4\text{He}} = \frac{1}{4\alpha^2} (\partial_t \theta + v_i \partial_i \theta)^2 - \frac{\rho_0}{2m} (\partial_i \theta)^2 \quad i = 1, 2, 3 \]

... identical to...

\[ \mathcal{L}'_{^4\text{He}} = \frac{1}{2} \sqrt{g} \ g^{\mu \nu} \partial_\mu \theta \partial_\nu \theta \quad \mu, \nu = 0, 1, 2, 4 \]

EFT for Superfluid $^4$He
To 2nd order in $\delta \theta$:
\[ v_i = (1/m) \partial_i \theta \]

Massless scalar field in curved spacetime!

\[ g_{\mu \nu} dx^\mu dx^\nu = \frac{\rho}{cm} \left\{ -c^2 dt^2 + \delta_{ij} (dx^i - v^i dt)(dx^j - v^j dt) \right\} \]
\[ c^2 = 2\alpha^2 \rho/m \]

Can now model black hole physics:

speed of light = speed of low-energy oscillations (i.e., "sound" modes)

\textbf{Hence:} "acoustic" spacetimes and "acoustic" black holes
2a. "Acoustic" Spacetimes and Superfluid $^4$He

(i) **What is the background structure of acoustic spacetimes?**

**Option #1: Minkowski spacetime**

\[ g_{\mu\nu} dx^\mu dx^\nu = (\rho/\text{cm})\{-c^2dt^2 + \delta_{ij}(dx^i - v^i dt)(dx^j - v^j dt)\} \]

\[ = \eta_{\mu\nu} dx^\mu dx^\nu + g'_{\mu\nu} dx^\mu dx^\nu \]

**Option #2: Neo-Newtonian spacetime**

- Superfluid $^4$He in Neo-Newtonian ST
- Massless scalar field in Minkowski ST

- 1st order $\delta\theta$
- 2nd order $\delta\theta$

- Massless scalar field in acoustic ST
(ii) *To what extent are "acoustic" spacetimes analogues of GR spacetimes?*

\[
\begin{align*}
\text{Einstein equations cannot} & \quad \Rightarrow \quad \text{Not dynamical analogues!} \\
\text{be derived from} & \quad \text{Not dynamical analogues!} \\
^4\text{He EFT.} & \quad \text{Not dynamical analogues!}
\end{align*}
\]

*"Kinematic" analogues of GR?*

... the features of general relativity that one typically captures in an “analogue model” are the *kinematic* features that have to do with how fields (classical or quantum) are defined on curved spacetime, and the *sine qua non* of any analogue model is the existence of some “effective metric” that captures the notion of the curved spacetimes that arise in general relativity. (Barceló, Liberati, Visser 2005, pg. 10.)
(ii) To what extent are "acoustic" spacetimes analogues of GR spacetimes?

\[
\begin{align*}
\text{Einstein equations cannot be derived from } ^4\text{He EFT.} & \quad \Rightarrow \quad \text{Not dynamical analogues!}
\end{align*}
\]

"Kinematic" analogues of GR?

The acoustic analogue for black-hole physics accurately reflects half of general relativity -- the kinematics due to the fact that general relativity takes place in a Lorentzian spacetime. The aspect of general relativity that does not carry over to the acoustic model is the dynamics -- the Einstein equations. Thus the acoustic model provides a very concrete and specific model for separating the kinematic aspects of general relativity from the dynamic aspects. (Visser 1998, pg. 1790.)
2a. "Acoustic" Spacetimes and Superfluid $^4$He

(ii) To what extent are "acoustic" spacetimes analogues of GR spacetimes?

\[
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\Rightarrow & \quad \text{Not dynamical analogues!}
\end{align*}
\]

"Kinematic" analogues of GR?

- If kinematics of GR = Minkowski ST, then \textit{No}!
- Kinematics of GR = ?

Some features that one normally thinks of as intrinsically aspects of gravity, both at the classical and semiclassical levels (such as horizons and Hawking radiation), can in the context of acoustic manifolds be instead seen to be rather generic features of curved spacetimes and quantum field theory in curved spacetimes, that have nothing to do with gravity \textit{per se}. (Barceló, Liberati, Sonego, Visser 2004, pg. 2.)
2b. Standard Model and Superfluid $^3$He-$A$

- Liquid of many $^3$He Cooper Pairs, all phases aligned
- Degrees of freedom: $S_z = 0, \pm 1; \ell_z = 0, \pm 1$
- $A$-phase: no $S_z = 0$ substates, $\hat{d} \parallel \hat{l}$

\[
H_{3He-A} = \chi^\dagger_{\alpha\beta} \{ (\epsilon - \mu) \sigma_3 + \tilde{V}_{\alpha\beta}(\hat{l}, \hat{d}, \tilde{k}) \} \chi_{\alpha\beta}
\]

\[
E(\tilde{k}) = 0, \text{ for 2 values of } \tilde{k}
\]

*Low-energy limit*

- Expand $E(\tilde{k})$ to 2nd order about zero points:

\[
E^2(\tilde{k}) \approx g_{ij}(k_i - qA_i)(k_j - qA_j)
\]

\[
g_{ij} \sim l_i l_j, A_i \sim l_i
\]

- Effective Lagrangian:

\[
\mathcal{L}'_{3He-A} = \bar{\Psi} g_{\mu\nu} \gamma^\nu (\partial_\mu - qA_\mu) \Psi
\]

*Potential field $A_\mu$ interacting with matter field $\Psi$ in curved spacetime*

\[
g_{\mu\nu} \sim (l_i, v_i), A_0 \sim l_i v_i
\]
2b. Standard Model and Superfluid $^3$He-$A$

- "Induced QED" (Zeldovich 1967): expand to 2nd order in fluctuations in $A_\mu$

$$\mathcal{L}'_{3He-A} = \bar{\Psi} g_{\mu\nu} \gamma^\nu (\partial_\mu - qA_\mu) \Psi + \frac{1}{4k^2} \sqrt{-g} g^{\mu\nu} F_{\mu\alpha} F_{\nu\beta}$$

- In-principle extension to $SU(n)$ gauge fields $\Rightarrow$ Standard Model

- Similar treatment of effective metric ("induced gravity" Sakharov 1967) fails to reproduce Einstein-Hilbert term

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**Condensed matter system**

- **QFT**
- **GR**

**low-energy approximation**
2c. Condensed Matter Approach to Quantum Gravity

**Research programme:**
- Determine appropriate condensed matter system that produces relevant matter, gauge and metric fields in low-energy limit. (Volovik 2003, Wen 2004.)

- **Background-dependent** approach to GR and Standard Model

**Literal Interpretation**

- *Gal*-invariant rest frame of condensate $\Leftrightarrow$ background structure

- low-energy fluctuations (quasiparticles, collective modes) $\Leftrightarrow$ matter/potential/metric fields $(\Psi, A_\mu, g_{\mu\nu})$

- "induced" vacuum corrections to interactions between $\Psi$ and $A_\mu$ $\Leftrightarrow$ gauge fields $(F_{\mu\nu})$
2c. Condensed Matter Approach to Quantum Gravity

**Relationalist Option (just condensate)**

1. background structure = high-energy properties of condensate
2. physical fields = low-energy fluctuations
3. relativistic ST structure = properties of low-energy fluctuations

**Substantivalist Options (condensate vs spacetime)**

A. "Conservative"

1'. background structure = properties of substantival Neo-Newt ST
2), 3)

B. "Intrepid"

1'), 2)
3') relativistic ST structure = properties of low-energy "emergent" substantival ST
3. Emergence

**Claim:** Novel phenomena (fields, particles, symmetries, spacetime, *etc.*) *emerge* in low-energy limit of certain condensed matter systems

*Emergence in the low-energy limit*

(1) *Distinct from emergence via symmetry breaking.*

non-relativistic liquid helium \(\xrightarrow{\text{lower temp}}\) non-relativistic superfluid helium \(\xrightarrow{\text{lower temp}}\) relativistic system

\(\mathcal{L} \xrightarrow{\text{spontaneous symmetry breaking}} \mathcal{L}'\)

(2) *Epistemological Emergence*

- Unpredictability
- Irreducibility
- Unexplainability
4. Universality, Dynamical Structure and Structural Realism

*Why does $^3$He-A reproduce the Standard Model?*

- $^3$He-A and Standard Model (sector above electroweak symmetry breaking) belong to same *universality class*.

- *Universality class* = characterized by common low-energy EFT

- Well-defined in Renormalization Group (RG) Theory:
  
  *universality class* = fixed point of RG flow

- Common "universal properties" = "generic" properties of EFT:
  - *decay behavior of correlation functions*
  - *gapless energy spectrum*
  - *symmetries of low-energy fluctuations* (i.e., *symmetries of EFT*)
4. Universality, Dynamical Structure and Structural Realism

- Universality classes of fermionic ground states are characterized by momentum space topology.
  - stable regions in k-space where quasiparticle energies \( \rightarrow 0 \)

- \( ^3\text{He}-A \) and the Standard Model have ground states characterized by the same momentum space topology.
  - stable point defects ("Fermi points")

\[
\begin{align*}
\text{Same k-space topology} & \quad \Rightarrow \quad \text{Same low-energy dynamics}
\end{align*}
\]

Irrespective of microscopic details:
- Standard Model
- \( ^3\text{He}-A \)
- any condensed matter system with "Fermi points"

\[\text{Same low-energy dynamical structure}\]
4. Universality, Dynamical Structure and Structural Realism

*(Epistemological) Structural Realism:*

1. The phenomena of experience are low-energy emergent.
2. Theories of such phenomena are EFTs of a "fundamental" theory \( T \).
3. As EFTs, such theories only provide us with knowledge of the low-energy dynamical structure of \( T \) (*i.e.*, the universality class of which \( T \) is a member).
4. Universality, Dynamical Structure and Structural Realism

Structural Realist interpretation of spacetime:

1. The spatiotemporal aspects of the phenomena of experience are low-energy emergent.
   - These are the spatiotemporal aspects of QFT and GR.

2. The spatiotemporal aspects of the fundamental condensate are structural.
   - These are the spatiotemporal properties of the universality class to which the fundamental condensate belongs.

Qualification: Universality class that best describes spacetime structure still unknown.